April 26, 1940

Dear Professor Fréchet,

Looking at the P.S. of your letter of April 24th in order to give you a quick reply, I see that the paradox is partly my own fault. I had sent the solution of the distribution of

$$\frac{S(\gamma x)}{\cancel{x}-y}$$

where x is distributed normally about zero, without noticing that your problem with $S(\gamma) = 0$ introduces a restriction which diminishes the degrees of freedom by one. For the common form of the analysis of variance we then have

	Sum of Squares	Mean Square
1	$S^2(\gamma x)/S(y^2)$	$s'^2 t^2$
$n-2$	$S(x-\bar{x})^2 - S^2(\gamma x)/S(y^2)$	s'^2
$n-1$	$S(x-\bar{x})^2$	s^2

so that the ratio you enquire about may be equated to

$$\frac{S(\gamma x)}{s\sqrt{S(y^2)}} \quad \cancel{\frac{S(\gamma x)}{s\sqrt{S(y^2)}}} = \cancel{t} \quad \frac{t\sqrt{n-1}}{\sqrt{n-2+t^2}}$$

the distribution of which is ~~equally~~ easily derived from that of t, being that of the sine instead of the tangent of an arbitrary angle. t has now, of course, n-2 degrees of freedom.

Yours sincerely,