

April 18, 1940

Dear Professor Fréchet,

I have your letter of April 13th and mine of March 18th before me. As I stated the case in my letter, I do not think it is correct to say that in my proof I "equalise without comment, as a matter of course, the probability of one same inequality in two different populations." The only population which I consider in that proof is that of events as defined in the first paragraph.

The question I proceed to discuss is whether, when confronted with a concrete sample, we may legitimately regard it as an event chosen at random from that population. In that population I ought to insist that it is indifferent whether λ is the same from sample to sample, or varies from sample to sample, that is, from event to event, for samples from populations having any value λ are, in fact, members of the population of events defined. I therefore demur at your comment that the value of λ , though arbitrary, is given and fixed. I go on to suggest that, when we have a concrete sample for which we accept the belief that it has been drawn from a normal population of which nothing is known except what the sample tells us of the parameters

λ and σ we may legitimately regard it as an event drawn at random from the population investigated. This step seems to me strictly analogous to that which is made in all applications of the theory of probability; for example, when a man whose personal and family history in no way differentiate him from the actuarial population accepted as healthy lives is accepted by a Life Assurance Company at its standard rates. This is certainly an act of ~~estimation~~ ^{judgment}, and not a deduction from any axiomatic basis, or at least from none that I should be prepared to put forward. Your amendment involving the words "from one of the populations, the theory of each of which has been investigated" does not seem to me necessary, if the full extent of the original population is once grasped.

If I am not mistaken as to your meaning, this reservation applies to the other difficulties which you feel, which turn, I think, always on the idea that the population of events ~~depend~~ defined refers to drawings from normal populations having the same mean. The distribution of \underline{t} is certainly independent of any possible variation in the parameters λ and σ .

Under section ^{III} 4 of your letter you raise a distinct point, which I have discussed in some of my addresses on the subject, namely, why I reject analagous arguments based on inefficient statistics. This point is fundamental, and I think quite easily explained. It is that, whereas in deductive reasoning we may make any selection we please from our axiomatic basis, and, reasoning from the ~~accepted~~ ^{selected} axioms only, it may be possible to

derive certain rigorously justifiable consequences, it is a characteristic of inductive reasoning that the whole of the information available must be utilised. Every statistician is aware that by an arbitrary selection from his data, and the subsequent use of this selected portion as though it were the whole of the information available, he could make a show of justifying any number of false conclusions. The use of inefficient statistics appears to be indistinguishable from a selection of ^{out} part/of the whole of the data available (See The logic of inductive inference. J.R.S.B. 98: 39-82). Of course there is also a sense, elaborated by Neyman and his colleagues, in which formal inferences may be drawn from inefficient statistics, but such formal inferences when properly understood, or when fully stated, are seen to be irrelevant to the objects of scientific research.

Yours sincerely,