March 18, 1940

Dear Professor Fréchet,

I think the naive proof that you want could run as follows: what we choose to call an "event" consists of <u>n</u> values <u>x</u> drawn at random from a normal population. The mean of the population we shall designate by μ, the mean of the sample by $\bar{x}$, and the estimated variance by

$$s^2 = \frac{1}{n-1} \quad S(x-\bar{x})^2.$$

These events may be divided into two classes, (a) the successful events for which $\bar{x} - \mu$ exceeds $\frac{st}{\sqrt{n}}$, and the unsuccessful events in which it is equal to or less than $\frac{st}{\sqrt{n}}$. Considering the population of all events that occur, without selection, the probability of success is the 'Student' integral from <u>t</u> to infinity, e.g., <u>t</u> can be chosen so that the probability of success is 5%, 2%, 1%, etc. So far this is pure mathematics, and, I believe, unexceptionable. Now, suppose we are confronted with a concrete sample for which we accept the belief that it has been drawn from a normal population of which nothing is known except what the sample tells us of the parameters μ and σ, we say "here is an event which <u>may be ligitimately regarded</u> as one chosen at random from the population of which the theory was investigated above'. The probability that it is a success is 2%. If this is so, the value of μ is less than a quantity which I can calculate from the sample. In this sense, therefore, the probability that μ is less than this quantity is 2%. The same argument applied to other percentile values gives a consistent series of values of μ, i.e., one in which μ ~~[illegible]~~ increases when the percentage is increased. The statement that the event with which we are confronted is one chosen at random from all such events is therefore made a basis from which we deduce a frequency distribution for μ, and which may be called its fiducial frequency distribution. In relation to its frequency distribution μ is, properly speaking, a random variable, whatever may be the physical origin of the sample considered.

The stipulation that nothing beyond the sample is known of the population sampled is relevant to our decision to regard the concrete sample as one chosen at random from the population of all such samples. It is, of course, only in the sense implied by that decision that the distribution of μ exists.

Yours sincerely,