August 4, 1942

Dear Jackson,

I have received your 5 Airgraph letters, which were a success for speed, for they arrived in about a month, but not so much so for legibility, chiefly perhaps because my eyes are worse than ever. Here is a note on the life table relationships which may be useful for reference.

If l_x is the proportion of flies surviving from emergence to age x, one can usefully introduce a second function

$$\int_x^\infty l_t \, dt = L_x$$

Then the expectation of life at emergence is

$$\frac{L_0}{l_0} = L_0, \text{ since } l_0 \text{ is unity;}$$

and the expectation of life at age x is

$$\frac{L_x}{l_x} .$$

Now, if you have a life table population flying about, i.e., flies of age x with frequency proportional to l_x, the death rate of a random sample of all ages will be

$$\frac{l_0}{L_0} = \frac{1}{L_0}$$

but the death rate of the same flies one week later will be

$$\frac{1}{L} \cdot$$

These relations will be disturbed, however, since an increasing population will have a high proportion of young flies, and a decreasing population a high proportion of old ones. But, for a stationary population, your values, p, were constant, i.e. independent of age (the early life-table actuaries made an exactly similar assumption) you would have

$$\frac{l_x}{L_x} = \frac{1}{L_o} e^{kx} \qquad p = e^{-k}$$

whence

$$\frac{-1}{L_x} \cdot \frac{dL_x}{dx} = \frac{1}{L_o} e^{kx}$$

$$\text{or} \quad -\log_e L_x = \frac{1}{kL_o} e^{kx} + \text{constant}$$

and

$$L_x = Ce^{-\frac{1}{kL_o} e^{kx}}$$

and l_x is the same thing multiplied by $\frac{1}{L_o} e^{kx}$

Putting $x = 0$ and $l_x =$, this comes to

$$\frac{C}{L_o} e^{-\frac{1}{kL_o}} = 1 \quad \text{or} \quad C = L_o e^{\frac{1}{kL_o}}$$

So

$$l_x = e^{kx} e^{-\frac{1}{kL_o}(e^{kx}-1)}$$

the death rate at any age x is thus

$$\mu_x = -\frac{d}{dx} \log l_x = -k + \frac{1}{L_o} e^{kx}$$

from which it appears that, as μ_x cannot be , the factor $\frac{1}{kL_o}$ must be not less than unity.

This gives an eminently reasonable life table, e.g., it fits the human data over a considerable range of ages, so that I do not think you ought to say that there is any theoretical reason to think that p_x rises. Of course your data may show it to rise, but that is another matter.

I am not quite clear what you mean by the span of life as contrasted with the mean life, perhaps it is the mean expectation of a random sample of all ages. I think this does not matter as in any case it can be expressed in terms of the two constants.

A point I am not clear about in letter 2 is, if you take 5 successive values of y, and add them to make Y, which particular values of y, do you add to make Ys ?

I should be immensely interested to have these ideas developed in a paper for the Annals. The whole problem of the interpretation of recapture data is so intricate and of such immense research importance that the very ~~existence~~ extensive experience you have gathered with tsetse flies ought, at all costs, to be put on record.

Yours sincerely,