26 April 1934.

Dear Jeffreys,

I am sending you back the type-scripts, so that you can see my last alterations, at least, I hope they may be the last. If you think they will do, will you send them both in to the Royal, as they are agreed documents. I think, on the whole, the suggestion of the Mathematical Committee has been justified and that our notes, within the limits of spaceassigned, will be of more general interest than if we had written independently.

Yours sincerely,

It is difficult to understand the difficulty expressed by Jeffreys as to the definition of probability when incommensurable, as the limit of the ratio of two numbers, when these both become infinite or increase without limit. All the sampling properties of hypothetical infinite populations can be expressed rigorously as limits of the sampling properties of finite populations, if as these are increased indefinitely the frequency ratios of their elements tend to the values assigned in the hypothetical infinite population. This is quite another matter from the difficulty experienced by those who attempted to define probability as the limit of the frequency ratio of experimental events, for we can have no direct knowledge of the existence of the nature of the limits approached when any experimental procedure is repeated indefinitely. In contrast the limits approached by repeating mathematical operations may be investigated with precision.

The logical situation which arises in the Theory of Estimation is of quite a different character. Here we are provided with a definite hypothesis, involving one or more unknown parameters, the values of which we wish to estimate from the data. We are either devoid of knowledge of the probabilities <u>a priori</u> of different values of these parameters or we are unwilling to introduce such vague knowledge as we possess into the basis of a rigorous mathematical argument.

Knowledge a priori may be , and often is, used in arriving at the specification of the ,forms of population we shall consider. The chief logical characteristic of this line of approach is that it separates the question of specification from the subsequent question of estimation, which can arise only when a sp specification is agreed on.

When a definite specification has been adopted, we can obtain a function of the parameters proportionate to the probability that, had these been the true values, the observations would have been those actually observed. This function is known as the likelihood of any value of a single parameter or of a set of values if the parameters are more than one. With respect to the parametric values the likelihood is not a probability and does not obey the laws of probability. Maximising the likelihood provides a method of estimation, which has been shown to possess the following relevant properties:

1. In certain cases an estimate is possible which, even from finite samples, contains the whole of the information contained in the sample. Such estimates are known as "sufficient". The method of maximum likelihood provides such "sufficient" estimates when they exist.

Dr. Jeffreys attempts the more difficult task of justifying our procedure in arriving at particular specifications by means of the Theory of Probability. It is not, however, obvious that probability provides our only or chief guide in this matter. Simpler specifications are preferred to more complicated ones, not, I think, necessarily because they are more probable or more likely, but because they are simpler. As more abundant data are accumulated certain simplifications are found to be very unlikely or to be significantly contradicted by the facts and are, in consequence rejected, but among the theoretical possibilities which are not in conflict with any existing body of fact. The calculation of probabilities, if it were possible, would not, in the writer's opinion, afford any satisfactory ground for choice .