

St John College

Cambridge

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Dear Fisher,

I find your genetic problem rather hard. I think it might be tackled like this. Suppose you have a population of one type & breed from them; & that the probability of any new one arising & mutation is  $\epsilon$ . Then I should say that the prior probability of  $\epsilon$  is uniformly distributed if you know nothing about  $\epsilon$  to begin with: i.e., if  $f(\epsilon) d\epsilon$  is the p.p. that  $\epsilon$  is in range  $d\epsilon$ ,  $f(\epsilon) = 1$ . More accurately perhaps, if you are testing the occurrence of mutation in a particular factor, the prior that  $\epsilon = 0$  is  $\frac{1}{3}$ ,  $\epsilon = 1$  is  $\frac{1}{3}$ , &  $f(\epsilon) = \frac{1}{3}$  otherwise. For another factor the same values; & as far as we know to begin with the two are independent so that  $f(\epsilon_1, \epsilon_2) d\epsilon_1 d\epsilon_2 = f(\epsilon_1) d\epsilon_1 f(\epsilon_2) d\epsilon_2$ . I can't see what would make your biologist think that  $\epsilon$  is the same for all factors, but if he is sure of it then do some  $f(\epsilon)$  tests for all. I think that if he knows this he must know a lot of other things too & this soon may be quite wrong. We might have to allow for our knowledge that in fact most things breed true, which would concentrate  $f(\epsilon)$  towards the lower values; but I think the above is right if he really knows nothing.

Newman once asked me the following: a man arrives at a railway junction in a town & in a foreign country, which he has never heard of before. The first thing he sees is a trolley numbered 100. Can he infer anything about the number of trolleys in the town? Newman thought the question was significant & so did I, & we both had a feeling that there were probably about 200. I tried it on M.S. Bartlett, who thought it was meaningless but had the same feeling about 200. This may have some analogy with the question of the existence of a further factor

liable to mutation. My very doubtful solution is: Let  $f(n)$  be the prior probability that there are  $n$  cars; the probability that number 100 would be observed, given  $n$ , is  $1/n$  for  $n \geq 100$ , 0 for  $n < 100$ . Hence the posterior probability of  $n$  is  $f(n)/n$ . If  $f(n)$  is constant  $\int f(n)/n$  diverges & we can infer nothing; but if  $f(n) \propto 1/n$ ;  $f(n)/n \propto 1/n^2$  & the probability that  $n$  exceeds  $m$  ( $> 100$ ) is  $\sum_{n=m}^{\infty} \frac{1}{n^2} / \sum_{n=100}^{\infty} \frac{1}{n^2} \approx \frac{1}{m} \div \frac{1}{100}$  & it is about as likely as not that  $n > 200$ .

I have just read Bayes's paper again for the first time since 1919. I see he defines probability in terms of expectation of value, as Ramsey does; R. has really given only conditions for consistency. Bayes gets  $P(p, q | h) = P(p | h) P(q | p, h)$  quite correctly; I can understand his reader better than Ramsey but that is probably my fault. What bothered me about R. was that I did not see that people could agree in their ordering of values any better than in their ordering of probabilities; it really involves an ideal man who would always choose out on the alternative that gives him the greatest expectation of value. But it's really no worse than mathematics, which really assumes an ideal man that always gets his arithmetic right.

Towards the end of your letter you speak again of unknown a priori probabilities as opposed to those arising with dice. My attitude is that you are asking the wrong question. My question is, what distribution of probability corresponds to absence of previous knowledge? I think you are regarding a probability as a statement about the composition of the world as a whole, which it is not & on a scientific procedure could not be until there was nothing more to do.

I have just been away cycling for ten days.

Yours sincerely  
Harold Jeffreys