St John's College
Cambridge

1940 April 15.

Dear Fisher,

Sorry I did not thank you for your letter before. The work got shelved owing partly to extra lecturing, but I am getting down to it again. I think that my problem can be tackled by a method something like one that was in my Biometrika paper originally, but withdrawn because Neyman had got as near it as didn't matter for the immediate purpose. The median standard error for the 10° squares (free-air anomaly reduced to mean height of the square) is 14 mgal, presumably genuine. But the scatter of the reduced values corresponds to an extra variation with mean square 22 mgal. The latter will include disturbances by harmonics of all orders up to 36 or so. The lack of knowledge is about how to divide it between any 3rd and 36th harmonics. Now this is very like the problem of sampling a population of several types, where we have direct information about some of the types, but others are not sampled and can be estimated only from the mean and scatter of the observed ones; we want the mean of the entire population. Let n_1 be the number of sampled types and σ_1 the sampling s.e. for each, n_2 the number of types not sampled and σ_2 the estimated extra scatter between types. Then neglecting σ_2 the s.e. for the mean of the whole would be $\sigma_1/\sqrt{n_1}$. But the unobserved types give an extra $\sigma_2\sqrt{n_2}$ in the sum and therefore $\frac{\sigma_2\sqrt{n_2}}{n_1+n_2}$ in the mean. The proper s.e. for the mean is, therefore given by

$$\sigma_m^2 = \frac{\sigma_1^2}{n_1} + \frac{n_2\sigma_2^2}{(n_1+n_2)^2}$$

This suggests that we should be able to take the standard error of an observed type as $\sigma_1^2 + \frac{n_1 n_2 \sigma_2^2}{(n_1+n_2)^2}$, and then treat the problem as a straight least squares one applied to the known 5° squares. It doesn't seem that the gravity problem is worse in principle, except that the present estimate of σ_2 includes all the variation and will have to be reduced as successive terms are removed from it.

What I mean by a 10° square is a region bounded by parallels and meridians 10° apart. There will be some variation of σ_2 with latitude in consequence, but it won't be more than in a ratio of 1 to $\sqrt{2}$, I think, and can be dealt with fairly easily.

By the way, now that Yates's paper is out are you ready to join the Camb.Phil.Soc. again ?

Yours sincerely,

Harold Jeffreys