

July 11, 1941

Dear Peters,

I hope the enclosed will be clear. My previous estimate had been based on a bad value for sheep, but the general indication is obvious, that the large Sheep-Day interaction requires more frequent sampling of each animal.

Yours sincerely,

Enclosure

Enclosure:

July 11.
Enclosure to Dr. *R. F. [unclear]*

The estimated variance after adjustment for pre-treatment counts is 23,035. This is on a whole sheep basis; to put it on the basis of one count unit divide by 27, obtaining 853.15.

The mean square is then comparable with those obtained from Table VII (expanded below)

	Degrees of freedom	Sum of squares	Mean square
Sheep	17	32841.64	1931.86
Days	5	16920.92	3384.18
SD	{ 17	12078.16	710.48 }
	{ 68	34348.38	505.12 }
Counters	2	116.49	58.245
CS	34	495.25	14.566
CD	10	314.04	31.404
SCD	170	2958.44	17.403
Parallel	648	6495.67	10.024

If no previous count had been made, a major source of error in the comparison would consist in the permanent differences in infestation of different sheep. Using the previous counts, however, we may expect it to be more like SD using the component for differences between the ^{periods} counts before and after treatment. As the latter figure involves different treatments, having perhaps a small effect, and does not take account of the actual process of adjustment, we may adopt the former figure 853.15.

Note too that the mean square for CS is actually less than that for SCD, as it ~~may~~ could not reasonably be, apart from chance fluctuations, so that we may pool these 204 degrees of freedom, giving them the mean square 16.930. Note also that the interaction CD does not affect the comparisons made.

The ingredients contributing to the actual error of the comparisons are then:

	Mean square	Divisor, no. of unit counts	Equivalent mean square of unit counts
S	853.15	27	31.524
SD	505.12	9	56.125
CSD	16.930	3	5.643
Parallels (pCSD)	10.024	1	10.024 .

I understand there is nothing to be gained by counting less than three units, so I will set up the calculations first on the supposition that we can have at will s sheep, d days, and c counters.

Of the total 31.524 ascribed to sheep, a portion, namely $\frac{1}{3}(56.125) = 18.708$ is due to the interaction SD. This leaves $\frac{12.816}{s}$ as the contribution to the error variance of mean count due to the limitation in the number of sheep in each group. Similarly of the portion ascribed to SD a small amount, namely $\frac{1}{3}(5.643) = 1.881$, is due to CDS. This leaves $\frac{54.246}{sd}$ as the contribution due to the limited number of samples taken. Finally, using the fixed number of parallels we have $\frac{5.643}{cds}$ as the final portion. The costs corresponding to these three items are supposed to be in the ratio $s = 180$, $sd = 10$, $cds = 3$. If we divide each component of variance by the corresponding cost and take the square root, we shall have ideal numbers for minimising the cost at a given precision.

In this case notice at once that the cds component is smaller than the sd component ~~for~~ ^{at} a higher ratio than 3:10, so that calculation will give cds smaller than sd , which must be interpreted as meaning that a single counter is sufficient. If this is

adopted we must replace the two latter components by the single one

$$\frac{54.244+16.93}{sd} = 71.174 \quad \text{with a corresponding cost of 13d.}$$

Now $\frac{12.816}{180}$ is .0712, of which the square root is .2668,
and $\frac{71.174}{13}$ is 5.4749, of which the square root is 2.3399.
Hence sd is 8.77 times as great as g , showing that the number of
samples to be taken from each sheep after treatment should be more
like 8 than 3, with minimum laboratory work applied to each sample.
Once the number of sampling occasions is fixed, the precision will
increase simply with the number of sheep, being for 8 occasions
 $\frac{21.713}{6}$ for as many sheep as are used. Also, of course, the cost
increases proportionately with the number of sheep. If the
laboratory work on each sheep is fixed, it comes, on these figures,
to £25.13.10 per unit of information! Using 9 occasions it is
£25.12.11.

July 10, 1941