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Since the ages of men and women are affected similarly by the selection of eye colours, showing respectively high and low sex ratios, we may make a more comprehensive examination of the situation by a bi-variate analysis involving sex ratio and mean age, within groups having the same eye colour.

For any chosen class of persons three quantities will then be calculated:

- 1) The square of the number of men divided by the total number, or in other words the number of men multiplied by the sex ratio.
- 2) the sex ratio multiplied by the total age of both sexes in the class, and
- 3) the total age multiplied by the average age.

In whatever way the aggregate of persons observed is divided into classes, the sum of these contributions added from all classes may be obtained. Thus, if all 500 persons observed are treated as a single class, of which 220 are male and the total age is 20006. then the three entries will be

$$\frac{1}{500} (220)^2 = 96.800$$

$$\frac{1}{500} (220)(20006) = 8802.640$$

$$\frac{1}{500} (20006)^2 = 800480.07.$$

Equally, if each individual were given a separate class, the first entry would be the total number of men, 220, the second entry the total age of all men, or 8044 years, and the third entry the sum of squares of all ages of men or women recorded, 1016942 (years).²

The results of applying this analysis of two classifications based on our are shown in the table. The first of these, shown in the second line

uses 14 classes only, namely the 13 classes to which 10 or more persons have been assigned, and the residue including all persons assigned to eye colour classes with less than 10 occupants. In the second classification all 61 eye colour classes in which any persons have been recorded have been treated separately;

Table. Sums of Squares and Products for four Classifications.

No. of classes	(x^2)	(xy)	(y^2)
1	96,800000	8802.640	800480.07
14	102.837449	8634.679	821686.88
61	118.201851	8392.729	845120.05
500	220.	8044.0	1016942.0

By subtracting ⁱⁿ this table each row from that following it we obtain an analysis of covariance for the two variates, sex ratio and mean age, the subdivision giving the contribution of:

- the 13 degrees of freedom between the 14 classes of the first classification;
- the 47 degrees of freedom between the 48 rare classes, which in the first classification were thrown together;
- the 439 degrees of freedom within eye colour classes.

We thus have:

Table. Analysis of Covariance of Sex Ratio and Age.

	D.F.	(x^2)	(xy)	(y^2)
Among common eye colours	13	6.037449	- 167.961	21206.81
Among rare eye colours	47	15.364402	- 241.950	23433.17
Within colour classes	439	101.798149	- 348.729	171821.95
	499	123.2	- 758.640	216461.93

The first column is a simple analysis of variance for sex ratio, giving a test of significance essentially equivalent to that for χ^2 . Significant variations in sex ratio occur both among the common and among the rare classes of eye colour.

The negative sign in the second column shows that in all comparisons a higher proportion of men is associated with lower age. In the totals the regression of age on sex ratio, - 6.158 years, is simply the difference between the mean ages of the ^{men}women and the ^{women}men examined. Within eye colour classes the regression, - 3.426 years, shows that the mean difference in age for a man and a woman in the same class is materially lower. Between classes we have the regressions - 27.820 years for the commoner eye colours and - 15.747 years for the rarer colours. The significance of the differences between these three regressions may be exhibited by analysing only the variance of age when allowance is made for the variation in the sex ratio within each class. We then have :

Table . Variance of age for given sex ratio.

	D.F.	S.S.	M.S.
Among common colours	12	16534.16	1377.8
Among rare colours	46	19623.08	426.6
Within classes	438	170627.31	389.6
Between regressions	2	5005.83	2502.9
Total	498	211790.38	

The higher mean square for deviation from regression among the commoner colours is to be expected, since the numbers in these classes vary greatly, and doubtless higher regressions would be appropriate for the commoner than for the rarer classes. The analysis, however, confirms clearly the significance of the lower ages of persons in eye colour classes having higher sex ratios.

One natural interpretation of this fact is that some changes in eye colour normally take place throughout life, and that corresponding changes take place, on the whole, earlier in women than in men, so that, in general, the subjects of these changes are transferred from more masculine to more feminine eye colour classes. To obtain an idea of the

time interval associated with this sex difference we may ask what discriminant function of the form $\text{Age} - \lambda(\text{Sex ratio})$ will be most closely associated with the classification of the iris. λ will then represent the number of years that a man must be older than a woman for their expectations in respect of eye colour to be as nearly as possible equivalent. If we choose λ so as to maximise the fraction of the sum of squares assigned to the 60 degrees of freedom between classes, the maximum value of this fraction is found to be .23735 with corresponding age difference = 29.316 years. On this interpretation, therefore, of the association between sex ratio and age, it would appear that changes commonly supervening in women early in life, perhaps before they are 30, commonly occur in men nearly 29 years later.