11th. April, 1930.

Professor V. Romanovsky,
Tashkend University,
TASHKEND,
Russian Turkestan.

Dear Professor Romanovsky,

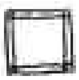
I am afraid I cannot manage the trip to Kharkoff in June next, as I seem to have in other ways a very busy year in front of me. Many thanks for the suggestion. I hope you will have a successful meeting.

I am glad to hear of the new class of integral equations; it is a subject that I admire from a distance. The combinatorial procedure for evaluating the higher moments of algebraic statistics may, however, be intimately of interest in this regard. It was a long while before I could see the reason for all the simplifications which the method introduces. Indeed it is still a mystery to me why the algebraic coefficients corresponding to the "patterns" should be so simple.

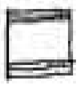
I worked out the other day the coefficients corresponding to the three symbolical figures

which are all that are wanted (in the case of a normal population) for anything like the 4th. semi-invariant of the distribution of k_4, such as (with two variates) any 4th. order semi-invariant of the simultaneous distribution of k_{40}, k_{31}, k_{22}, k_{13}, k_{04}. Well, the patterns have eight rows each, and the number of separations of eight parts is very large, so that it was very heavy work before I had the coefficients; but when all is done they are simply

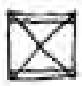
There seem to be 34 such bivariate formulae



$$\frac{n(n+1)(n^4-8n^3+21n^2-14n+4)}{(n-1)^3(n-2)^3(n-3)^3}.$$



$$\frac{n^2(n+1)^2(n-2)(n-3)}{(n-1)^3\ (n-2)^3\ (n-3)^3}$$



$$\frac{n(n+1)(n^4-9n^3+23n^2-11n+4)}{(n-1)^3\ (n-2)^3\ (n-3)^3}$$

So that letting N_1, N_2, N_3 stand for these three expressions the 4th. semi-invariant of k_4 is simply

$$4 \cdot 12^3\left(9N_1+8N_2+36N_3\right)\kappa_2^8$$

and, for example, for the 4th. semi-invariant of k_{22} in the bivariate problem, we have only to subdivide the numerical factors by supposing that the four rods which meet at each point are two black and two red, and enumerating the number of ways of linking them up with 0, 2, 4, 6, or 8 black-red junctions (as opposed to black-black or red-red junctions which must be equal in number and supply the factors κ_{20}, κ_{02}). Thus in every problem the algebraic coefficients are the same, and they are so simple that one feels that one ought to be able to write them down by inspection of the pattern, or of its symbolical diagram.

I am glad you liked the Sieve. I feel that Eratosthenes has been too long exposed to the patronising remarks of his critics!

Yours sincerely,