

25 September 1930.

Dr K. Tjebbes,
Box 82,
LANDSKRONA,
Sweden.

Dear Dr Tjebbes,

I have had a try at your problem of assessing the amount of inbreeding according to the method of planting out sugar beet.

The main difficulty in respect of data is that it is not known experimentally what proportion of the ovules of any plant are fertilised by its immediate neighbours, and what proportion from more distant plants. If the law connecting frequency of cross pollination with distance were known it would be certainly possible to calculate numerically the function you require, though the mathematical form of the function would probably be ^{un-}manageable except in rather special cases.

I have, therefore, chosen a form of function which is mathematically manageable, and which for special values of the variable parameter should give a good approximation to the facts. Assuming the frequency of cross pollination between two plants at a distance r to be x^{-r^2} , where x is

the parameter, the functions can be evaluated in terms of special functions which arise in the study of the Elliptic θ functions, and which can be determined numerically.

The practical assumption underlying the choice of any particular value of x may perhaps be best expressed by giving the percentage of the ovules of any plant which are fertilised by pollen from the eight adjoining plants, as opposed to the rest of the field. If x is small this percentage will be large, if x is larger the percentage will be smaller. I have chosen three values of x which make this percentage to be 93.96, 73.77 and 47.91 per cent.. I do not know if I ought to consider lower values.

In any case the procedure of mixing all four lots of seed must give 25 per cent. inbred. The smaller x is, the smaller will be the proportion inbred under perfectly regular planting, which therefore ranges from 0 to 25 per cent. For sowing mixed seed in alternate rows the percentage will again not exceed 25 per cent. but this value is attained at both extremes, either when $x = 1$, and distance makes no difference, or when $x \rightarrow 0$, and cross fertilisation is confined to the four nearest plants, for two of these, in the same row will each have a half chance of being of the same line as the central plant. Consequently, the proportion inbred must have a minimum for some special value of x .

The numerical values found appear in the table

Per cent of pollen from 8 adjoining plants	<i>Per cent. inbred.</i>		
	Regular planting	Alternate rows	4 lines mixed
93.96	3.48	19.55	25.00
73.77	11.67	20.70	25.00
47.91	18.52	22.84	25.00

It will be seen that the method of alternate rows will not give much less than 20 per cent. inbred. It is, however, questionable if the method of regular planting does much better than this for it is as high as 18.52 per cent. even when nearly 50 per cent. of the pollen comes from neighbouring plants.

If, as I should be inclined to guess, as much as 50 per cent. or more of the pollen comes from outside the 3 x 3 square; the only effective way of diminishing inbreeding would be to increase the number of strains to 8 or 9, which would certainly cut it down, even if all the seed were mixed.

Yours sincerely,