3 August 1933.

Dr. J. Wishart, F.R.S.E.,
School of Agriculture,
CAMBRIDGE.

Dear Wishart:

If q is the sum of squares for n_1 degrees of freedom expressed as a fraction of the total $n_1 + n_2$ degrees of freedom then when n_1 is even one can use the formula

$$P = (1-q)^{\frac{1}{2}n_2}\left\{1 + \frac{n_2}{2}q + \frac{n_2(n_2+2)}{2.4}q^2 + \ldots\ldots\ldots\ldots\ q^{\frac{1}{2}(n_1-2)}\right\}$$

and of course this is workable if either n_1 or n_2 is even. This is tedious for n_1 or n_2 = 60, with 30 terms in the expansion, but for these values one can start with a good approximation from the old table or other sources such as the Incomplete Gamma function.

If $\frac{q}{1-q} = r$ the alternative expression

$$(1+r)^{-\frac{1}{2}(n_1+n_2-2)}\left(1 + \frac{n_1+n_2-2}{2}r + \ldots\ldots\ r^{\frac{1}{2}(n_1-2)}\right)$$

is sometimes equally convenient, the coefficients inside the bracket being in this case those of a positive binomial with integer ~~coefficient~~ index, if n_1 and n_2 are both even. In fact the z distribution has as its integral the exact partial sum of a positive or negative binomial, and if De Moivre had been a better analyst he would have arrived at the z distribution instead of the normal curve, or at least at its mathematical expression.

When both n_1 and n_2 are odd the exact formula has a trigonometric form in terms of an angle α where $\tan^2\alpha$ is the ~~variance~~ s.s. ratio $\frac{n_1}{n_2}e^{2z}$ for

$$P = 1 - \frac{2\alpha}{\pi} - \frac{2\sin\alpha}{\pi}\left\{\cos\alpha + \frac{2}{3}\cos^3\alpha + \ldots\ldots + \frac{2.4\ldots.(n_2-3)}{3.5\ldots.(n_2-2)}\cos^{n_2-2}\alpha\right\}$$

$$+ \frac{2}{\pi}\cdot\frac{2.4\ldots.(n_2-1)}{1.3\ldots.(n_2-2)}\sin\alpha\cos^{n_2}\alpha\left\{1 + \frac{n_2+1}{3}\sin^2\alpha + \ldots. + \frac{(n_2+1)\ldots.(n_1+n_2-4)}{3.5\ldots.(n_1-2)}\sin^{n_1-3}\alpha\right\}$$

which looks formidable in general, but at the highest value for which it is needed, $n_1 = 15$, $n_2 = 15$, the brackets only have seven terms each and should not be unmanageable.

I have given the fundamental and exact formulae because it is those I am now using. I know in making the original tables, I used all sorts of dodges in different places, utilising existing tables where I felt sure they would give what was wanted, to the accuracy I then had in view, but I believe, that except for getting good trial values, which we now have available easily all over the table, it is not worth while using anything but the exact formulae in pegging out the framework.

Yours sincerely,

R. A. Fisher