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On the Segmentation of Markets

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This paper endogenizes the market structure of an economy with heterogeneous agents who want to form bilateral matches in the presence of search frictions and when utility is nontransferable. There exist infinitely many marketplaces, and each agent chooses which marketplace to be in: agents get to choose not only whom to match with but also whom they meet with. Perfect segmentation is obtained in equilibrium, where agents match with the first person they meet. All equilibria have the same matching pattern. Although perfect assortative matching is not obtained in equilibrium, the degree of assortativeness is greater than in standard models.

I. Introduction

The issue of how heterogeneous agents form matches was first examined in the works of Gale and Shapley (1962) and Becker (1973) in a fric-

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tionless environment. Recent work has extended this analysis by incorporating search frictions so as to capture the idea that it takes time to meet someone.[1]

Implicit in these models is the assumption that agents are all searching within a marketplace. If one considers a model with homogeneous agents and nontransferable utility, this assumption is natural: in the marriage market context, since no male (female) is better than any other male (female) a priori, the expected payoff to a female (male) from matching with any male (female) is the same. However, once ex ante heterogeneity is introduced, assuming that search is conducted in one marketplace is no longer innocuous.

Suppose that there is an objective ranking of both males and females. Since agents use a reservation strategy, that is, each male (female) will have a lowest female (male) type he (she) will agree to match with, an agent typically spends time meeting people he will never match with, because either the person he meets is below his reservation type or his own type is below the reservation type of the person encountered. As agents do not necessarily match with the first person they meet, they impose a congestion externality on each other. Standard matching models, by assuming that all heterogeneous agents search randomly in one marketplace, do not address this congestion externality.

Take a standard search model, which has been examined in McNamara and Collins (1990), Burdett and Coles (1997), and Bloch and Ryder (2000). They show that when agents' types are distributed over the interval $[\underline{x}, \bar{x}]$ and the utility a type x agent receives from matching with a type y agent is y, in equilibrium, agents partition themselves into distinct classes where agents in each class match only with other agents in their class.[2] That is, a *class structure* is formed. It is clear that agents from each class are imposing a congestion externality on all other classes. Intuitively, an easy way to get around this externality will be to take all agents in a given class and put them in their own marketplace, so that they meet only other agents from their class. This way, each agent faces the same expected quality of a match, but will now be matching at a faster rate.

However, this reasoning assumes that agents' reservation strategies are left unchanged, which is not true in equilibrium. Take agents from the first class who are now in their own marketplace. The highest-type

[1] A nonexhaustive list includes McNamara and Collins (1990), Lu and McAfee (1996), Burdett and Coles (1997), Burdett and Wright (1998), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), and Smith (2006).

[2] Eeckhout (1999) and Smith (2006) show that more generally a class structure is also obtained when the utility an agent gets from matching is such that $u(x_1, y_1)u(x_2, y_2) = u(x_1, y_2)u(x_2, y_1)$, which is true if and only if u is multiplicatively separable, i.e., $u(x, y) = f(x)g(y)$ (see Topkis 1998, sec. 2.6).

agents in this new marketplace now have a different reservation strategy because the composition of agents in this new marketplace is different. In fact, they will be more picky about whom they agree to match with by increasing their reservation type. In other words, in this new marketplace consisting only of class 1 agents, we get the result that a class structure is obtained once again. Now take the new class 1 agents in this marketplace. They are again in a marketplace with agents they would not agree to match with if they were to meet. Why not form their own marketplace to increase their matching rate? If we continue reasoning this way, a natural question is whether this process goes on ad infinitum so that each type of agent is in its own marketplace.

This paper builds on the existing literature by assuming the aforementioned utility specification and extends it by allowing for the existence of infinitely many marketplaces. We characterize steady-state equilibria when the distribution of types of agents in each marketplace is known to everyone, each agent is free to decide which marketplace to search in, and once he has decided to go to a particular marketplace, search is conducted randomly there. Moreover, an equilibrium has to be such that it is not possible to create a new marketplace that would attract people.

In our paper, when we allow for many marketplaces and for each agent to choose which marketplace to position himself in, segmentation arises as an equilibrium phenomenon. Second, because the distribution of agents in each marketplace is public knowledge, agents can make an informed decision as to which marketplace to search in. Hence, search is no longer forced to be completely random and each agent can direct his search toward the best marketplace for himself. Agents not only get to choose whom to match with, but also choose whom they meet with.

We show that all equilibria feature *perfect segmentation* in that agents match with the first person they meet, and all equilibria have the same class structure, which is finite. This implies that perfect assortative matching cannot be obtained in equilibrium. Since agents match with the first person they meet in equilibrium, the congestion externality in standard search models is not present here. And since there is only one equilibrium class structure, the value of search for each type of agent in any equilibrium is the same.

In light of our result, standard matching models can be interpreted in two ways: the entire economy is studied, as we do, but everyone is searching in a unique marketplace; or one can think of these models as studying one marketplace in isolation with the types of agents taken as given. In the former, our results imply that standard results with partial segmentation like those of Burdett and Coles (1997) are no longer equilibria when segmentation is allowed. In the latter case, our model, by endogenizing the market structure, pins down the set of types in each active marketplace.

Examples of segmentation of markets are plentiful. In the marriage market, the rich and beautiful tend to search for a partner among the rich and beautiful, and do so by going to fancy bars and clubs, whereas the Average Joe goes to the neighborhood pub to meet his Average Jane rather than fancy clubs because he knows he would not be able to find a match there.[3] Although our paper is closest to the marriage market because of the nontransferable utility assumption, segmentation of markets is observed in other markets, and our analysis is thus relevant. In the labor market for fresh graduates, for instance, the most desirable employers do not search randomly but among a subset of students from the best schools, and the best students from the best schools tend to search among these firms, thereby forming their own search market; the next-best firms and students, knowing they cannot match with the best students and firms respectively, form their own marketplace, and so on. By obtaining segmentation as an equilibrium outcome, our paper is able to shed some light on the mechanism behind the segmentation observed in these markets.

Our paper is organized as follows. Section II presents the standard model in which everyone is in one marketplace. Section III sets up the model in which there are multiple marketplaces and agents are allowed to choose where to search. In Section IV we construct an equilibrium with perfect segmentation. Section V characterizes all the other equilibria. Section VI relates our paper to the literature and discusses the impact of having increasing returns to the meeting technology. Section VII presents conclusions.

II. The Standard One-Marketplace Model

A. *Setup*

Time is continuous. The economy is populated by a unit mass of infinitely lived agents who discount the future at rate $r > 0$ and who wish to form bilateral matches. Agents are characterized by their type x, which belongs to $X = [\underline{x}, \bar{x}]$, with $\underline{x} > 0$. All agents agree on how to rank one another, and agents' types are revealed upon meeting each other. When a type x agent matches with a type y agent, the former receives utility

[3] Some might say that fancy bars charge higher prices than local pubs for drinks; segmentation is thus achieved at least partially through price discrimination. We would argue that a man looking for a match can have two beers at a fancy bar rather than four beers at his local pub if he is really searching for a match.

y; the utility an agent receives therefore depends entirely on his partner.[4] This implies that all agents prefer to match with the highest possible type. When unmatched, an agent enjoys a flow utility normalized to zero so that being matched yields a higher flow payoff than being unmatched.

All agents start their life unmatched, and because of search frictions it takes time for agents to meet. More specifically, we assume that agents meet according to a Poisson process with parameter λ; this parameter is constant, that is, independent of the measure of unmatched agents searching for a partner.[5] Although being unmatched is undesirable, it does not necessarily mean that an agent will match with the first person he meets. The reason is that he may prefer to wait for a better match, so "meeting" is distinct from "matching."

When an agent matches, he leaves the pool of searching agents forever, and we assume that he is replaced by a clone. This clones assumption ensures that G, the cumulative distribution function (cdf) of singles' types, is stationary and independent of the matching pattern.[6] Although this is not essential for our results, we assume that G has full support and is differentiable on X, and g denotes its density function.

Like Becker (1973), Shimer and Smith (2000), and Smith (2006), among others, this paper is a partnership model. Although numerous matching models consider two sides of the market, a partnership model is equivalent to a model with two sides being symmetric and restricted to using symmetric strategies. Obviously the partnership model employed in this paper cannot explain nonsymmetric behavior, but we leave it for future research.

B. *The Steady-State Equilibrium*

We focus on steady-state equilibria in which agents use stationary strategies. For two agents with types x and y to match when they meet, x needs to accept y and y needs to accept x. That is, the match has to be mutually agreeable. The problem a type x agent faces is as follows: Given the set of types of agents $\Omega(x)$ who accept a match with him, what types

[4] We assume this utility specification to simplify the exposition; a more general utility function can be adopted in which a type x agent receives $u(x, y)$ when matching with a type y agent, with $u(x, y)$ multiplicatively separable. Jacquet and Tan (2004) adopt this more general specification and show that the results are unaffected.

[5] Section VI.*B* discusses the implications of increasing returns in the meeting technology.

[6] This assumption can be relaxed by introducing an exogenous inflow of new agents to endogenize the distribution of unmatched agents as in Burdett and Coles (1997) and Smith (2006). In the former, it is shown that this can result in multiple steady-state distributions of singles.

of agents should he accept? By standard arguments, $u(x)$, the value of being unmatched for agent x, satisfies

$$ru(x) = \lambda \int_{\Omega(x)} \max\{y - u(x), 0\} dG(y). \tag{1}$$

An agent x will accept a match with y if and only if $y \geq u(x)$;[7] that is, he uses a reservation strategy. If we consider two agents with types x_1 and x_2, $x_1 > x_2$, agents accepting a type x_2 agent will also accept a type x_1 agent. Therefore, an agent with type x_1 cannot fare worse than an agent with type x_2, meaning that $u(x_1) \geq u(x_2)$; therefore, reservation strategies are nondecreasing in type. Proposition 1 states the existence of a unique equilibrium, which we call the Burdett-Coles equilibrium, and characterizes its structure.

Proposition 1. For any G, when there is one marketplace, an equilibrium exists and is unique. It is characterized by a finite, and strictly decreasing, sequence of reservation strategies $\{u_j^{bc}\}_{j=1}^{J^{bc}}$, $u_{J^{bc}}^{bc} \leq \underline{x}$, where $u_0^{bc} \equiv \bar{x}$, and for all $j \geq 1$, u_j^{bc} solves

$$u_j^{bc} = \frac{\lambda}{r} \int_{u_j^{bc}}^{u_{j-1}^{bc}} [G(u_{j-1}^{bc}) - G(y)] dy. \tag{2}$$

All agents with attributes in $X_1 = [u_1^{bc}, \bar{x}]$ share the same reservation type u_1^{bc}, and for all $j > 1$, all agents with attributes in $X_j = [u_j^{bc}, u_{j-1}^{bc})$ share the same reservation type u_j^{bc}.

We will not formally prove the proposition since it will only reproduce proofs available in the literature, but we will lay out the intuition. Consider first the $\bar{x}$ agents. They are accepted by everyone since they are the highest type. As reservation strategies are nondecreasing and $\bar{x}$ agents have $u(\bar{x})$ as their reservation type, all agents whose types are in the interval $[u(\bar{x}), \bar{x}]$ are also accepted by everyone. But since one's type affects one's payoff only through whom one can match with, agents who have the same opportunity set will have identical reservation strategies, and thus $u(x) = u(\bar{x}) \equiv u_1^{bc}$ for all $x \in [u_1^{bc}, \bar{x}]$. These agents therefore match only with each other, and we say that they are class 1 agents. Now consider agents of type $u_1^{bc} - \epsilon$ for $\epsilon > 0$ arbitrarily small. They will like to match with agents in class 1 but will not be accepted since their types are below class 1 agents' reservation type. However, all other agents not in class 1 will accept them since they are the best agents not in class 1. By the same argument as above, one can find a reservation type u_2^{bc} such that all agents with attributes in $[u_2^{bc}, u_1^{bc})$ choose the same reservation type u_2^{bc} and therefore match only with each other, thereby forming the

[7] We use the convention that if an agent is indifferent between staying unmatched and matching with an agent, he chooses to match.

second class. It is easy to see from here that this reasoning applies to agents with types in $[u_3^{bc}, u_2^{bc})$ and so on, until one reaches a class with reservation type equal to or less than $\underline{x}$.

We therefore obtain a class structure $\{X_j\}_{j=1}^{J^{bc}}$ in this Burdett-Coles economy in which agents whose types fall in the interval X_j belong to the same class, and they are class j agents: two agents from that class will match if they meet, and none of these agents will match with any agent not in their class, either because they are below the reservation type of the agent met or that agent's type is below their own reservation type. Note that agents in the last class, J^{bc}, have a reservation type weakly less than $\underline{x}$.

This class structure makes it clear that with search frictions and ex ante heterogeneity, agents will not generally be able to match with all types of agents in the economy, but only with a subset. And agents who will never match with each other impose an externality on each other by reducing the rate at which they meet agents they will actually match with. In the present case of a class structure, if one could partition the economy into several marketplaces with each marketplace populated by agents of a given class, keeping the acceptance strategy of the members of each class unchanged, there would be a Pareto improvement: each agent's expected quality of a match is unchanged, and he matches at a higher rate since he then matches with the first person he meets. Hence, agents of different classes no longer impose externalities on each other since they are searching in different marketplaces. However, when agents are split this way, their reservation strategies will correspondingly change to reflect the new distribution of types in their new marketplaces.

III. The Generalized Model with Multiple Marketplaces

From now on the economy consists of countably many marketplaces.[8] We assume that agents can move freely between marketplaces at no cost, and agents of the same type are allowed to search in different marketplaces. The distribution of types in each marketplace is assumed to be public knowledge so that each agent makes an informed decision when choosing which marketplace to go to. We maintain the assumption of random search within each marketplace.

We denote by X^n the set of types searching in marketplace n, with supremum $\bar{x}^n$ and infimum $\underline{x}^n$, and by G^n and ω^n the cdf of types and

[8] We will show that there is only a finite number of classes in equilibrium, and thus allowing for the set of marketplaces to be a *continuum* would not modify the results about the class structure.

the mass of agents searching in marketplace n respectively.[9] A market structure M is defined as the sequence $\{X^n, G^n, \omega^n\}_{n=1}^N$, where N is the number of active marketplaces. Given a market structure, each type of agent has a value of search in each of the active marketplaces, and we denote by $u^n(x)$ the value of search in marketplace n for a type x agent. Hence, $u^n(x)$ satisfies

$$ru^n(x) = \lambda \int_{\Omega^n(x)} \max\{y - u^n(x), 0\} dG^n(y), \tag{3}$$

where $\Omega^n(x)$ is the set of types of agents in marketplace n willing to match with an agent of type x. We define an equilibrium as follows.

Definition 1. A steady-state equilibrium is a market structure $M = \{X^n, G^n, \omega^n\}_{n=1}^N$ and a level of utility $u(x)$ for each type $x \in X$ such that

1. $u^n(x) \le u(x)$ for all n, and $u^n(x) = u(x)$ if $x \in \mathrm{Supp}(G^n)$, where $u^n(x)$ solves (3);
2. $G(x) = \sum_{n=1}^N \omega^n G^n(x)$; and
3. it is not possible to find a distribution G^0 and a function $\alpha : X \rightarrow [0, 1]$ such that (a) for all x,

$$\alpha(x) = \begin{cases} 1 & \text{if } u^0(x) > u(x) \\ 0 & \text{if } u^0(x) < u(x), \end{cases}$$

and $\alpha(x) \in [0, 1]$ if $u^0(x) = u(x)$, where $u^0(x)$ satisfies (3) for $n = 0$ with

$$G^0(x) = \frac{\int_{\underline{x}}^{x} \alpha(y) dG(y)}{\int_{\underline{x}}^{\bar{x}} \alpha(y) dG(y)};$$

and (b) there exists an x where $u^0(x) > u(x)$.

Part 1 of the definition says that in equilibrium each agent searches in the best marketplace for himself. As agents of the same type need not be searching in the same marketplace, two or more marketplaces can yield the same value of search to agents of a given type. And one can have $u^n(x) = u(x)$ with no type x agent searching in marketplace n. The second equilibrium condition is simply a market-clearing condition in that if we were to put all agents searching in all the active marketplaces into one marketplace, we would get back the population's distribution of types.

Since this paper is interested in segmentation as an equilibrium phenomenon, we allow new marketplaces to be created. So in equilibrium,

[9] In this paper we use superscripts to index the marketplace and subscripts to index the class.

it must be true that there is no payoff to creating a new marketplace 0, which is dealt with in part 3 of the definition of equilibrium. More precisely, in equilibrium it is not possible to select a group of agents with types in $X^0 \equiv \{x | \alpha(x) > 0\}$ such that if they are put in the new marketplace 0 with cdf G^0 over X^0, all of them are no worse off than in their original marketplaces, and if agents of a given type $x \in X^0$ are strictly better off there, then all type x agents are searching in marketplace 0, that is, $\alpha(x) = 1$. The last requirement, 3*b*, imposes a nonredundancy condition since the new marketplace must make at least one type of agent strictly better off.

One can think of this economy as having countably many competitive market makers who each own a marketplace and who decide whether to make it active. Each market maker can advertise the desired types he would like to have in his marketplace,[10] but just as he cannot force an agent to go to his marketplace, he also cannot exclude agents from his marketplace. Although an agent can choose to search in any marketplace, he will self-select into the one that he expects to be the best for himself, and in equilibrium this marketplace is one a market maker intended him to go to. If agents of a given type are indifferent between several marketplaces in which they have been invited to search by the market makers, they choose randomly between them, but in such a way that in equilibrium the distribution of types across these marketplaces is indeed such that they are indifferent. Furthermore, in equilibrium it must not be possible for a market maker to be able to open up a new marketplace that will attract agents.[11]

As an example, think of marketplaces as bars or clubs and market makers as bar or club owners. In this economy with multiple clubs, agents of the same type need not all be in the same club. But in equilibrium, it must be true that no agent will like to switch clubs. At the same time, if there is room for a club owner to step in to open up a new club that would attract some agents because at least some of them can be made strictly better off and all other agents intended to be there no worse off, then this cannot be an equilibrium. So equilibrium in this economy also requires that there is no payoff in opening up a new marketplace that will attract some agents.

In the standard model with one marketplace, a class was defined as the set of agents who would match with each other if they were to meet. Once we introduce many marketplaces, this definition still holds, but now two agents of the same class need not be in the same marketplace.

[10] One could allow market makers to charge a fee for entry, but since there are countably many market makers, competition would drive down to zero the fee each of them can charge.

[11] This is related to Moen's (1997) competitive search equilibrium concept.

Given an equilibrium market structure M, there will be a class structure within each marketplace n by applying proposition 1.[12]

IV. A Perfect Segmentation Equilibrium

Since one important motivation of this paper is to address the inefficiencies in standard random search papers resulting from the search externalities that agents who meet but will never match impose on one another, we first want to find out if there exists an equilibrium in which agents do not suffer such externalities. We call these equilibria *perfect segmentation equilibria* or PSE. In this section we show that a PSE indeed exists, and we do so by constructing one. We focus here on constructing an equilibrium with the simplest possible market structure in that all agents of the same type search in the same marketplace (assumption 1) and the set of types of agents searching in a given marketplace is either a singleton or an interval (assumption 2).

Under assumptions 1 and 2, if the set of types searching in marketplace n, X^n, is an interval, the distribution of types in marketplace n is

$$G^n(x) = \frac{G(x) - G(\underline{x}^n)}{G(\bar{x}^n) - G(\underline{x}^n)} \quad \text{for all } x \in X^n,$$

and the mass of agents in marketplace n is $\omega^n = G(\bar{x}^n) - G(\underline{x}^n)$. Since both the cdf and the mass of agents can be recovered from X^n for each active marketplace n, a market structure in this case is simply defined as $M = \{X^n\}_{n=1}^N$. Our strategy in constructing an equilibrium satisfying assumptions 1 and 2 is to characterize market structures in which all agents search in the best marketplace for themselves before checking for which of these there is no payoff to creating a new marketplace.

A. *Characterizing the Market Structure*

Let us first consider agents of type $\bar{x}$, and let us assume that they are in a marketplace by themselves, say marketplace 1. From equation (3), when $X^1 = \{\bar{x}\}$, the reservation type of $\bar{x}$ agents in this marketplace is

$$u^1(\bar{x}) = \frac{\lambda}{r + \lambda}\bar{x},$$

which is strictly less than $\bar{x}$ as long as λ is finite, that is, as long as there are search frictions. Given that $\bar{x}$ agents are the best agents around and that the first marketplace is populated exclusively by agents of this type,

[12] The proof of proposition 1 in this case needs to be generalized to allow the set of types in a given marketplace to be nonconnected. We do not provide the formal proof, but the intuition should suffice to convince the reader that the result carries over.

among all active marketplaces, the value of search for an agent is the greatest in marketplace 1, provided that he can match there. Hence, for any agent with type $x \in [u^1(\bar{x}), \bar{x}]$, his type is higher than the reservation type of the best agents, and he can therefore match in marketplace 1. It follows that if $X^1 = \{\bar{x}\}$, agents with types in $[u^1(\bar{x}), \bar{x}]$ are not searching in their best marketplace, which cannot be true in equilibrium.

More generally, under assumptions 1 and 2, a PSE must be such that the marketplace in which the top agents search, say marketplace 1, is made up of one class, that is, $u^1(x) = u_1 \leq \underline{x}^1$ for all $x \in X^1$, and either it contains the whole population, in which case $\underline{x}^1 = \underline{x} \geq u_1$, or not, in which case $\underline{x}^1 = u_1 > \underline{x}$. To show this, first note that following the argument used to derive the class structure in the Burdett-Coles economy, all agents with types in $[u^1(\bar{x}), \bar{x}]$ and who search in marketplace 1 accept to match with one another, and hence all belong to the same class.

If $u_1 \leq \underline{x}$, then there is only one class in the economy, all agents have a value of search of u_1, and $\underline{x}^1 = \underline{x}$. Then we have already found a market structure $M = \{X\}$ inducing perfect segmentation and in which all agents search in the best marketplace for themselves.

The more interesting case is the one in which $u_1 > \underline{x}$. Then there is more than one class in the economy, and since we are constructing a PSE, $\underline{x}^1 \geq u_1$. Moreover, $\underline{x}^1 \leq u_1$, because if not, an agent with type $x \in [u_1, \underline{x}^1)$ would obtain a value of search there of u_1, which is the greatest value of search he can hope for since the distribution of types in marketplace 1 first-order stochastically dominates the distribution of types of any other marketplace. Therefore, it must be that $\underline{x}^1 = u_1$. In both cases u_1 must satisfy

$$ru_1 = \lambda \int_{u_1}^{\bar{x}} (y - u_1) \frac{dG(y)}{1 - G(u_1)}. \qquad (4)$$

Suppose that there exists a $u_1 > \underline{x}$. Then given this u_1, the marketplace containing the best agents not in the first class, say they are in marketplace 2, must also contain one class since we are looking for a PSE. The common reservation type of agents in this second marketplace, and class, is thus $u_2 \leq \underline{x}^2$. Following the reasoning we used earlier in constructing the first class, it must be clear that if $\underline{x}^2 > u_2 \geq \underline{x}$, then agents with types in $[u_2, \underline{x}^2)$ would not be searching in the best marketplace for themselves. Hence, for market structures satisfying assumptions 1 and 2, if the second marketplace is to contain one class, then the set of types of agents searching in marketplace 2 is either $[u_2, u_1)$ when $u_2 > \underline{x}$ or $[\underline{x}, u_1)$ when $u_2 \leq \underline{x}$; that is, $X^2 = [\max\{u_2; \underline{x}\}, u_1)$, where in both cases u_2 solves

$$ru_2 = \lambda \int_{u_2}^{u_1} (y - u_2) \frac{dG(y)}{G(u_1) - G(u_2)}.$$

If we apply the same reasoning recursively, it is clear that a market structure under assumptions 1 and 2 featuring perfect segmentation and in which all agents search in the best marketplaces for themselves is characterized by a finite sequence $\{u_j\}_{j=1}^{J}$, $u_J \leq \underline{x}$, where for all j, u_j solves

$$ru_j = \lambda \int_{u_j}^{u_{j-1}} (y - u_j) \frac{dG(y)}{G(u_{j-1}) - G(u_j)}, \tag{5}$$

with $u_0 \equiv \bar{x}$. Agents with types in X_j form the jth class of the economy, where $X_1 = [u_1, \bar{x}]$, and $X_j = [u_j, u_{j-1})$ for all $1 < j \leq J$. The sequence $\{u_j\}_{j=1}^{J}$ is finite because for all $j \geq 2$,

$$u_j < \left(\frac{\lambda}{r + \lambda}\right) u_{j-1} < \left(\frac{\lambda}{r + \lambda}\right)^j \bar{x},$$

where the first inequality comes from (5) and the second inequality from successively applying the former. As long as λ is finite, there exists a finite K such that $[\lambda/(r + \lambda)]^K \bar{x} \leq \underline{x}$, and this implies that there is a finite $J \leq K$ such that $u_J \leq \underline{x} < u_{J-1}$.

The next lemma (whose proof is in the Appendix) establishes the existence of such a sequence of reservation types, gives a sufficient condition for uniqueness, and compares these reservation types with those in the Burdett-Coles economy.

LEMMA 1. For any G, in a PSE in which the market structure satisfies assumptions 1 and 2, the following conditions hold:

1. there exists a finite sequence $\{u_j\}_{j=1}^{J}$, $u_J \leq \underline{x}$, where, for all j, u_j solves (5), with $u_0 \equiv \bar{x}$;
2. if G is such that $d\hat{x}(z;\ y)/dz \leq 1$, where $\hat{x}(z;\ y) \equiv \int_z^y xdG(x)/[G(y) - G(z)]$, then the sequence is unique; and
3. if $J^{bc} > 1$, then $u_j > u_j^{bc}$ for all $j \leq J^{bc}$, and therefore $J \geq J^{bc}$.

The intuition behind the possible multiplicity of classes' lower bounds is as follows. Suppose that the right tail of g is very "thin" so that few people are at the top of the distribution, and that we have found u_1 such that all agents with types in $X^1 = [u_1, \bar{x}]$ form exactly one class when in one marketplace by themselves. Thus u_1 solves (4), which can be rewritten as

$$u_1 = \frac{\lambda}{r + \lambda} \hat{x}(u_1;\ \bar{x}),$$

where $\hat{x}(z;\ \bar{x}) = \int_z^{\bar{x}} ydG(y)/[1 - G(z)]$ is the average type in marketplace

1 when its infimum type is z and all agents with types above z search in marketplace 1. Suppose that g is such that there are a lot of agents with types just below u_1; when those agents are added to marketplace 1, the marginal type $\underline{x}^1$ decreases; and the average type will at first decrease by less. The reservation type of the agents originally in the first marketplace thus falls, but by less than the marginal type, and therefore two classes appear. If a large enough mass of agents with lower attributes are being added, the average type in marketplace 1, $\hat{x}(\underline{x}^1; \bar{x})$, will eventually drop faster than $\underline{x}^1$ does. In this case the reservation type of the top agents decreases faster than $\underline{x}^1$, and therefore another value $\tilde{u}_1 < u_1$ can be found such that all agents with types in $[\tilde{u}_1, \bar{x}]$ form exactly one class when in one marketplace by themselves. From here one can intuit that if G is such that the average type $\hat{x}(\underline{x}^1; \bar{x})$ always decreases by less than the marginal type $\underline{x}^1$, then there is a unique u_1 such that $u_1 = [\lambda/(\lambda + r)]\hat{x}(u_1; \bar{x})$.[13] This is the condition given in part 2 of lemma 1. It turns out that a sufficient condition for this to be true[14] is that $1 - G$ is log-concave.[15]

When there is more than one class in the Burdett-Coles economy, part 3 of lemma 1 establishes that the reservation type of a class in the Burdett-Coles economy is strictly lower than the reservation type of the corresponding class in a market structure that is part of a PSE. Intuitively, in the Burdett-Coles economy with more than one class, agents meet other agents they will not match with, whereas in a PSE, each marketplace consists of only one class, implying that class 1 agents can be more picky about whom they match with. For all other class j agents, their reservation types are increasing in the upper bound of their marketplaces, so $u_j > u_j^{bc}$ for all $j \leq J^{bc}$. Therefore, there are at least as many classes in an economy with perfect segmentation as in the Burdett-Coles economy.

When there is only one class in the Burdett-Coles economy, it might be possible to find a $u_1 > \underline{x}$ such that if all agents with types in $[u_1, \bar{x}]$ search in one marketplace, then they all have the same reservation type u_1, but this cannot be guaranteed. For instance, as established earlier, if $1 - G$ is log-concave, the average type $\hat{x}(\underline{x}^1; \bar{x})$ always decreases by less

[13] We would like to thank Espen Moen for suggesting to us this interpretation using marginal and average types.

[14] The proof is available from the authors on request.

[15] A function is said to be log-concave if its log is concave.

With an endogenous distribution of singles and a unique marketplace, Burdett and Coles (1997) show that a sufficient condition for uniqueness of the class structure is that $1 - G$ be log-concave. It is worth noting that this is also sufficient for uniqueness when the overall distribution of unmatched agents is exogenous but there are many marketplaces and there is perfect segmentation. In their paper the endogeneity of the distribution of singles comes from the matching pattern, whereas in this paper it originates from the endogeneity of the set of types in a given marketplace.

than the marginal type $\underline{x}^1$. Hence, since $[\lambda/(\lambda + r)]\hat{x}(\bar{x}; \bar{x}) = [\lambda/(\lambda + r)]\bar{x} < \bar{x}$ and $u_1^{bc} \leq \underline{x}$, that is, $[\lambda/(\lambda + r)]\hat{x}(\underline{x}; \bar{x}) \leq \underline{x}$, we have $u_1 = [\lambda/(\lambda + r)]\hat{x}(\underline{x}^1; \bar{x}) < \underline{x}^1$ for all $\underline{x}^1 \in (\underline{x}, \bar{x}]$, implying that the unique solution to (4) is $u_1 = u_1^{bc} \leq \underline{x}$.

B. *Creation of a New Marketplace*

Denoting by u_1^* the highest first-class lower bound solving (4), we will now show that among all the lower bounds solving (4), only u_1^* is such that no class 1 agent can be made strictly better off from the creation of a new marketplace 0. Consider a market structure $M = \{X^n\}_{n=1}^N$ such that $X^1 = [u_1, \bar{x}]$, with u_1 solving (4) and $u_1 < u_1^*$. It is clear that there is a payoff from creating a new marketplace 0 with G^0 and α such that

$$\alpha(x) = \begin{cases} 1 & \text{for } x \in X^0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$G^0(x) = \frac{G(x) - G(u_1^*)}{1 - G(u_1^*)} \quad \text{for all } x \in X^0,$$

where $X^0 = [u_1^*, \bar{x}]$. All agents meant to search in marketplace 0, that is, those with types in X^0, enjoy a value of search there of u_1^*, which is strictly greater than the value of search in marketplace 1, u_1. And since the lower bound of marketplace 0 is also the reservation type of its inhabitants, the value of search there for all other agents is zero since they would not be able to match there. As $\underline{x} > 0$, the value of search for all agents must be strictly positive in equilibrium, and hence agents with types not in X^0 have a greater value of search in their original marketplaces, which is consistent with $\alpha(x) = 0$ for all $x \notin X^0$.

We now show that if $X^1 = [u_1^*, \bar{x}]$, no class 1 agent can obtain a greater value of search than u_1^* in a new marketplace 0. We have two cases to consider, whether marketplace 0 contains one or more classes. Let us first consider the former case and suppose that it is possible to find a G^0 and a function α such that all agents initially in marketplace 1 and whose types are in X^0 are better off, with some of these agents strictly better off; that is, $u^0(x) \geq u_1^*$ for all x and $u^0(x) > u_1^*$ for some x, $x \in X^0 \cap [u_1^*, \bar{x}]$. But since marketplace 0 contains one class, $u_1^* < u^0(x) = u^0$ for all $x \in X^0$. It is straightforward to see then that all agents with types in $[u^0, \bar{x}]$ must be intended to search in marketplace 0: they would all be accepted by all agents with types in $[u^0, \bar{x}] \cap X^0$, and they would enjoy a value of search of $u^0 > u_1^*$ in the new marketplace. Hence, given that the set of types in this new marketplace must include all agents with types in $[u^0, \bar{x}]$, X^0 must be $[u^0, \bar{x}]$ and $\alpha(x) = 1$ for all $x \in [u^0, \bar{x}]$.

But since u_1^* is the highest solution to (4), it is not possible to have $u^0 > u_1^*$ for all $x \in X^0$ where this new marketplace contains only one class.

If, however, the new marketplace were to contain two classes or more, it can also be shown (see the proof of lemma 2 below, which is in the Appendix) that no agent with type in $[u_1^*, \bar{x}]$ would fare strictly better in the new marketplace than in marketplace 1. The reason is that when $X^1 = [u_1^*, \bar{x}]$, class 1 agents cannot fare strictly better in the new marketplace since they now meet agents they will not match with, so they must be less picky about whom they match with; that is, their reservation type is no higher than u_1^*. We thus have the following lemma.

LEMMA 2. If a market structure $M = \{X^n\}_{n=1}^N$ is such that $X^1 = [u_1^*, \bar{x}]$, then there does not exist a pair (G^0, α) satisfying parts 3a and 3b of the equilibrium definition for which $u^0(x) > u_1^*$ for some $x \in (X^0 \cap X^1)$.

We now turn our attention to the other classes, and marketplaces. If we denote by u_2^* the highest of the u_2 solving (5) for $u_1 = u_1^*$, it is clear that for all other possible reservation types for the second class a new marketplace can be created for all agents with types in $[u_2^*, u_1^*)$, and all these agents are strictly better off there. So, given that u_1^* is the only solution to (4) that can be an equilibrium lower bound for the first marketplace, and class, only u_2^* can be an equilibrium lower bound.

Since lemma 2 established that if a market structure is such that $X^1 = [u_1^*, \bar{x}]$ it is not possible to create a new marketplace that would make some class 1 agents strictly better off, we will now show that if a market structure is such that $X^1 = [u_1^*, \bar{x}]$ and $X^2 = [u_2^*, u_1^*)$, then it is not possible to create a new marketplace 0 that can make some of these class 2 agents strictly better off with none of them being worse off. If we first consider the possibility of creating a new marketplace not containing any agent whose type is in $[u_1^*, \bar{x}]$, we can follow the reasoning used for class 1 to show that it is not possible to create a new marketplace 0 yielding a value of search greater than u_2^* to any agent with type in $[u_2^*, u_1^*)$.

But if we now consider creating a new marketplace 0 that contains some class 1 agents, could some class 2 agents be made better off in the new marketplace? It can be shown in this case (see the proof of lemma 3 below, which is in the Appendix) that this is not possible. The intuition is the following. If some class 1 agents were to search in this new marketplace 0, then they should be no worse off in it than in marketplace 1, implying that they would not match with any agent of type below u_1^* in the new marketplace. But if this were so, class 2 agents would not get to match with better-type agents than in marketplace 2 by searching in the new marketplace; on the contrary, they would get to meet agents they would not match with, and this can only drive down their reservation type. We then have the following lemma.

LEMMA 3. If a market structure $M = \{X^n\}_{n=1}^N$ is such that $X^1 = [u_1^*, \bar{x}]$ and $X^2 = [u_2^*, u_1^*)$, then there does not exist a pair (G^0, α) satisfying parts 3*a* and 3*b* of the equilibrium definition for which $u^0(x) > u_2^*$ for some $x \in (X^0 \cap X^2)$.

Having shown that class 2 agents cannot do better than being in their own marketplace where the reservation type of these agents is u_2^*, given u_2^*, we can solve for class 3 agents' highest lower bound u_3^*, and so on, until we reach the lower bound $u_{J^*}^* \leq \underline{x}$. This sequence of lower bounds $\{u_j^*\}_{j=1}^{J^*}$ for the J^* classes is such that each agent searches in his best marketplace, and there are no payoffs from creating a new marketplace. We thus have the following proposition.

PROPOSITION 2. The market structure $M^* = \{X^{n*}\}_{n=1}^{J^*}$ and levels of utility $u(x) = u_n^*$ for $x \in X^{n*}$, all n, where $X^{1*} = [u_1^*, \bar{x}]$, $X^{n*} = [u_n^*, u_{n-1}^*)$ for all $n = 2, \ldots, J^* - 1$, $X^{J^**} = [\underline{x}, u_{J^*-1}^*)$, and such that u_n^* is the highest solution to

$$ru_n = \lambda \int_{u_n}^{u_{n-1}^*} (y - u_n) \frac{dG(y)}{G(u_{n-1}^*) - G(u_n)},$$

$u_{J^*}^* \leq \underline{x} < u_{J^*-1}^*$ with $u_0^* \equiv \bar{x}$, constitute a PSE. The number of active marketplaces J^* is finite, and $J^* \geq J^{bc}$.

V. Other Equilibria

We have constructed a PSE satisfying assumptions 1 and 2. We now want to consider all other possible market structures, that is, those not satisfying assumptions 1 and 2 and also those in which some agents do not necessarily match with the first person they meet. Before showing which market structures can be equilibrium market structures, we show that even when assumptions 1 and 2 are relaxed, all market structures in which agents search in their best marketplace induce an economywide class structure; that is, there exists a sequence of disjoint intervals $\{X_j\}_{j=1}^J$ such that agents with types in X_j would match with each other if they were to meet, but they would not match with any agent whose type is not in X_j. Let us start with the first class. It must naturally contain the $\bar{x}$ agents, and since reservation strategies are nondecreasing, $u(x) \leq u(\bar{x})$ for all x. If an agent with type $x \in [u(\bar{x}), \bar{x}]$ searches in a marketplace with value of search $u(x) < u(\bar{x})$, then he has a profitable deviation by joining a marketplace containing $\bar{x}$ agents since he will be accepted by everyone with types in $[u(\bar{x}), \bar{x}]$ and will therefore enjoy a value of search of $u(\bar{x})$ there. Denoting $u(\bar{x})$ by $\tilde{u}_1$, it follows that all agents with types in $X_1 = [\tilde{u}_1, \bar{x}]$ must share the same reservation type

$\tilde{u}_1$, and they therefore all belong to the same class. If we apply the same argument recursively, it is clear that a market structure M in which no agent wishes to change marketplace must display an economywide class structure $\{X_j\}_{j=1}^J$ such that $X_1 = [\tilde{u}_1, \bar{x}]$, $X_j = [\tilde{u}_j, \tilde{u}_{j-1})$ for all $j \geq 2$, and $\tilde{u}_J \leq \underline{x}$. We then have the following proposition, which establishes that in equilibrium all agents match with the first person they meet (the proof is in the Appendix).

Proposition 3. An equilibrium must feature perfect segmentation.

To understand this result, let us, for simplicity, consider market structures satisfying assumptions 1 and 2 in which at least one marketplace has more than one class. Consider $M = \{X^n\}_{n=1}^N$ such that $X^n = X^{n*}$ for all $n < K < J^*$, and marketplace K of M contains more than one class. If the relative mass of agents in the classes of K, other than the first, is zero, then all these agents have a value of search in K that is zero because the probability they will meet someone they can match with is zero, and they must prefer another marketplace. So the relative mass of agents in the classes of K, other than the first, must be strictly positive. In this case, the reservation type of the first class in K, u_1^K, is strictly lower than u_K^* since these agents have a strictly positive probability of meeting agents they will not match with and hence are less picky than if there had been one class. Hence, a marketplace 0 can be created for agents with types in $[u_K^*, u_{K-1}^*)$, which makes all these agents strictly better off.

Market structures in which some agents do not match with the first person they meet have been ruled out. We are thus left with considering market structures in which agents of the same type can search in different marketplaces and type sets are not necessarily intervals (i.e., assumptions 1 and 2 do not hold), and yet agents match with the first person they meet.

Lemma 4. Let $M = \{X^n, G^n, \omega^n\}_{n=1}^N$ be a perfectly segmented market structure satisfying parts 1 and 2 of definition 1. Then there exists a market structure $\hat{M} = \{\hat{X}^n\}_{n=1}^{\hat{N}}$, also satisfying parts 1 and 2 of definition 1, that is equivalent to M in that each type x agent has the same level of utility $u(x)$ in both cases.

Let Γ_j denote the set of marketplaces containing agents in class j. Because we are considering market structures featuring perfect segmentation, the sets of types of the marketplaces class 1 agents search in are such that, for all $n \in \Gamma_1$, $\underline{x}^n \geq \tilde{u}_1$, $\cup_{n\in\Gamma_1} X^n = [\tilde{u}_1, \bar{x}]$, and $\sum_{n\in\Gamma_1} \omega^n G^n(x) = G(x) - G(\tilde{u}_1)$ for all $x \in [\tilde{u}_1, \bar{x}]$. From (3), $\tilde{u}_1$ is such that, for all marketplaces $n \in \Gamma_1$,

$$r\tilde{u}_1 = \lambda \int_{X^n} (y - \tilde{u}_1) dG^n(y).$$

Since $\sum_{n\in\Gamma_1} \omega^n G^n(x) = G(x) - G(\tilde{u}_1)$, multiplying both sides of the above equation by ω^n and summing up over all marketplaces in Γ_1, we obtain (4). Hence, a market structure M in which class 1 agents search in more than one marketplace is equivalent to another market structure M' in which all agents from class 1 of M are searching in one marketplace and all the other marketplaces are identical. These two market structures are equivalent since all classes are identical, and therefore they yield the same level of welfare for each type of agent.

When we repeat the argument recursively, it is clear that for a perfectly segmented market structure M such that $\cup_{n\in\Gamma_1}X^n = [\tilde{u}_1, \bar{x}]$, $\cup_{n\in\Gamma_j}X^n = [\tilde{u}_j, \tilde{u}_{j-1})$ for all $j = 2, \ldots, J-1$, and $\cup_{n\in\Gamma_J}X^n = [\underline{x}, \tilde{u}_{J-1}) \subseteq [\tilde{u}_J, \tilde{u}_{J-1})$, where $\tilde{u}_k$ solves

$$r\tilde{u}_k = \lambda \int_{X^n} (y - \tilde{u}_k) dG^n(y), \tag{6}$$

for $n \in \Gamma_k$, we have that, given $\tilde{u}_{k-1}$, $\tilde{u}_k$ also solves (5), which proves the equivalence result.

The intuition behind this result is simple. Rewrite equation (6) for $k = 1$ as

$$\tilde{u}_1 = \frac{\lambda}{r+\lambda}\int_{X^n} x dG^n(x),$$

where $\int_{X^n} x dG^n(x)$ is the average type in marketplace n, $n \in \Gamma_1$. This implies that all marketplaces containing class 1 agents have the same average type. Hence, if we were to put all class 1 agents together in one marketplace, the average type does not change, and therefore their reservation type stays the same.

There are potentially many market structures that are equivalent in that they imply the same class structure. For instance, consider a market structure with marketplaces 1 and 2 in which X^2 is an interval $(\underline{x}^2, \bar{x}^2)$ and $X^1 = [\tilde{u}_1, \underline{x}^2] \cup [\bar{x}^2, \bar{x}]$, they both share the same reservation type $\tilde{u}_1$, and all agents of the same type search in the same marketplace. If we assume that G is uniform, then

$$\tilde{u}_1 = \left(\frac{\lambda}{r+\lambda}\right)\left(\frac{\underline{x}^2 + \bar{x}^2}{2}\right) = \left(\frac{\lambda}{r+\lambda}\right)\left(\frac{\tilde{u}_1 + \bar{x}}{2}\right),$$

and all $\bar{x}^2 = \bar{x} - \gamma$ and $\underline{x}^2 = \tilde{u}_1 + \gamma$ for some $\gamma \in (0, (\bar{x} - \tilde{u}_1)/2)$ are possible boundaries for X^2. In this case, there are a *continuum* of ways to split the first class into two marketplaces, each having the same average type as the original marketplace. And if we further consider splitting the first class into more than two marketplaces and allowing agents of the same type to search in different marketplaces, the possibilities expand even more.

We have shown that there is no payoff to creating a new marketplace when the market structure is M^*. But this must also be true for market structures equivalent to M^*, implying that they are also equilibrium market structures. Take a market structure $M = \{X^n, G^n, \omega^n\}_{n=1}^N$ such that, for any active marketplace k, $X^k \subseteq X^{j*}$ and $[\lambda/(r+\lambda)]\int x dG^n(x) = u_j^*$ for some j, and $G(x) = \sum_{n=1}^N \omega^n G^n(x)$ for all x. Then, following the line of argument used in lemmas 2 and 3, it is clear that it is not possible to create a new marketplace 0 that would attract some agents, with some of them made strictly better off. Hence, M is an equilibrium market structure. On the contrary, all other market structures not satisfying assumption 1 or 2 cannot be part of an equilibrium. The reason is that they are equivalent to perfectly segmented market structures that are not equilibrium market structures since there is a payoff to creating a new marketplace.

Hence, even though the set of equilibrium market structures is potentially large, there is a unique equilibrium class structure and all agents fare the same in all equilibria. These results are summarized in the following proposition.

Proposition 4. A market structure $M = \{X^n, G^n, \omega^n\}_{n=1}^N$ is an equilibrium market structure if and only if its implied class structure is that implied by M^*, and therefore, for all $x \in X^{j*}$, $j = 1, \ldots, J^*$, $u(x) = u_j^*$.

VI. Discussion

A. *Comparison to the Search and Matching Literature*

In frictionless assignment models, which were first studied by Gale and Shapley (1962), when agents have the same objective ranking over types, as is the case in this paper, the solution features perfect assortative matching. For instance, the solution to an assignment problem with symmetrically distributed males and females features the top male matching with the top female, the next-best male matching with the next-best female, and so on.[16]

Standard search and matching models do not deliver this result of perfect assortative matching. In particular, for search models that have multiplicatively separable utility functions (our paper uses a special case of this), a class structure is obtained in which matching sets are intervals. Hence, there is some form of positive assortative matching among agents, but it is no longer perfect, in contrast to frictionless assignment

[16] Formally, the only stable assignment rule Ψ is such that for each male agent $i \in I^m$ with type $x(i)$, the female agent he is assigned to is $j = \Psi(i) \in I^f$ such that $x(j) = x(i)$. Obviously it must be that $\Psi(j) = i$ or $\Psi(\Psi(i)) = i$; i.e., an agent is his or her partner's partner. This implies that the matching sets $\Lambda(\cdot)$ are singletons and for all $x \in X$, $\Lambda(x) = x$. In fact, in this case Ψ is a measure preserving bijection between I^m and I^f.

models. When we introduce segmentation, the degree of positive assortativeness is greater than that of standard one-marketplace matching models. But because of the presence of search frictions, sorting is nevertheless less than perfect.

We view as interesting the result in proposition 4 that, in equilibrium, a number of active marketplaces can yield the same value of search to some types of agents. This can help explain why, for example, there are so many different bars or clubs that are very similar to one another and people are indifferent to going to any of them and seem to randomize on different nights where to go.

Several other papers have dealt with market segmentation, but in different ways. Bose's (2003) paper also considers segmentation of markets and is therefore related. His model has only two types of agents (traders), with heterogeneity in the number of units of goods the traders can hold on to (either one or two units), and he does not endogenize the market structure. Mortensen and Pissarides (1999) consider a model with segmentation of the labor market in which segmentation arises because of technological constraints: To be productive in a job, a worker needs a minimum level of skills; and if a worker is employed in a firm with a technology below his skill level, he is no more productive than if he had the minimum skill required. This perfect complementarity between skills and technology ensures that low-type agents do not find it profitable to deviate to marketplaces with agents of types above theirs, and hence they obtain perfect assortative matching. Lang, Manove, and Dickens' (2005) model of labor market segmentation is a directed search model and thus is not directly comparable to ours.

Competitive search equilibrium models of Moen (1997) and Mortensen and Wright (2002) feature a market maker who creates marketplaces through posting wages/prices, thereby allowing each agent to choose which marketplace to go to. As the waiting time is implicitly priced, agents sort themselves into the correct marketplaces, with the market tightness adjusting to internalize the search externalities. Search externalities are internalized in our model just as in competitive search models because each agent is able to direct his search toward the right marketplace.[17]

The article by Bloch and Ryder (2000) is also related to our work. The authors consider a matchmaker who, for a fee, can match agents instantaneously. Agents choose either to pay the fee to be matched by

[17] Our paper is also related to the recent literature that has considered the use of money, either in the context of a cooperative matching game as in Corbae, Temzelides, and Wright (2003) or with more than one marketplace as in Matsui and Shimizu (2005). In these models, preferences are heterogeneous in the sense that when there are K types of agents, an agent typically consumes good k and produces good $k+1$ (mod K), so there is no objective ranking of types, in contrast to our paper.

the matchmaker or to search for a partner in a marketplace with search frictions. The two main differences between their approach and ours are that we allow for many marketplaces whereas they consider only two, and in our model search frictions are present in all marketplaces. Damiano and Li (2007, forthcoming) also consider a matchmaking model, but in their case the matchmaker(s) can create different marketplaces and charge a different fee for each of them.

Our paper differs crucially from sectoral models such as the ones of Davidson, Martin, and Matusz (1987, 1988), Hosios (1990), and Uren (2006). In these models, each firm belongs to one of two sectors, and only workers are mobile. The spirit of our model is to allow *all* agents to move freely between marketplaces.

B. *The Meeting Technology*

In order to focus on the sorting effects, it was assumed throughout the paper that the meeting technology displays constant returns to scale so that the measure of agents in each marketplace does not matter for the meeting rate. It is natural to ask whether the results obtained are robust to the introduction of increasing returns, as segmentation then becomes less attractive since the meeting rate is lower than if everyone is searching in one marketplace.

To answer this, let us deal with a market structure in which all agents in the same class are in the same marketplace. As long as the increasing returns are not too strong, the results obtained hold. Consider the case in which $\lambda'(\mu^n) > 0$, where $\lambda(\mu^n)$ is the meeting rate in marketplace n when μ^n is the measure of agents searching in n. If we try to construct a PSE, u_1, the lower bound of the first marketplace and of the first class, must satisfy

$$u_1 = \frac{\lambda(1 - G(u_1))}{r} \int_{u_1}^{\bar{x}} (y - u_1) dG^1(y),$$

which can be rewritten as

$$u_1 = \frac{\lambda(\mu^1)}{r\mu^1} \int_{u_1}^{\bar{x}} [1 - G(y)] dy.$$

In the Burdett-Coles economy $\lambda(\mu^1)/\mu^1$ is replaced by $\lambda(1)$ in equation (2). It is then clear that the ratio $\lambda(\mu^1)/\mu^1$ is crucial. If it is decreasing in μ^1, then all the possible equilibrium lower bounds for the first marketplace, which are also the equilibrium reservation types of the first class, are greater than the lower bound of the first class when everyone meets in one marketplace. And since the maximum equilibrium lower bound of a marketplace with one class is still increasing in its upper bound, the only equilibrium must be perfectly segmented, and it displays a larger number of classes than the Burdett-Coles economy. It is easy to show that this analysis carries over for all other classes. This case corresponds to a meeting technology being less than quadratic.[18]

It is clear that with increasing returns in the meeting technology the equivalence result of lemma 4 no longer holds, and market structures in which agents of the same class search in more than one marketplace cannot be part of the equilibrium. To see this, consider our previous example in which the first class is split between marketplaces 1 and 2, where X^2 is an interval $(\underline{x}^2, \bar{x}^2)$ and $X^1 = [u_1, \underline{x}^2] \cup [\bar{x}^2, \bar{x}]$, and u_1 is the common reservation type of these agents. If we regroup them in marketplace 1, the meeting rate increases since there are now more agents searching there. Hence, the top agents of the first class will increase their reservation type above u_1, and two classes appear. So the highest possible lower bound for the first class is obtained when all the agents of that class search in the same marketplace.

It is worth noting that if $\lambda(\mu)/\mu$ is constant, which corresponds to the case in which the meeting technology is quadratic, then the class structure obtained for a PSE satisfying assumptions 1 and 2 is identical to the Burdett-Coles class structure. In that case the increasing returns are just enough to offset the congestion externality, in that the rate at which agents meet the "right" types is unchanged whether there is segmentation or not.[19] More generally, one can see that with a quadratic matching function, all market structures satisfying assumptions 1 and 2 display the same class structure.

VII. Conclusion

We endogenize the market structure of an economy with heterogeneous agents who want to form bilateral matches in the presence of search frictions and when utility is nontransferable. We allow for the existence of infinitely many marketplaces and for each agent to choose which marketplace to be in. In equilibrium all market structures feature perfect segmentation. Although perfect assortative matching cannot be obtained in equilibrium, the degree of assortativeness is nevertheless greater than in standard models. All these market structures have the same class structure, implying that all agents fare the same in all equilibria.

[18] The meeting function is quadratic if the number of matches quadruples when the measure of searching agents doubles. In this case the probability that an agent meets someone doubles.

[19] See Teulings and Gautier (2004) for a search and matching model with heterogeneous agents and a quadratic matching function.

Appendix

Proof of Lemma 1

From (2) and integrating by parts the right-hand side of (5), we have that u_j^{bc} and u_j solve $\hat{\Phi}(u_j^{bc};\ u_{j-1}^{bc}) = 0$ and $\Phi(u_j;\ u_{j-1}) = 0$, respectively, where

$$\hat{\Phi}(z;\ y) \equiv z - \frac{\lambda}{r}\int_z^y [G(y) - G(x)]dx$$

and

$$\Phi(z;\ y) \equiv z - \frac{\lambda}{r}\int_z^y \frac{G(y) - G(x)}{G(y) - G(z)}dx.$$

1. We have that $\Phi(0;\ \bar{x}) = -(\lambda/r)\int_0^{\bar{x}}[1 - G(x)]dx < 0$ and, by L'Hospital, that $\lim_{z\to\bar{x}}\Phi(z;\ \bar{x}) = \bar{x} > 0$. Hence, since Φ is continuous, the intermediate value theorem implies that there exists a $u_1 = \phi(\bar{x}) \in (0,\ \bar{x})$, where $\phi(x)$ is such that $\Phi(\phi(x);\ x) = 0$. Applying the reasoning recursively, we have that, given $u_{j-1} > \underline{x}$, there exists a $u_j = \phi(u_{j-1})$, and the sequence ends once we find $u_J = \phi(u_{J-1}) \leq \underline{x}$. The sequence is finite because $u_j < [\lambda/(\lambda + r)]u_{j-1} < [\lambda/(\lambda + r)]^j\bar{x}$ for $j \geq 2$.

2. The term u_j solves (5), which can be rewritten as $u_j = [\lambda/(\lambda + r)]\hat{x}(u_j;\ u_{j-1})$. We then have

$$T(0;\ u_{j-1}) = -\left(\frac{\lambda}{\lambda + r}\right)\hat{x}(0;\ u_{j-1}) < 0$$

and

$$\lim_{z\to u_{j-1}} T(z;\ u_{j-1}) = \left(\frac{r}{\lambda + r}\right)u_{j-1} > 0,$$

where

$$T(z;\ y) \equiv z - \frac{\lambda}{\lambda + r}\hat{x}(z;\ y);$$

therefore, if T is strictly increasing, there exists a unique u_j such that $T(u_j;\ u_{j-1}) = 0$. Since

$$\frac{dT(z;\ y)}{dz} = 1 - \left(\frac{\lambda}{\lambda + r}\right)\frac{d\hat{x}(z;\ y)}{dz},$$

a sufficient condition for this is $d\hat{x}(z;\ y)/dz \leq 1$. It follows that a unique sequence $\{u_j\}_{j=1}^J$ exists if $d\hat{x}(z;\ y)/dz \leq 1$ for all $y \in (\underline{x},\ \bar{x}]$ and $z < y$.

3. It is clear that $\hat{\Phi}(z;\ y) > \Phi(z;\ y)$ for all $(z,\ y) \in ([\underline{x},\ y) \times X)\backslash(\underline{x},\ \bar{x})$, and $\hat{\Phi}(z;\ y) = \Phi(z;\ y)$ for $(z,\ y) = (\underline{x},\ \bar{x})$. Hence, if $u_1^{bc} > \underline{x}$, $u_1 > u_1^{bc} = \hat{\phi}(\bar{x})$, where $\hat{\phi}(\bar{x})$ is such that $\hat{\Phi}(\hat{\phi}(\bar{x});\ \bar{x}) = 0$. Since $\Phi(z;\ y)$ is strictly decreasing in y, $\hat{\Phi}(z;\ y) \geq \Phi(z;\ y) > \Phi(z;\ x)$ for $x > y$, all $y \geq \underline{x}$. It follows that if $u_1^{bc} > \underline{x}$, then $u_2 = \phi(u_1) > \phi(u_1^{bc}) > \hat{\phi}(u_1^{bc}) = u_2^{bc}$, and so on. QED

Proof of Lemma 2

If there is only one class in marketplace 0, the result has been proved in the text. For the case in which there are two or more classes, suppose $u^0(\bar{x}^0) > u_1^*$. It must then be that $\alpha(x) = 1$ for all $x \geq u^0(\bar{x}^0)$, and hence, X^0 has the form $[u^0(\bar{x}^0), \bar{x}] \cup Y$ for some set Y such that $Y \cap [u^0(\bar{x}^0), \bar{x}] = \emptyset$. But if we integrate by parts the right-hand side of (3), $u^0(\bar{x}^0)$ solves $\tilde{\Phi}(u^0(\bar{x}^0); \bar{x}, \omega^0(Y)) = 0$, where

$$\tilde{\Phi}(z;\ y,\ \omega) \equiv z - \frac{\lambda}{r}\int_z^y \frac{G(y) - G(x)}{G(y) - G(z) + \omega}\,dx,$$

$\omega^0(Y) \equiv \int_Y \alpha(y)dG(y)$ being the mass of agents with types in Y searching in 0. We have $\tilde{\Phi}(0;\ y,\ \omega) < 0$ and $\tilde{\Phi}(y;\ y,\ \omega) > 0$ for $y > \underline{x}$, and $\tilde{\Phi}(z;\ y,\ \omega)$ is continuous in z. So by the intermediate value theorem, there exists a solution $\tilde{\phi}(y,\ \omega)$ to $\tilde{\Phi}(z;\ y,\ \omega) = 0$. Since

$$\tilde{\Phi}(z;\ y,\ \omega) - \Phi(z;\ y) = \frac{\lambda}{r}\int_z^y \frac{[G(y) - G(x)]\omega}{[G(y) - G(z) + \omega][G(y) - G(z)]}\,dx,$$

it is clear that $\tilde{\Phi}(z;\ y,\ \omega) \geq \Phi(z;\ y)$ for all $\omega \geq 0$, with strict inequality for $\omega > 0$. Since u_1^* is the highest u_1 solving $\Phi(u_1;\ \bar{x}) = 0$, it follows that, for all $\omega^0(Y) \geq 0$, $u^0(\bar{x}^0) \leq u_1^*$, and for all $\omega^0(Y) > 0$, $u^0(\bar{x}^0) < u_1^*$, which finishes the proof. QED

Proof of Lemma 3

It has been shown in the text that if the new marketplace 0 does not contain any class 1 agent, then $u^0(x) \leq u_2^*$ for all $x \in (X^0 \cap X^2)$, all G^0 and α satisfying part 3 of our definition of equilibrium. Now suppose that some class 1 agents search in the new marketplace 0. Because class 1 agents cannot be made strictly better off, as proven in lemma 2, for them to be in 0 it must be that $u^0(x) = u_1^*$ for all $x \in (X^0 \cap X^1)$. Now suppose that at least some agents originally in the second marketplace are strictly better off in the new marketplace and enjoy a value of search of $u^0 > u_2^*$ there so that $\alpha(x) = 1$ for all $x \in [u^0,\ u_1^*)$. However, none of these agents with types above u_1^* will match with agents of types less than u_1^*. Thus, from (3) the value of search in marketplace 0 for agents with type $x \in [u^0,\ u_1^*)$ is

$$ru^0 = \lambda \int_{u^0}^{u_1^*} (y - u^0)dG^0(y),$$

which, after we integrate the right-hand side by parts, implies that u^0 solves $\tilde{\Phi}(u^0;\ u_1^*, \tilde{\omega}^0) = 0$, where $\tilde{\omega}^0 = \omega^0(X^0 \backslash [u^0,\ u_1^*))$ is the mass of agents in marketplace 0 with types not in $[u^0,\ u_1^*)$. But there does not exist a $u^0 > u_2^*$ solving $\tilde{\Phi}(u^0;\ u_1^*, \tilde{\omega}^0) = 0$, which contradicts our assumption that some agents originally in the second marketplace are strictly better off in the new marketplace and finishes the proof. QED

Proof of Proposition 3

Consider M such that some agents do not match with the first person they meet. Take the first marketplace in which agents belong to at least two different classes. Let us consider the class consisting of the better agents of the lot, say class j. The value of search for these agents, $\tilde{u}_j$, must solve

$$r\tilde{u}_j = \lambda \int_{\tilde{u}_j}^{\tilde{u}_{j-1}} (y - \tilde{u}_j) dG^n(y)$$

for all $n \in \Gamma_j$, Γ_j being the set of marketplaces containing agents of the jth class. If the market structure satisfies assumptions 1 and 2, then Γ_j is simply a singleton. If we denote by ω_{-j}^n the mass of agents in marketplace n not belonging to class j, multiplying both sides by ω^n for all n, summing up over n, and integrating by parts the right-hand side of the above equation, we obtain that $\tilde{u}_j$ solves $\tilde{\Phi}(\tilde{u}_j;\ \tilde{u}_{j-1},\ \sum_{n\in\Gamma_j}\omega_{-j}^n) = 0$. In fact, $\cup_{j\in\Gamma_j} X^j$ must contain the interval with upper bound $\tilde{u}_{j-1}$ and lower bound $\tilde{u}_j$. We have proven earlier (see the proof of lemma 2) that there exists a solution $\tilde{\phi}(z, \omega)$ to $\tilde{\Phi}(x;\ z, \omega) = 0$, and that for any nonzero ω, $\tilde{\Phi}(x;\ z, \omega) > \Phi(x;\ z)$ for all x, for a given z. Setting $z = \tilde{u}_{j-1}$, we have that $\tilde{u}_j^* = \phi(\tilde{u}_{j-1}) \geq \tilde{u}_j = \tilde{\phi}(\tilde{u}_{j-1},\ \sum_{n\in\Gamma_j}\omega_{-j}^n)$, with strict inequality if $\sum_{n\in\Gamma_j}\omega_{-j}^n > 0$. Since the relative mass of agents not in class j searching in marketplaces in Γ_j must be nonzero (otherwise these agents have a zero probability of matching in these marketplaces, which cannot be true in equilibrium since this entails a value of search of zero), we have $\tilde{u}_j < \tilde{u}_j^*$. Hence, there is a payoff from creating a new marketplace 0 since if we choose α and G^0 such that $\alpha(x) = 1$ for $x \in [\tilde{u}_j^*, \tilde{u}_{j-1})$ and zero otherwise, and $G^0(x) = [G(x) - G(\tilde{u}_j^*)]/[G(\tilde{u}_{j-1}) - G(\tilde{u}_j^*)]$, all agents in this new marketplace 0 are strictly better off there than in their original marketplace. And no other agent wishes to join. QED

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