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Exploring Excited States of the Nucleon in 2+1 Flavor Lattice QCD

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Abstract. Our recent first-principles lattice-QCD exploration of the ground and first eleven excited states of the nucleon in the positive parity channel is presented. Of particular interest is the first positive-parity excitation of the nucleon; the Roper resonance. Using correlation-matrix methods developed by the CSSM Lattice Collaboration, a low-lying Roper state is observed in our full QCD analysis using the PACS-CS gauge fields made available via the ILDG. Our results for the Roper display significant curvature as the chiral regime is approached.

Keywords: Roper resonance, dynamical fermions, Lattice QCD
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Lattice field theory is the first principles approach to Quantum Chromodynamics (QCD). It provides a non-perturbative approach to the fundamental quantum field theory governing the properties of hadrons. For example, the ground-state hadron spectrum is now well understood. However, gaining knowledge of the excited-state spectrum presents additional challenges, as the excited state masses are extracted from the sub-leading exponentials.

The first positive parity excitation of the nucleon, the $N\frac{1}{2}^{+}(1440)\,P_{11}$ or Roper resonance, has been a subject of extensive interest since its discovery in 1964 through a partial-wave analysis of pion-nucleon scattering data [1].

In constituent quark models with a harmonic oscillator potential this P_{11} state (with principal quantum number $N = 2$) appears above the lowest-lying odd-parity S_{11} (1535) state [2, 3], whereas in Nature the Roper resonance is almost 100 MeV below the S_{11} state. Due to its surprisingly low mass, the P_{11} state has held enormous curiosity and speculation in the nuclear and particle physics community. For example, the Roper resonance has been described as a hybrid baryon state with explicitly excited gluon field configurations [4, 5], as a breathing mode of the ground state [6] or a state which can be described in terms of a five quark (meson-baryon) state [7].

Several attempts have been made in the past to find the elusive low-lying Roper state in the lattice framework, however a low-lying Roper state has not been observed. The difficulties lie in finding effective methods to isolate the energy eigenstates of QCD and in accessing the light quark mass regime of QCD.

The 'Variational method' [8, 9] is the state-of-the-art approach for determining the excited state hadron spectrum. It is based on the creation of a matrix of correlation

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functions in which different superpositions of excited state contributions are linearly combined to isolate the energy eigenstates. A low-lying Roper resonance was identified with this method using a variety of source and sink smearings in constructing correlation matrices [10, 11] in quenched QCD. Here we bring these effective techniques to the dynamics of full QCD to explore the low-lying even-parity states of the nucleon using 2+1-flavor dynamical QCD gauge-field configurations [12].

In constructing our correlation matrix for the nucleon spectrum, we consider the two-point correlation function matrix with momentum $\vec{p}=0$

$$G_{ij}^{\pm}(t) = \sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}} \{\Gamma_{\pm} \langle \Omega | \chi_i(x) \bar{\chi}_j(0) | \Omega \rangle\} = \sum_{\alpha} \lambda_i^{\alpha} \bar{\lambda}_j^{\alpha} e^{-m_{\alpha} t}, \tag{1}$$

where Dirac indices are implicit. Here, λ_i^{α} and $\bar{\lambda}_j^{\alpha}$ are the couplings of the interpolators χ_i and $\bar{\chi}_j$ at the sink and source respectively and α enumerates the energy eigenstates with mass m_{α}. $\Gamma_{\pm} = \frac{1}{2}(\gamma_0 \pm 1)$ projects the parity of the eigenstates.

Since the only t dependence comes from the exponential term, one can seek a linear superposition of interpolators, $\bar{\chi}_j u_j^{\alpha}$, such that

$$G_{ij}(t_0 + \triangle t)\, u_j^{\alpha} = e^{-m_{\alpha} \triangle t}\, G_{ij}(t_0)\, u_j^{\alpha}, \tag{2}$$

for sufficiently large t_0 and $t_0 + \triangle t$. This leads to solving left and right eigenvalue equations with eigenvectors v_i^{α} and u_j^{α}, respectively. The vectors v_i^{α} and u_j^{α} diagonalize the correlation matrix at time t_0 and $t_0 + \triangle t$ providing the projected correlator, $v_i^{\alpha} G_{ij}^{\pm}(t)\, u_j^{\beta} \propto \delta^{\alpha\beta}$.

The parity and eigenstate projected correlator

$$G_{\pm}^{\alpha} \equiv v_i^{\alpha} G_{ij}^{\pm}(t)\, u_j^{\alpha}, \tag{3}$$

is then analyzed using standard techniques to obtain the masses of different states.

The PACS-CS $2+1$ flavor dynamical-fermion configurations [12] are used in this analysis. These configurations use the non-perturbatively $\mathcal{O}(a)$-improved Wilson fermion action and the Iwasaki-gauge action [13]. The lattice volume is $32^3 \times 64$, with $\beta = 1.90$ providing a lattice spacing $a = 0.0907$ fm.

Five values of the (degenerate) up and down quark masses are considered, with hopping parameter values of $\kappa_{ud} = 0.13700, 0.13727, 0.13754, 0.13770$ and 0.13781, providing pion masses in the range of 702 - 160 MeV; for the strange quark $\kappa_s = 0.13640$. Ensembles of 350 configurations are considered for the four heavier quarks and for the lightest quark an ensemble of 198 configurations is used with a total of 750 fermion sources.

We consider three standard nucleon interpolators χ_1, χ_2 and χ_4 [14]. The correlation matrices are constructed using an extensive sample of different levels of gauge-invariant Gaussian smearing [15] at the fermion source and sink, including 4, 9, 16, 25, 35, 50, 70, 100, 125, 200, 400, 800 and 1600 sweeps. These levels of smearing correspond to rms radii in lattice units ($a \simeq 0.09$ fm) of 1.20, 1.79, 2.37, 2.96, 3.50, 4.19, 4.95, 5.920, 6.63, 8.55, 12.67, 15.47 and 16.00.

It is evident from the effective mass plot, $m(t) = \ln\{G_{ij}^{+}(t)/G_{ij}^{+}(t+1)\}$ in Fig. 1, that the variation in the superposition of excited state contributions is revealed in the

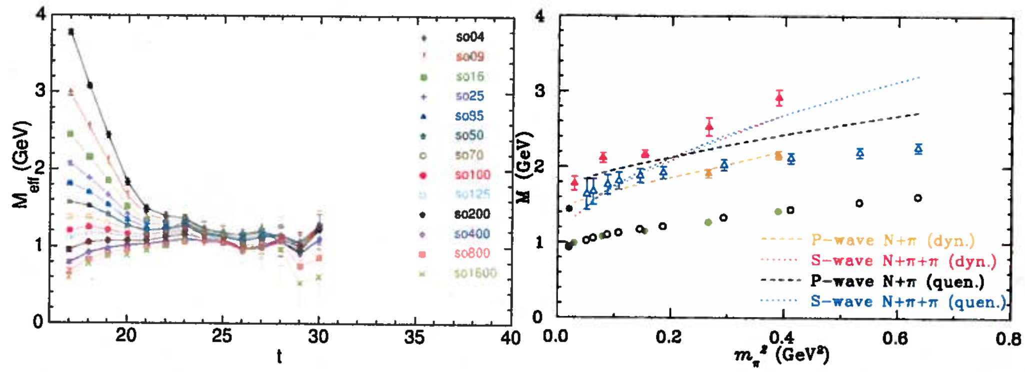

FIGURE 1. (Color online). (left) Effective mass from smeared-source to point-sink correlators for various levels of smearings at the source for $\kappa_{ud} = 0.13770$ ($m_\pi = 293$ MeV). (right) Ground and first excited (Roper) states of the nucleon are presented from dynamical and quenched QCD.

different approaches of the effective mass to the ground state plateau where the results converge. From this plot, it is clear that the correlation matrix analysis for excited state contributions will be most effective in the regime $t = 17-21$, following the source at $t = 16$.

In Fig. 1 (right), masses of the low-lying positive-parity states of the nucleon are presented with the scale set via the Sommer parameter [16]. The most significant result of this investigation is the manner in which the extracted Roper state (filled triangles) approaches the physical value. The significant curvature in the chiral regime indicates the important role played by mesonic dressings of the Roper. Although both the quenched (where the effects of dynamical quark loops are not considered) and full QCD results for the ground state nucleon are in reasonable agreement, significant differences are observed for the Roper in the light quark mass regime indicative of the effects of dynamical fermion loops in creating the mesonic dressings of the Roper.

In Fig. 2, results for different dimensions of the correlation matrices with different spin-flavor interpolators are presented. It is noted that, the full spin-flavor combination of the 12×12 correlation matrix is required in order to account for all the low-lying energy states.

A comparison of our results with the HSC Collaboration's results (using $2+1$ flavor anisotropic Clover lattice incorporating derivative operators) [17] for the nucleon mass spectrum reveals the same number of energy states below 3 GeV. The nature of the spectra are qualitatively in agreement. The s-wave $N\pi\pi$ scattering states, which have no volume dependence, are in excellent agreement. The volume dependent two particle p wave $N\pi$ scattering states are in accord with expectations. Therefore, it will be interesting to explore the first excited state on a large volume lattice.

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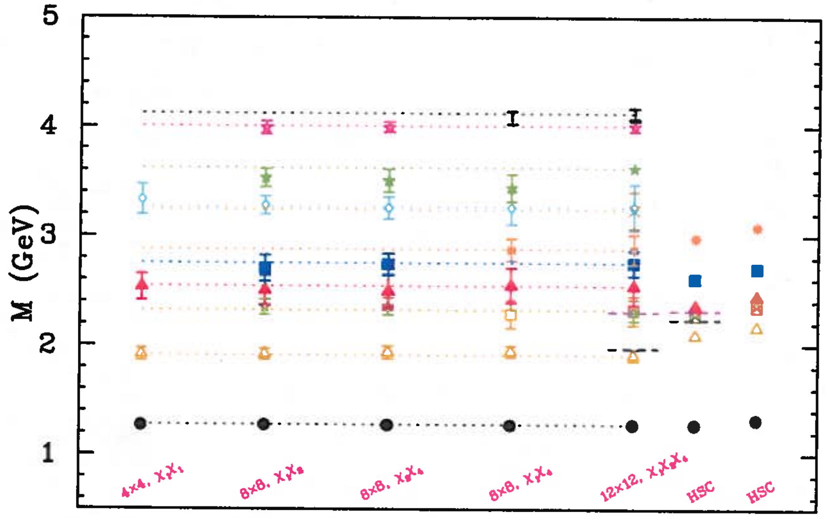

FIGURE 2. (Color online). Nucleon, $N\frac{1}{2}^{+}$, mass spectrum for different dimensions and different spin-flavor combination of interpolating fields, for the 2nd heaviest PACS-CS quark mass. Non-interacting p wave $N\pi$ and s wave $N\pi\pi$ threshold states are presented by short dashed lines. The HSC Collaboration results (Ref. [17]) of lattice volume $\sim (1.97\,\text{fm})^3$ are shown in right two columns, in which the left column corresponds to the rescaled results scaled to match our ground state mass. The horizontal dotted lines are reference lines.

REFERENCES

1. L. D. Roper, *Phys. Rev. Lett.* **12**, 340–342 (1964).
2. N. Isgur, and G. Karl, *Phys. Lett.* **B72**, 109 (1977).
3. N. Isgur, and G. Karl, *Phys. Rev.* **D19**, 2653 (1979).
4. Z.-p. Li, V. Burkert, and Z.-j. Li, *Phys. Rev.* **D46**, 70–74 (1992).
5. C. E. Carlson, and N. C. Mukhopadhyay, *Phys. Rev. Lett.* **67**, 3745–3748 (1991).
6. P. A. M. Guichon, *Phys. Lett.* **B164**, 361 (1985).
7. O. Krehl, C. Hanhart, S. Krewald, and J. Speth, *Phys. Rev.* **C62**, 025207 (2000), `nucl-th/9911080`.
8. C. Michael, *Nucl. Phys.* **B259**, 58 (1985).
9. M. Luscher, and U. Wolff, *Nucl. Phys.* **B339**, 222–252 (1990).
10. M. S. Mahbub, et al., *Phys. Lett.* **B679**, 418–422 (2009), `0906.5433`.
11. M. S. Mahbub, A. O. Cais, W. Kamleh, D. B. Leinweber, and A. G. Williams, *Phys. Rev.* **D82**, 094504 (2010), `1004.5455`.
12. S. Aoki, et al., *Phys. Rev. D* **79**, 034503 (2009), `0807.1661`.
13. Y. Iwasaki (1983), uTHEP-118.
14. D. B. Leinweber, R. M. Woloshyn, and T. Draper, *Phys. Rev.* **D43**, 1659–1678 (1991).
15. S. Gusken, *Nucl. Phys. Proc. Suppl.* **17**, 361–364 (1990).
16. R. Sommer, *Nucl. Phys.* **B411**, 839–854 (1994), `hep-lat/9310022`.
17. R. G. Edwards, J. J. Dudek, D. G. Richards, and S. J. Wallace (2011), `1104.5152`.