Twisted Analytic Torsion

Ryan Mickler

Supervisors: Prof. Varghese Mathai and Prof. Michael Murray

THE UNIVERSITY OF ADELAIDE

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Abstract

In [28], Mathai and Wu extended the notion of analytic torsion, as first conceived by Ray and Singer [34], to $\mathbb{Z}_2$-graded complexes. The main example of this is the de Rham complex with the flux-twisted differential $d_H = d + H$, where H is a closed three form, a complex that arises in geometric situations where there is twisting by a gerbe. We review the formalism required to construct this torsion, and present the key results. We generalise the analysis found in Farber [12] and Forman [14] to the $\mathbb{Z}_2$-graded situation to study the behaviour of the torsion of families of complexes near points at which the cohomology jumps. By studying analytical deformations of these complexes, we provide results showing that in some cases the torsion and some related invariants of this twisted operator are related to the untwisted torsion only through maps of a cohomological nature.

Signed Statement

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