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Numerical Modeling of Column Separation with Large Pressure Pulses

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Abstract

Liquid transients involving column separation occur in pipelines when the pressure drops to the vapor pressure of the liquid. The collapse of vapor cavities may result in short duration pressure pulses superimposed on water hammer waves, that exceed the Joukowsky or instantaneous valve closure pressure rise. This paper presents the analytical development of an improved mathematical model for describing column separation in pipelines. The model treats distributed vaporous cavitation regions and localized vapor cavities including intermediate cavities. The results from the numerical model are compared with some experimental results for a reservoir–pipeline–valve system.

Introduction

The purpose of this paper is to present the development of an improved numerical simulation modeling approach to quantitatively predict the occurrence of liquid column separation and vaporous cavitation in pipelines. The mathematical equations for the separate description of liquid column separations or intermediate cavities, and vaporous cavitation regions is presented. Intermediate cavities result from the interaction of two low pressure waves traveling in opposite directions, and for example, may be caused by the interaction of a pressure wave due to valve closure and a wave due to the later collapse of a vapor cavity at the valve. As a result, an intermediate cavity can form anywhere in the pipeline including horizontal reaches or pipes of constant slope. Previous investigators (Lupton [9], 1953, O'Neill [10], 1959 and Sharp [13], 1960) recognized the possibility of the formation of intermediate cavities associated with the solution of water hammer problems involving water column separation using the graphical method. However, the graphical method was not suited to describing water hammer involving distributed vaporous cavitation as noted by Knapp [6], 1939. The comparison of the results from the numerical model and the results from an experimental apparatus at The University of Michigan (Simpson [15], 1986) are also presented.

Literature Review

An extensive review of the literature relating to water hammer involving column separation was presented in a Ph.D. thesis by Simpson [15], 1986. A summary will be presented here. The 1930's marked the beginning of the intensive study of column separation that occurred during fluid transients or water hammer events. The dangers of the parting and abrupt rejoining of the water column related to high head penstock design were addressed by Billings at al. [4], 1933. Angus [1], 1935 presented one of the first mathematical models of a single localized vapor cavity at a boundary due to the rapid closure of a valve at the downstream end of a pipeline. A graphical method was presented for solving water hammer problems that involved column separation. Le Conte [8], 1937 presented some of the first experimental results for a local liquid column separation at a rapidly closing valve at the downstream end of a pipeline. Knapp [5], 1937 is believed to be the first investigator to recognize the possibility of the formation of distributed vaporous cavitation and clearly distinguishes it from local liquid column separations. Knapp [5], 1937 further clarified his concept of the formation of vaporous cavitation by presenting an example of a rarefaction wave traveling up a sloping pipe following a rupture in the pipe. The paper by Lupton [9], 1953, in which he describes the events associated with distributed vaporous cavitation, is an important contribution to the understanding of column separation.

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The growth and collapse of a vapor cavity at a boundary following column separation was investigated by Bergeron [2], 1939. He is believed to be the first researcher to detail the growth and collapse of the cavity in terms of a flow versus time. The time of existence of the cavity was recognized as being important to the subsequent pressure variations during the transient. The collapse of a boundary cavity at an exact multiple of $2L/a$ was compared with the collapse at a time that is not a multiple of $2L/a$. The possibility of the formation of cavities at both boundaries and high points in a pipe system have long been recognized. Lupton [9], 1953 introduced the possibility of the formation of "internal gaps" or intermediate cavities as the result of the interaction of two low pressure waves.

The 1960's marked the development of digital computer methods of water hammer analysis. Many different modeling approaches for column separation were proposed. More recent modeling attempts include the discrete free gas cavity models(Provoost [11], 1976, Provoost and Wylie [12], 1981), and combined cavity–vaporous cavitation modeling approach (Kranenburg [7], 1974 and Streeter [17], 1983). The discrete vapor cavity modeling approach remains a popular technique due to its simplicity of application. The combined cavity–vaporous cavitation approach of Streeter [17], 1983 forms the basis of the present modeling development presented in this paper.

Analytical Development

The analytical development of a proposed simulation model for liquid transients, which involves the formation of localized vapor cavities and distributed vaporous cavitation regions, is presented in this section. The model is referred to as the "combined cavity–vaporous cavitation model." Governing equations for intermediate cavities and distributed vaporous cavitation regions in each of the different flow regimes for pipe flow with column separation are presented. The locations of the interfaces that separate water hammer and distributed vaporous cavitation regions are determined in this modeling approach, and the equations for interface movement are presented. New interface conditions arise when intermediate cavities are considered, which necessitates the development of the governing equations for the collapse of both an intermediate cavity between two vaporous zones, and collapse of a boundary vapor cavity adjacent to a vaporous cavitation zone.

A different approach to modeling the distributed vaporous cavitation region has been developed, in which the direction of expansion of each separate zone is determined. This is an improvement upon the previous multi–zone vaporous cavitation model of Streeter [17], 1983. The new approach enables the use of an improved method for the computation of the incipient velocity in the vaporous cavitation region when vapor pressure head first occurs at a location in the pipe.

The Water Hammer Region

Modeling of the water hammer region uses the standard method of characteristics solution. Details of the application of this method appears in many standard textbooks (for example, Wylie and Streeter [19], 1978). The evaluation of the friction term using an integration by parts technique is detailed by Simpson [15], 1986.

Boundary Vapor Cavities and Intermediate Cavities

Prior to the introduction of digital computer techniques the graphical method did not lend itself to easy simulation of column separation. Consideration of only the ends of a pipeline system led to an oversight of the possible formation of intermediate cavities in the pipeline due to the interaction of two low pressure waves. O'Neill [10], 1959 and Sharp [14], 1965 pointed out that Bergeron's work ([2], 1939 and [3], 1961) did not account for an intermediate vapor cavity that formed away from the control valve, following the first collapse of the cavity at the valve. The presence of an intermediate cavity imposes an internal boundary condition in the pipe (O'Neill [10], 1959).

Once the pressure at a computing section is calculated to be below the vapor pressure of the liquid either a vaporous cavitation zone or an vapor cavity forms. The pressure head then becomes equal to the vapor pressure head of the liquid and becomes a constant pressure boundary condition in the pipeline. If the location of the column separation is at a valve or boundary of the system then a localized vapor cavity will begin to grow. The volume of the cavity grows and diminishes according to (Wylie and Streeter [19], 1978):

$$\mathcal{V}_v = \int_{t_{in}}^{t} -Q_{P_u} dt \tag{1}$$

in which $\mathcal{V}_v$ is the volume of the vapor cavity at the valve, t_{in} is the time of inception of the vapor cavity, and Q_{P_u} is the inflow to the valve.

Alternatively for a location in the pipeline away from the boundaries if the pressure head is computed to be below the vapor pressure head in a water hammer region then an intermediate cavity will form. The necessary condition for the formation of an intermediate cavity is for the pressure to drop to the vapor pressure of the liquid due to the interaction of two opposite traveling waves. Thus it is necessary to distinguish between the formation of an intermediate cavity and the expansion of an existing vaporous cavitation zone. A distributed vaporous cavitation zone develops when a single pressure rarefaction wave travels into a zone of decreasing pressure. An intermediate cavity is assumed to form if the pressure falls to the vapor pressure of the liquid within a waterhammer region. If an adjacent computational section was computed to drop to the vapor pressure head of the liquid at the previous time step then it is assumed that a distributed vaporous cavitation region has expanded to the section currently under consideration.

The growth of the intermediate cavity with time is represented by:

$$\mathcal{V}_v = \int_{t_{in}}^{t} (Q_P - Q_{P_u}) dt \tag{2}$$

in which $\mathcal{V}_v$ is the volume of the intermediate cavity, t_{in} is the time of inception of vapor at the intermediate cavity location, Q_{P_u} is the inflow on the upstream side of the section, and Q_P is the outflow from the section on the downstream side during the time of existence of the cavity.

Distributed Vaporous Cavitation Zones

A distributed vaporous cavitation zone has been distinguished from an intermediate vapor cavity in the section above. A distributed vaporous cavitation region usually extends over a long portion of the pipeline. The pressure in the pipeline in the vaporous cavitation region is equal to the vapor pressure head of the liquid. The void fraction of vapor content is usually much less than unity. A rarefaction wave traveling into a region of decreasing pressure causes the liquid to pull apart leaving adjacent liquid particles with slightly different velocities and thereby forms a vaporous cavitation zone.

The differentiation between the formation of an intermediate vapor cavity and the expansion of an existing vaporous cavitation zone has pointed to a new approach for the computation of the incipient velocity in an expanding distributed vaporous cavitation zone. The direction of the expansion of each distributed vaporous cavitation zone is determined in this new method and followed until the zone stops growing. The appropriate characteristic equation may now be used to compute the incipient velocity in the vaporous cavitation zone at the time just following the passage of the rarefaction wave that is responsible for the formation of the vaporous cavitation zone. This technique represents an improvement over the interpolation technique proposed by Streeter in his 1983 paper [17].

The governing equations for describing the velocity V_{mix} and variation of void fraction α_v in the vaporous cavitation region in an upward sloping pipe may be derived beginning with the general integral expressions for the conservation of mass and linear momentum. The continuity equation becomes:

$$\frac{d\alpha_v}{dt} = \frac{\partial V_{mix}}{\partial x} \tag{3}$$

Manipulating the equation for the conservation of momentum leads to:

$$\frac{dV_{mix}}{dt} = -\frac{f V_{mix} |V_{mix}|}{2D} - g \sin\theta \tag{4}$$

in which f is the Darcy–Weisbach friction factor, g is the gravitational acceleration, D is the pipe diameter and θ is the angle of the pipe with the horizontal. This equation has only one dependent

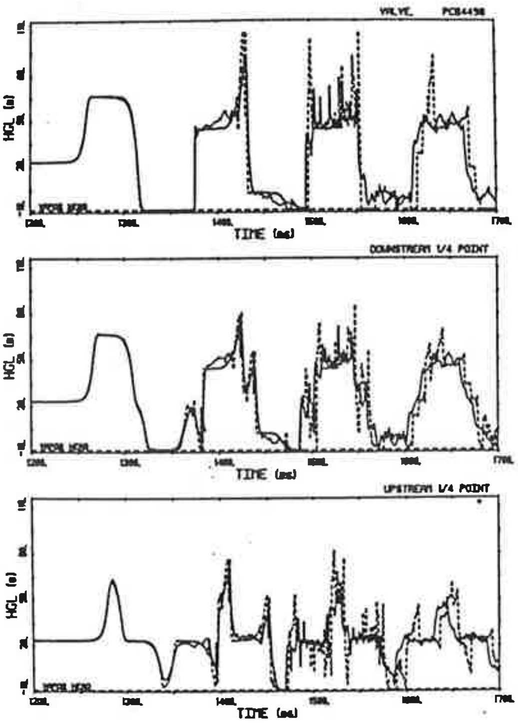

Figure 1: Pressure Head Variation at Valve, Downstream 1/4 Point and Upstream 1/4 Point for $V_o = 0.332\ m/s$, $H_R = 23.41\ m$, $h_f = 0.33\ m$, Numerical Model (dashed), Experimental Results (solid)

variable V_{mix} and is independent of the void fraction α_v. As a result, Equation 4 may be integrated to obtain the velocity of the vaporous mixture. Once the vaporous velocity has been determined the continuity equation (Equation 3) may be used to determine the void fraction.

Equations for Movement of Transition Separating Water Hammer and Cavitation Regions

A vaporous region expands in size by propagating into a water hammer region. Eventually the distributed vaporous cavitation region stops expanding and the boundary separating the water hammer and vaporous cavitation regions commences to move back into the cavitation region. A shock wave forms between the two regions, which penetrates the vaporous cavitation region. Shock conditions were derived by Wylie and Streeter [20], 1978, and Streeter [17], 1983, applying the principles of unsteady continuity and momentum across the shock. Manipulation of the equation for the conservation of mass leads to:

$$a_{sh} = \frac{V - V_{mix}}{\alpha_v + \frac{g}{a^2}(H_{sh} - H^v_{sh})} \tag{5}$$

in which a_{sh} is the celerity of the shock wave, V is the velocity on the water hammer side of the shock, a is the wave speed, H_{sh} is the head rise after the shock has passed. The general integral equation for the conservation of linear momentum becomes:

$$a_{sh} = \frac{g(H_{sh} - H^v_{sh}) + V^2 - V^2_{mix}}{V - V_{mix}} \tag{6}$$

These two equations may be solved to compute the celerity of the propagation of the shock or consolidation wave as well as the pressure head rise after the passage of the shock. Some of the basic equations for an analytical model have been presented. The details of the implementation of the numerical model are presented by Simpson [15], 1986.

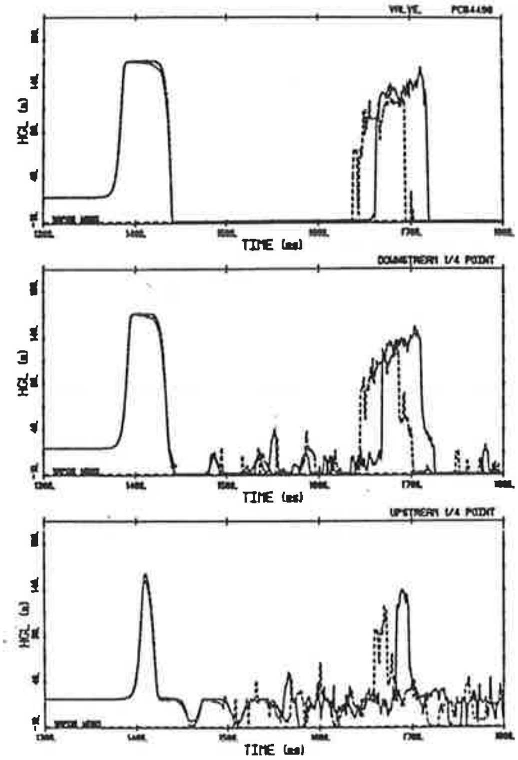

Figure 2: Pressure Head Variation at Valve, Downstream 1/4 Point and Upstream 1/4 Point for $V_o = 1.125\ m/s$, $H_R = 21.74\ m$, $h_f = 2.80\ m$, Numerical Model (dashed), Experimental Results (solid)

Comparison of Numerical Results with Experimental Results

The results of the combined cavity–vaporous cavitation model are compared to the experimental results of Simpson [15], 1986 and Simpson and Wylie [16], 1987. The experimental apparatus considered consists of a 36.0 meter long, 19.05 mm diameter copper pipe with an upward slope such that the ends have a 1.0 meter elevation difference. A tank that is capable of being pressurized by compressed air is located at the upstream end of the pipe . A fast–closing valve is located at the downstream end. Four transducers measured the pressure at 3 locations along the pipe. Piezoelectric quartz transducers were located at the upstream one–quarter point, downstream one–quarter point and at the valve. In addition a strain–gage transducer was also located at the valve. Experiments were conducted at The University of Michigan for a range of velocities. The results in this paper consider initial steady state velocities of 0.332 m/s and 1.125 m/s. The comparison between experimental results and numerical model are reasonable. The match of timing is especially good for the 0.332 m/s case. The large pressure pulse due to the collapse of a vapor cavity at the valve is predicted. The attenuation of the first pressure rise at the valve for the 1.125 m/s case is also predicted. Another common numerical model—the discrete cavity model does not predict this attenuation. Instead a large pressure rise is incorrectly predicted following the cavity collapse. More details of these studies will be presented in a follow–up paper.

Summary and Conclusions

The elements of an improved numerical model for column separation that describes localized vapor cavity formation including boundary cavities and intermediate cavities along with distributed vaporous cavitation has been presented. The results from the numerical model compare favorably with experimentally measured transients involving column separation for two different velocities.

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