

Magnetic Properties of the Nucleon in a Uniform Background Field

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Abstract

We present a calculation of the magnetic moment and magnetic polarisability of the nucleon. The calculation is performed using the background field method of lattice QCD. Dynamical results are from $32^3 \times 64$ configurations with 2+1 flavours of quark provided by the PACS-CS group through the ILDG. These lattices use a clover fermion action and Iwasaki gauge action with $\beta = 1.9$ and physical lattice spacing $a = 0.0907(13)$ fm. Quenched results come from $32^3 \times 40$ lattices using a FLIC fermion action and Symanzik improved gauge action with $\beta = 3.2$ and $a = 0.127$ fm.

The Landau energy is a crucial effect in the calculation of magnetic polarisabilities for charged particles. We derive the Landau levels and show their effect using examples of proton energy shifts in a background field.

Next we investigate the effects of moving the origin of the background gauge potential. This procedure looks similar to the technique of twisted boundary conditions, but we explain how for a quantised background field there is no change in the physical states, and show evidence using tree level calculations.

We present magnetic moment calculations for the proton and neutron, with a comparison between quenched and dynamical background field results as well as three point function results. We use the variational method in order to isolate excited states so that we can present results for the magnetic moment of the lowest lying odd-parity proton and neutron states.

Finally we present a calculation of the magnetic polarisability of the neutron. We investigate ways of improving the plateau behaviour of the energy shift, including the use of a variational analysis with a variety of source and sink smearings. Results are compared with experimental values.

Statement of originality

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Contents

1	Introduction	1
2	Quantum Chromodynamics	4
2.1	The Standard Model	4
2.2	Gauge field theory	5
2.3	The quark model	6
2.4	The QCD Lagrangian	7
2.5	Expectation values	8
3	Lattice QCD	10
3.1	Lattice gauge action	10
3.2	Lattice fermion action	12
3.2.1	Wilson action	13
3.2.2	Clover action	14
3.3	Correlation functions	15
3.4	Interpolating fields	17
3.5	Gauge field ensemble	18
4	The Background Field Method	20
4.1	Introduction	20
4.2	Formulation	21
4.3	Quantising the magnetic field	22
5	Simulation Details	28
5.1	Error analysis	30
6	Landau Levels	32
6.1	Introduction	32

6.2	Derivation	34
6.3	Examples	37
7	Twisted Boundary Conditions	43
7.1	Introduction	43
7.2	Twisted boundary conditions and momentum shifts	45
7.2.1	Examples	46
7.3	Twist-like phases with a background field	50
7.3.1	Explanation	54
7.4	Summary	57
8	Magnetic Moment	58
8.1	Introduction	58
8.2	Quark model prediction	59
8.3	Method	61
8.4	Results	64
8.4.1	Effective energies	64
8.4.2	Magnetic field dependence	68
8.4.3	Magnetic moment as a function of pion mass	69
8.5	Odd Parity Nucleon	72
8.5.1	Initial results	76
8.5.2	Variational method	78
8.5.3	Results	80
8.5.4	Effective energies	83
8.5.5	Fits to the field	89
8.5.6	Magnetic moment as a function of pion mass	93
8.5.7	Quark model prediction	98
9	Magnetic Polarisability	101
9.1	Introduction	101
9.2	Method	102
9.3	Results	104
9.3.1	Effective energies	104
9.3.2	Source smearing	104
9.3.3	Magnetic field dependence	113
9.3.4	Magnetic polarisability as a function of pion mass	114
10	Conclusion	118

A	Quenched Results	126
A.1	Simulation details	126
A.2	Magnetic moment	126
A.3	Magnetic polarisability	136

List of Figures

4.1	Top: Plaquette definition. Bottom: Background field plaquette.	24
4.2	Plaquettes at the edge of the lattice	25
4.3	Corner plaquette.	26
6.1	Example of Landau levels from quenched calculation.	38
6.2	Example of Landau levels from dynamical calculation.	40
7.1	Twisted boundary effective masses at tree level.	47
7.2	Neutron effective mass twisted boundary conditions comparison.	49
7.3	Background field and twisted boundary eff. masses at tree level.	51
7.4	Energies with background field and TBCs at different smearings.	53
7.5	Example 4×4 lattice with shifted origin background field. . . .	56
8.1	Neutron effective mass plot with background field on.	63
8.2	Spin-difference effective energy plot at $\kappa = 0.13700$	64
8.3	Spin-difference effective energy plot at $\kappa = 0.13727$	65
8.4	Spin-difference effective energy plot at $\kappa = 0.13754$	66
8.5	Spin-difference effective energy plot at $\kappa = 0.13770$	67
8.6	Neutron spin-difference mass shift vs field strength.	70
8.7	Proton spin-difference mass shift vs field strength.	71
8.8	Proton magnetic moment as a function of pion mass squared. .	73
8.9	Neutron magnetic moment as a function of pion mass squared.	74
8.10	Odd parity proton spin-difference energy shift.	77
8.11	Odd parity neutron spin-difference energy shift.	77
8.12	Eigenvector values from the odd-parity variational analysis. . .	82
8.13	Projected spin-down effective mass for the odd-parity neutron.	84
8.14	Odd parity proton spin-difference energy shift at $\kappa = 0.13700$.	85
8.15	Odd parity proton spin-difference energy shift at $\kappa = 0.13727$.	86
8.16	Odd parity proton spin-difference energy shift at $\kappa = 0.13754$.	87

8.17	Odd parity proton spin-difference energy shift at $\kappa = 0.13770$.	88
8.18	Odd parity neutron spin-difference energy shift at $\kappa = 0.13700$.	89
8.19	Odd parity neutron spin-difference energy shift at $\kappa = 0.13727$.	90
8.20	Odd parity neutron spin-difference energy shift at $\kappa = 0.13754$.	91
8.21	Odd parity neutron spin-difference energy shift at $\kappa = 0.13770$.	92
8.22	Odd parity proton spin-diff. mass shift vs field strength. . . .	94
8.23	Odd parity neutron spin-diff. mass shift vs field strength. . . .	95
8.24	Magnetic moment of the proton and its odd-parity excitations.	96
8.25	Magnetic moment of the neutron and its odd-parity excitations.	97
9.1	Spin-averaged neutron effective mass including bare mass. . .	103
9.2	Spin-averaged energy shift at $\kappa = 0.13700$	105
9.3	Spin-averaged energy shift at $\kappa = 0.13727$	106
9.4	Spin-averaged energy shift at $\kappa = 0.13754$	107
9.5	Spin-averaged energy shift at $\kappa = 0.13770$	108
9.6	Proton spin-averaged energy shift at $\kappa = 0.13700$	109
9.7	Spin-up and spin-down energy shifts at different smearings. . .	110
9.8	Spin-averaged energy shift with combined smearings.	111
9.9	Spin-averaged energy shift from variational analysis.	112
9.10	Spin-averaged mass shift vs field strength.	115
9.11	Neutron magnetic polarisability as a function of m_π^2	116
A.1	Quenched spin-difference energy shift at $m_\pi = 840$ MeV. . . .	128
A.2	Quenched spin-difference energy shift at $m_\pi = 775$ MeV. . . .	128
A.3	Quenched spin-difference energy shift at $m_\pi = 693$ MeV. . . .	129
A.4	Quenched spin-difference energy shift at $m_\pi = 626$ MeV. . . .	129
A.5	Quenched spin-difference energy shift at $m_\pi = 540$ MeV. . . .	130
A.6	Quenched spin-difference energy shift at $m_\pi = 435$ MeV. . . .	130
A.7	Quenched spin-difference energy shift at $m_\pi = 275$ MeV. . . .	131
A.8	Proton spin-difference energy shift vs field strength.	132
A.9	Neutron spin-difference energy shift vs field strength.	133
A.10	Quenched proton magnetic moment as a function of m_π^2	134
A.11	Quenched neutron magnetic moment as a function of m_π^2	135
A.12	Quenched spin-average energy shift at $m_\pi = 840$ MeV.	137
A.13	Quenched spin-average energy shift at $m_\pi = 775$ MeV.	137
A.14	Quenched spin-average energy shift at $m_\pi = 693$ MeV.	138
A.15	Quenched spin-average energy shift at $m_\pi = 626$ MeV.	138
A.16	Quenched spin-average energy shift at $m_\pi = 540$ MeV.	139

A.17 Quenched spin-average energy shift at $m_\pi = 435$ MeV.	139
A.18 Quenched spin-average energy shift at $m_\pi = 275$ MeV.	140
A.19 Neutron spin-average energy shift vs field strength.	141

List of Tables

5.1	Simulation details.	28
8.1	Neutron magnetic moment values for different fit windows. . .	68
8.2	Neutron magnetic moment values from 1 and 2 parameter fits.	70
8.3	Magnetic moment values for the proton and neutron.	73
8.4	Magnetic moment values for odd-parity states.	100
9.1	Neutron magnetic polarisability values from 1 and 2 parameter fits.	114