NON-LINEAR MODELS FOR EVALUATING THE RESIDUAL OPENING OF HYDRAULICALLY STIMULATED FRACTURES AND ITS IMPACT ON WELL PERFORMANCE

by

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Abstract

Hydraulic stimulation techniques have been employed successfully over the past 60 years to enhance the productivity of oil and gas reservoirs. These techniques work by injecting a pressurised fluid into the wellbore to initiate and propagate an artificial fracture or to open a network of existing fractures. These techniques are also commonly known as hydraulic fracturing or fracking. The main objective of hydraulic stimulation is to create highly conductive pathways, which can significantly increase the permeability of the reservoir and, subsequently, improve the well productivity. An injection of small particles (usually known as propping agents or proppants) with the fracturing fluid is the most common method to prevent the stimulated fractures from full closure during the production stage because of confining stresses.

To date, research has largely focused on the assessment of conditions and characteristics of fluid-driven fractures, as well as proppant transport and settlement mechanisms. The modern theory of hydraulic fractures is based on linear elastic fracture mechanics and theories of poro-elasticity, fluid flow in narrow openings and suspension flow in porous media. Despite numerous studies being carried out, few are devoted to the residual opening of hydraulic fractures, which has a significant effect on well productivity. There are many exciting potential applications and developments of hydraulic stimulation techniques for geothermal reservoirs and coal seam gas production. These all require new and more comprehensive theories, supported by analytical and numerical solutions capable of describing the non-linear effects of proppant placement and compressibility on the fracture residual opening profile and, ultimately, on the reservoir permeability and well performance.

In order to address these needs and gaps, this thesis aims to develop:

- a new mechanical model for predicting the mechanical response of saturated and unsaturated low-consolidated granular particles to compressive loading;
- a new mathematical method and non-linear solutions for evaluating the residual aperture of fractures partially filled with unconsolidated compressible particles (proppant) and subjected to compressive loading;
- a new mathematical model for evaluating the production rate of hydraulically stimulated wells taking into account the residual closure and various regions of distinct permeability along the fracture.

These new models are all based on the classical theories of solid, fluid, contact, fracture, rock and soil mechanics, which provide a framework for evaluating the residual opening profiles (aperture) of hydraulically stimulated fractures, as well as the influence of the fracture residual aperture on the well performance.

A number of simplifications are used to formulate the mathematical models and develop non-linear solutions. Many of these simplifications, such as two-dimensional problem geometry, plane strain conditions and linear elastic behaviour of the medium, represent a well-established foundation for analytical and numerical modelling in reservoir engineering. Accounting for other important phenomena, such as proppant flow-back and secondary cracking, is beyond the scope of this thesis but may be included in future work. The numerical results obtained within the developed models indicate that the residual openings and distribution of proppant along the fracture have a significant effect on well productivity (up to 50 per cent in the case of a relatively low level of confining stresses in the reservoir) and must be incorporated into the evaluation of the efficiency of hydraulic stimulation techniques and assessment of well productivity.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or tertiary institution without the prior approval of The University of Adelaide.

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Luiz Bortolan Neto

Thesis by Publication

This thesis is comprised of a combination of peer-reviewed publications and submitted journal articles in accordance with the Academic Program Rules 2013 of The University of Adelaide. The journals involved all deal with subject matter closely related to the research field of this thesis.

This thesis is based on the following publications:

1. Bortolan Neto, L, Kotousov, A & Bedrikovetsky, P 2011, 'Application of contact theory to evaluation of elastic properties of low consolidated porous media', *International Journal of Fracture*, vol. 168, no. 2, pp. 267–276. doi:10.1007/s10704-010-9574-6
2. Bortolan Neto, L, Kotousov, A & Bedrikovetsky, P 2011, 'Elastic properties of porous media in the vicinity of the percolation limit', *Journal of Petroleum Science and Engineering*, vol. 78, no. 2, pp. 328–333. doi:10.1016/j.petrol.2011.06.026
3. Bortolan Neto, L. & Kotousov, A 2012, 'On the residual opening of cracks with rough faces stimulated by shear slip', in A Kotousov, R Das & S Wildy (eds), *7th Australasian Congress on Applied Mechanics (ACAM 7)*, Engineers Australia, Adelaide, pp. 867–876.
4. Bortolan Neto, L & Kotousov, A 2012, 'Residual opening of hydraulically stimulated fractures filled with granular particles', *Journal of Petroleum Science and Engineering*, vol. 100, pp. 24–29. doi:10.1016/j.petrol.2012.11.014

5. Bortolan Neto, L & Kotousov, A 2013, ‘Residual opening of hydraulic fractures filled with compressible proppant’, *International Journal of Rock Mechanics and Mining Sciences*, vol. 61, pp. 223–230. doi:10.1016/j.ijrmms.2013.02.012
6. Bortolan Neto, L & Kotousov, A 2013, ‘On the residual opening of hydraulic fractures’, *International Journal of Fracture*, vol. 181, no. 1, pp. 127–137. doi:10.1007/s10704-013-9828-1
7. Bortolan Neto, L & Khanna, A 2013, ‘The performance of hydraulic fractures partially filled with compressible proppant’, *Australian Journal of Multi-disciplinary Engineering*, vol. 10, no. 2, pp 185–197. doi:10.7158/N13-AC08.2013.10.2

The following articles are relevant to the present work and are included as appendices:

8. Kotousov, A, Bortolan Neto, L & Rahman, SS 2011, ‘Theoretical model for roughness-induced opening of cracks subjected to compression and shear loading’, *International Journal of Fracture*, vol. 172, no. 1, pp. 9–18. doi:10.1007/s10704-011-9642-6
9. Bortolan Neto, L, Khanna, A & Kotousov, A 2013, ‘A new approach to evaluate the performance of partially propped hydraulic fractures’ in *APPEA 2013 Journal and Conference Proceedings*, vol. 53, pp. 355–362. APPEA 2013 Conference and Exhibition, Brisbane, Australia, APPEA and Media Dynamics.
10. Kotousov, A, Bortolan Neto, L & Khanna, A 2014, ‘On a rigid inclusion pressed between two elastic half spaces’, *Mechanics of Materials*, vol. 68, pp. 38–44. doi:10.1016/j.mechmat.2013.08.004
11. Khanna, A, Bortolan Neto, L & Kotousov, A 2014, ‘Effect of residual opening on the inflow performance of a hydraulic fracture’. *International Journal of Engineering Science*, vol. 74, pp. 80–90. doi:10.1016/j.ijengsci.2013.08.012

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Glossary

Bottomhole pressure: The pressure at the depth of the producing layer.

Breakdown pressure: The pressure at which a formation matrix fractures. The breakdown pressure is determined before establishing feasible reservoir stimulation techniques. Hydraulic fracturing treatments are performed above the breakdown pressure, whilst matrix treatments are carried out with the treatment pressure safely below the breakdown pressure.

Breaker: Any substance used to degrade the viscosity of polymer-based fracturing fluids, helping to enhance post-fracturing fluid recovery.

Damage zone: The area surrounding the wellbore that has been harmed by the drilling process.

Fluid leak-off: See *fluid loss*.

Fluid loss: The leakage of the stimulating fluid into the formation matrix.

Formation matrix: The rock mass around the borehole.

Fracture conductivity: The fracture capability to transmit fluids from the reservoir to the wellbore at the production stage.

Pay zone: A reservoir or portion of a reservoir that contains economically producible natural resources.

Penetration ratio: The ratio of the hydraulic fracture length to the equivalent reservoir length.

Percolation: Movement and filtering of fluids through porous materials.

Percolation limit: See *percolation threshold.*

Percolation threshold: The limit that defines the medium as frame supported or fluid supported. Below this threshold, no connections between solid particles exist and the medium is fluid supported. Above the percolation limit, the medium is frame supported because of the connections between solid particles.

Permeability: The ability of a porous medium to convey fluids.

Porosity: The percentage of pore volume or void space of a porous medium.

Pressure drawdown: The difference between the average reservoir pressure and the flowing bottomhole pressure.

Proppant: Portmanteau of the words ‘propping’ and ‘agent’. See *propping agent* for further details.

Proppant flow-back: Proppant being produced out of a hydraulically fractured well during the resource recovery phase.

Propping agent: Granular particles mixed with fracturing fluid to hold fractures open after a hydraulic fracturing treatment. In addition to naturally occurring sand grains, man-made or specially engineered proppants may also be used. Proppant materials are sorted for size and sphericity to provide an efficient conduit for production of fluid from the reservoir to the wellbore.

Reservoir: A subsurface body of rock with sufficient porosity and permeability to store and convey fluids.

Stimulation: A treatment carried out to reinstate or enhance well productivity. Stimulation treatments fall into two main groups: hydraulic fracturing treatments and matrix treatments. Fracturing treatments induce highly conductive flow paths between the reservoir and the wellbore. Matrix treatments are designed to restore the natural permeability of the reservoir following damage to the near-wellbore area.

Unconventional reservoir: Any reservoir that requires special recovery operations (e.g., stimulation treatments) outside the conventional operating practices. These operations must overcome economic constraints in order to make production from unconventional reservoirs monetarily viable.

Wellbore: The drilled hole or borehole, including the open-hole or uncased portion of the well.

Chapter 1

Introduction

Chapter 1

Introduction

1.1 Overview

Since the late 1940s hydraulic stimulation technologies have been employed by the oil industry to increase the permeability of hydrocarbon reservoirs, reduce production costs and speed up the investment return (Lhomme 2005; Nolte 2000; Rahim & Holditch 1995; Veatch & Moschovidis 1986). However, in the mid-1950s, hydraulic stimulation activities entered a long-term decline because of the sharp drop in oil prices and regulated gas price policies introduced in many western countries. A new rise of activities in this area resumed only in the mid-1970s, fostered by the world oil crisis and the natural gas shortage that affected the United States (US), which remains a major consumer of oil and gas (Economides & Nolte 2000; Lizardo & Mollick 2010). Nowadays, the application of these technologies has gone beyond oil reservoirs, and hydraulic stimulation techniques have been applied to improve permeability of geothermal reservoirs, recovery of coal seam gas (CSG), disposal of waste drill cuttings, fault reactivation in mining and measurement of *in situ* stresses (Adachi et al. 2007).

The conventional hydraulic fracture stimulation technique consists of injecting a pressurised fluid into a wellbore. This pressurised fluid acts as the driving force to separate the faces of interlocked natural joints or to initiate and propagate an artificial fracture. To keep artificial or natural fractures open after the stimulation stage, small particles (proppant) are usually injected with the fracturing fluid. At the production stage these particles are subjected to confining stresses, causing a reduction of the initial opening. The final aperture, which is strongly related to well productivity, is termed the ‘residual fracture opening’ (Barbour & Young 1980; Mayerhofer & Meehan 1998).

In many situations, application of hydraulic fracturing techniques does not automatically guarantee the expected result (an improvement of well performance). Such situations are particularly common for unconventional reservoirs, such as tight oil/gas, hot dry rock and coal-bed methane reservoirs (Nassir, Settari & Wan 2010). In these naturally fractured formations, proppant transport is limited and proppant injections are not very effective to keep fractures open far from the wellbore once fluid pressure is removed (Warpinski & Teufel 1987). Well completions have been reported to be successful (economically feasible) only for hydraulic fractures subjected to a high level of shear stresses for which little or no injection of proppants is normally required (Walker Jr. et al. 1998). This behaviour can be explained by the shear slip and roughness-induced opening mechanisms, which keep the fracture faces open at the production stage, having an effect similar to that of the propping agent (Mayerhofer et al. 1997).

As mentioned above, interest in hydraulic fracturing research was renewed in the mid-1970s when oil prices rocketed. During this period, oil industries across many countries actively supported further practical development, as well as theoretical investigations of mechanisms related to hydraulic fracture stimulations (Lhomme 2005). Many mathematical models were developed to predict the outcomes and guide the design of hydraulic fracturing stimulations. However, these models mainly focused on fracture initiation and propagation (Detournay & Carbonell 1997), fluid flow (Chekhonin & Levonyan 2012), fluid loss (leak-off)

(Mathias & Reeuwijk 2009) and proppant placement techniques (Mader 1989), proppant transport and settling (Clark 2006) and the flow-back phenomenon (Daneshy 2004; Parker, Weaver & van Batenburg 1999). At the same time, the existing mathematical approaches available to predict residual fracture openings (fracture openings after fluid pressure is removed) remain relatively primitive. These models normally disregard the mechanical response of the proppant as well as its distribution along the fracture length. The proppant is typically considered as incompressible and residual openings (fracture shapes) are usually approximated by elliptical or constant-thickness profiles (Entov & Murzenko 1994; Murzenko 1994; Kanevskaya & Kats 1996; Valkó & Economides 1996, 1998; Zazovskii & Todua 1990). However, the compressibility of the proppant and its distribution along the fracture can have a significant effect on the shape and aperture of stimulated fractures, which, in turn, can largely affect well productivity. The latter is confirmed by the present theoretical developments, along with a case study conducted for typical geological reservoirs (Bortolan Neto, Khanna & Kotousov 2013; Bortolan Neto & Khanna 2013; Khanna, Bortolan Neto & Kotousov 2014).

1.2 Summary of gaps

The significant financial benefits possible with hydraulic stimulation motivate further development and theoretical studies in this field. Further, the pursuit of renewable and environmentally friendly sources of energy has triggered the need for the development of non-conventional energy sources, such as shale gas, tight gas, coal-bed methane and geothermal energy. As an example, stimulated shale gas wells in the US in 2011 produced a total of 240,700 cubic metres (8.5 million cubic feet) of natural gas, which corresponds to an equivalent value of US$ 36 billion (Hassett & Mathur 2013). The well design of non-conventional reservoirs often incorporates hydraulic stimulations techniques (e.g., hydraulic fracturing or slick-water fracturing), which warrants further attention to this field.

Physical phenomena associated with hydraulic stimulation techniques are not fully described by existing mathematical models. For instance, even though the residual openings of hydraulic stimulated fractures strongly influence well productivity (Vincent 2002), past models largely disregarded this important phenomenon. The influence of proppant compressibility and its distribution along the fracture on fracture conductivity and well performance typically were ignored in previous analytical and numerical studies. Another important fracture opening mechanism specifically for geothermal and CSG reservoirs is the roughness-induced opening. This opening mechanism still lacks background theory and was previously modelled by adopting highly simplified approaches (e.g., Olsson & Brown 1993; Piggott & Elsworth 1991; Willis-Richards, Watanabe & Takahashi 1996). One exception could be the theoretical model of shear dilatation developed by Dyskin and Galybin (2001).

Further details of past studies and existing research gaps will be presented in Chapter 2. The aims and methodology for the proposed research addressing the identified gaps follow.

1.3 Main objective and methodology

The main objective of this research is to improve understanding of fracture opening mechanisms, as well as mechanical behaviour of unconsolidated compressible granular particles representing the propping agent often used in hydraulic stimulations. This understanding is necessary to develop a mathematical framework capable of guiding the design and application of hydraulic stimulations for both conventional and non-conventional reservoirs, including geothermal energy and CSG.

To achieve this objective, fracture-opening mechanisms, which control residual opening profiles after removal of pressurised fluid, will be analysed. This analysis will allow formulation of mathematical models and generation of the governing equations based on the theories of solid, fracture, contact, rock, soil, and fluid mechanics. The governing equations of the fracture opening mechanisms will be solved via the distributed dislocation technique (DDT) (Hills et al. 1996), which is often employed in fracture mechanics for analysis of crack problems.

Two propping mechanisms are considered in this thesis: those due to (1) roughness-induced opening and those due to (2) finite compressibility of low consolidated proppant pack distributed inside the fracture during the injection stage of the stimulation. These mechanisms will be investigated with the help of Goodman's (1976) asperity deformation model, Terzaghi's classical soil consolidation theory (Terzaghi, Peck & Mesri 1996), and a novel approach, developed in the present thesis by employing Hertz's (1896) contact theory. Finally, residual openings profiles will be coupled with a fluid flow model to evaluate the effect of various parameters and proppant distribution on the performance of hydraulically stimulated wells.

The complexity of the formulated problem does not allow an analytical formula or closed-form solution to be obtained, instead requiring the application of a numerical approach. Therefore, the solution procedure will employ the Gauss–Chebyshev quadrature and the Newton–Raphson iterative method. These approaches have been selected as the most simple, suitable and reliable for obtaining numerical solutions to the governing equations, which represent a system of non-linear singular equations with Cauchy-type kernel (Hills et al. 1996).

1.4 Specific aims and details of the publications

Several chapters of this thesis represent articles published by the candidate. The thesis is logically divided into three main parts. The first part (represented by Chapters 4 and 5) is dedicated to the analysis of the mechanical response of compressible low consolidated particles to compressive loading. The evaluation of the residual opening profiles of hydraulic fractures partially filled with compressible particles is the focus of the second part (Chapters 6 to 9). The third and final part of this thesis (Chapter 10) investigates the effect on well productivity of the fracture residual opening profile with distinct regions of conductivity.

The three parts of the thesis are cohesive and interconnected. The compressive behaviour of the proppant pack (low consolidated particles) studied in Part 1 affects the residual opening of the hydraulic fracture, which is considered in Part 2. The fracture residual aperture has a strong influence on the conductivity of the hydraulic fracture and, consequently, on the well performance, which is investigated in Part 3. Subsections 1.4.1 to 1.4.3 provide a brief overview of the specific aims and methodologies of each individual part and the link between the specific aims and main objective of this research.

1.4.1 Theory of low consolidated porous media

A large portion of natural resources is trapped in low consolidated reservoirs (Hagin & Zoback 2007). The particles that are typically injected into a hydraulic fracture or natural fracture network (Kern, Perkins & Wyant 1959) also form a low consolidated granular medium (Geehan et al. 1999). Therefore, the mechanical behaviour of a low consolidated pack of compressible particles is of significant interest in the area and is the focus of the first part of the thesis.

In the first part of this thesis the theory of packing of spherical particles and Hertz's contact theory (Hertz 1896; Johnson 1985) are exploited to derive the compression elastic modulus of a low consolidated pack of spherical particles with various packing arrangements. The effective medium is assumed to be formed by particles of constant size during a long-term consolidation process. The developed model takes into account the percolation limit and the initial and final porosities of the pack. Hertz's (1896) contact theory is considered quite accurate when the ratio of the contact radius to the radius of the spherical particles is sufficiently small (Johnson 1985). The unsaturated (dry) case of low consolidated pack is examined first, followed by the theoretical model describing mechanical behaviour of a low consolidated pack saturated with a fluid (water, brine). The results are compared with well-established micromechanical theories, as well as experimental data available from the literature. The latter are in good agreement with the developed theoretical models.

The publications that resulted in Chapters 4 and 5 are summarised below.

Chapter 4: Application of contact theory to evaluation of elastic properties of low consolidated porous media.

At the beginning of the Chapter 4, the diagenesis process (Foscolos, Powell & Gunther 1976) is introduced along with the theory of packing of spherical particles (Panayiotopoulos 1989). The bulk modulus of elasticity of the porous media relationship with the elastic properties of the particles and packing structure (particles arrangement, medium porosity, percolation threshold) is then derived, based on the classical Hertz's contact theory (Hertz 1896; Johnson 1985). The theoretical results are compared with experimental data (Nur et al. 1998) and with various popular micromechanical theories, such as the rule of mixtures (Nakamura, Wang & Sampath 2000), the self-consistent method (Eshelby 1957; Hill 1965), the Hashin–Shtrikman bounds (Hashin & Shtrikman 1961), and the Mori–Tanaka theory (Mori & Tanaka 1973). It is shown that micromechanical models fail to provide a reasonable assessment of mechanical behaviour of low consolidated pack of particles while the developed theory gives results in good agreement with experimental studies. Some differences between the obtained results and the experimental data are attributed to various limitations of the classical contact theory, as well as the exclusion of other important phenomena that might influence the elastic properties of the overall medium, such as non-elastic mechanisms of consolidation (e.g., creep), anisotropy and variation in size and shape of particles.

Chapter 5. Elastic properties of porous media in the vicinity of the percolation limit.

A generalisation of the theoretical method for a saturated low consolidated granular pack, which takes into account the fluid compressibility, is presented in

Chapter 5. Terzaghi's principle (Barenblatt, Entov & Ryzhik 1990) is used for the development of the relationship between the compressive response of the pack and the elastic properties of the particles, fluid compressibility and packing structure (particle arrangement, medium porosity, percolation threshold). The derived relationship is compared with several micromechanical theories – the rule of mixtures (Nakamura, Wang & Sampath 2000), the self-consistent method (Eshelby 1957; Hill 1965), the Hashin–Shtrikman bounds (Hashin & Shtrikman 1961), and the Mori–Tanaka theory (Mori & Tanaka 1973) – and experimental studies (Nur et al. 1998). Based on the comparison, it is concluded that the developed model provides the best evaluation of the mechanical properties of a low consolidated porous medium for saturated conditions near the percolation threshold. Other effects mentioned above (such as non-elastic mechanisms of consolidation, variation in size and shape of particles, anisotropy, fracture and repacking processes) may be incorporated into the developed models and could be the subject of further investigation.

1.4.2 Evaluation of the residual opening of hydraulic fractures

A new computational approach to calculate the residual opening of hydraulic fractures is developed in the second part of the thesis. This numerical approach is based on the DDT and can account for arbitrary models of the compressive behaviour of the proppant. It can also be applied to evaluate opening profiles because of the roughness-induced fracture opening mechanism.

The solution method employs the Gauss–Chebyshev quadrature to translate the analytical equations delivered within the DDT framework into a system of non-linear algebraic equations. The latter are solved numerically. The solution method is validated by a comparison with previous analytical results obtained by Cox and Rose (1996). These authors used a similar mathematical formulation to model the non-linear behaviour of composite patching repair of fatigue cracks. A very good agreement between the present method and the previously developed analytical procedure is observed, which validates the adopted solution method.

The residual fracture opening is studied considering the following four different cases (scenarios):

- a crack subjected to compression and shear slip in the presence of asperities on the crack faces (Chapter 6). The deformation of the asperities is modelled by Goodman's (1976) equation;
- a hydraulic fracture fully filled with a propping agent and subjected to compression (Chapter 7). Terzaghi's consolidation theory (Terzaghi, Peck & Mesri 1996) is used to simulate the proppant compressive behaviour;
- a hydraulic fracture partially filled with a proppant agent (Chapter 8). This chapter focuses on the investigation of the combined effect of the proppant distribution and compressibility on the residual fracture openings;
- a hydraulic fracture partially filled with a compressible pack of low consolidated particles (Chapter 9). The compressive response of the particles is modelled within the new theory of low consolidated porous medium developed in Chapters 5 and 6.

The tools developed in Chapters 6 to 9 can be used in practical situations to assist the evaluation of hydraulic fracture conductivity and well productivity. One possible application is discussed in Chapter 10. Detailed information regarding the background theory and the specific goals of each article in Chapters 6 to 9 is provided next.

Chapter 6: On the residual opening of cracks with rough faces stimulated by shear slip.

Chapter 6 develops a new advanced theoretical model to evaluate the residual opening profile of fractures subjected to compression and shear stresses because of the slippage and presence of asperities on the crack faces. The governing equations are derived based on the DDT (Hills et al. 1996). Goodman's (1976) model is embedded into the DDT formulation to appropriately simulate the mechanical behaviour of asperities under compression. The solution approach is based on the Gauss–Chebyshev quadrature method.

The developed model is then validated against previous analytical results published by Cox and Rose (1996). These results are related to a different problem: the composite patching repair of fatigue cracks in aluminium plates. The comparison is possible because of the same mathematical formulation of both problems. A significant part of Chapter 6 is dedicated to the analysis of the deformations of the fracture asperities, as well as their effect on the fracture aperture and stress intensity factors. The developed model can be further used to evaluate the fracture conductivity and the effect of shear slip stimulation on the well productivity

Chapter 7: Residual opening of hydraulically stimulated fractures filled with granular particles.

The compressibility of the proppant pack is expected to largely determine the residual opening of hydraulically stimulated fractures (Cutler et al. 1985; Montgomery & Steanson 1985). Nonetheless, the modelling of the residual opening of hydraulic fractures still relies on overly simplified assumptions of constant or elliptical shapes of fracture opening and disregards the proppant pack compressibility. Therefore, Chapter 7 presents a new mathematical model for evaluation of the residual opening profiles of hydraulic fractures, taking into account the proppant pack compressibility. The DDT (Hills et al. 1996) is used to derive the governing equations of the problem. The mechanical behaviour of a pack of unconsolidated particles (Woodworth & Miskimins 2007) is modelled using Terzaghi's soil consolidation theory (Terzaghi, Peck & Mesri 1996), which is incorporated into the DDT formulation. The Gauss–Chebyshev quadrature method is employed to solve the governing equations. Some analytical results are used for the validation of the developed model (Cox & Rose 1996).

The influence of proppant pack compressibility on the hydraulic fracture residual opening and stress intensity factor is investigated in detail. For the case of a fracture fully filled with a nearly incompressible proppant pack, a small change in the fracture opening is expected. In contrast to the above particular case, for a fracture filled (partially or not) with a highly compressible proppant pack, the residual fracture opening shall be insignificant. These two limiting cases are analysed to demonstrate that the developed model complies with the expected physical behaviour.

The model presented in Chapter 7 disregards the influence of the proppant distribution inside the fracture on the residual aperture, which can also affect the fracture conductivity (Cipolla et al. 2009). This issue prompted further development of the model.

Chapter 8: Residual opening of hydraulic fractures filled with compressible proppant.

The proppant transport and settling during the hydraulic fracture stimulation stage is complicated and the final proppant distribution inside the fracture is non-uniform (Cipolla et al. 2009; Kern, Perkins & Wyant 1959). Motivated by this fact, Chapter 8 presents a more comprehensive mathematical model capable of accommodating the effect of the proppant distribution inside the hydraulic fracture on the residual opening profile. The solution approach is based on the Gauss–Chebyshev quadrature and the Newton–Raphson iterative scheme. The iterative procedure is necessary, as the problem is highly non-linear. The numerical results are partially validated against previous analytical and numerical studies.

The model developed in this chapter allows the evaluation of the combined effect of the proppant pack compressibility and distribution on the residual opening of hydraulic fractures. It was observed for typical parameters of hydraulic stimulations and proppant properties that the residual opening profile is heavily influenced by the proppant compressibility and its distribution. Thus, the use of simplified profiles of residual fracture openings, as used in many previous studies, can lead to erroneous predictions of the productivity of hydraulically fractured wells (Cooke Jr. 1973; Davies & Kuiper 1988).

Chapter 9: On the residual opening of hydraulic fractures.

Chapter 9 further extends the developed method. The proppant response model used in Chapter 9 is based on the equations from the theory of low consolidated porous media developed in Chapters 4 and 5, which was specifically developed to describe the compressive response of a low consolidated pack of particles. The numerical solution of the problem, as in the previous studies, is based on the Gauss–Chebyshev quadrature.

The low consolidated proppant pack distribution and compressibility effect on the hydraulic fracture residual profile and stress intensity factor is investigated in detail. This chapter also analyses the proppant pack response to the confining stresses and offers some insights in to how this response may affect the proppant flow-back phenomenon. The results suggest that the partial filling of hydraulic fractures might reduce proppant back-production.

1.4.3 Investigation of the fracture residual opening influence on well productivity

Chapter 10: On the performance of hydraulic fractures partially filled with compressible proppant.

The final part of this thesis focuses on the evaluation of the productivity of hydraulically fractured wells. It specifically focuses on the effect of the residual fracture opening profiles on the productivity index, which is often used to characterise the efficiency of hydraulic stimulations and well performance. The assessment of well productivity is conducted by incorporating the shape of the residual opening profile calculated with the approach developed in Chapter 8 into the model of fluid flow in narrow fracture channels originally developed by Zazovskii and Todua (1990). The latter model was extended significantly to accommodate the proppant distribution, as well as the distinct permeability zones inside the hydraulic fracture.

Various scenarios of proppant pack distribution and compressibility are analysed. The calculated residual aperture of the hydraulic fracture for each scenario is used to find the productivity index. The combined model is then applied to conduct two case studies. The first case investigates the effect of the distribution of the proppant on the productivity index and the second focuses on the coupled influence of proppant distribution and compressibility on well productivity. It is verified that the fracture residual aperture significantly affects fracture conductivity and, consequently, well performance.

The investigations performed also imply that a hydraulic fracture with low penetration ratio would have its productivity maximised if it were fully filled with proppant. Moreover, it is observed that for situations of constant proppant pack permeability, fracture conductivity exhibits an inverse linear relationship with proppant pack compressibility. This observation confirms the benefits of reducing proppant pack compressibility, a common practice (Mader 1989) for improving well productivity.

The case studies also suggest that in some situations (relatively low confining stresses, for example), decreasing fracture propped length may increase well production rate. This phenomenon can take place because of the poorer permeability of the proppant pack in comparison with the un-propped region of the fracture.

1.5 Organisation of the thesis

The previous sections described the motivation, objectives and methodology of this research. The research background and a brief overview of the existing literature regarding the modelling of hydraulic stimulations are presented in Chapter 2. A brief introduction to the theory of linear elastic fracture mechanics and to the DDT is given in Chapter 3. Chapters 4 to 10 represent a cohesive set of articles published, accepted or submitted for publication, which describe the theoretical framework and the main outcomes of the research. The articles included in this thesis demonstrate the development, implementation and validation of the new approach for evaluation of the effect of residual openings on well productivity. The conclusions and main contributions delivered by this thesis are discussed in Chapter 11 together with some suggestions for possible applications and future work.

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Chapter 2

Basic Aspects of Design and Efficiency Evaluation of Hydraulic Stimulations

Chapter 2

Basic Aspects of Design and Efficiency Evaluation of Hydraulic Stimulations

2.1 Introduction

Hydraulic fracturing technologies are widely employed by the oil and gas industry to allow or increase hydrocarbon production from low-permeability reservoirs, maximising the investment return (Daneshy 2010). A standard hydraulic stimulation treatment starts with the initiation of a crack because of the pressure exerted by a stimulating viscous fluid pumped into the wellbore, as illustrated in Figure 2.1. This crack propagates until reaching a hydrocarbon-bearing layer (pay zone). Usually, the viscous fluid alone is injected first to open the fracture sufficiently to allow ingress of granular particles added to the stimulating fluid in subsequent mining stages. These granular particles are termed proppants (propping agents) and are responsible for preventing the fracture from full closure after pumping stops and the injected fluid leaks off to the surrounding rock formation at the end of the fracturing treatment. The created fracture forms a highly permeable path that facilitates the flow of fluids from the reservoir towards the wellbore.

The successful hydraulic fracturing completions of reservoirs located in porous rock formations (sandstones, limestones, dolomite formations) encouraged the application of this technique in reservoirs with geological discontinuities, such as shale gas, tight oil/gas, coal-bed methane and hot dry rocks. Early attempts of applying hydraulic fracture techniques to these new fields have often failed to achieve the expected outcomes, leading to financial losses. This is because geological discontinuities (natural fractures) induce the propagation of various short and non-continuous cracks, rather than a single massive fracture, thereby preventing and limiting proppant mobility (Warpinski & Teufel 1987). This fact motivated the development of new techniques of hydraulic stimulation (Mayerhofer et al. 1997; Walker Jr. et al. 1998), usually called slick-water fracturing or roughness-induced opening, where little or no injection of proppant is required to support the openings (Kotousov, Bortolan Neto & Rahman 2011; Palisch, Vincent & Handren 2010; Zhang, Jeffrey & Thiercelin 2009).

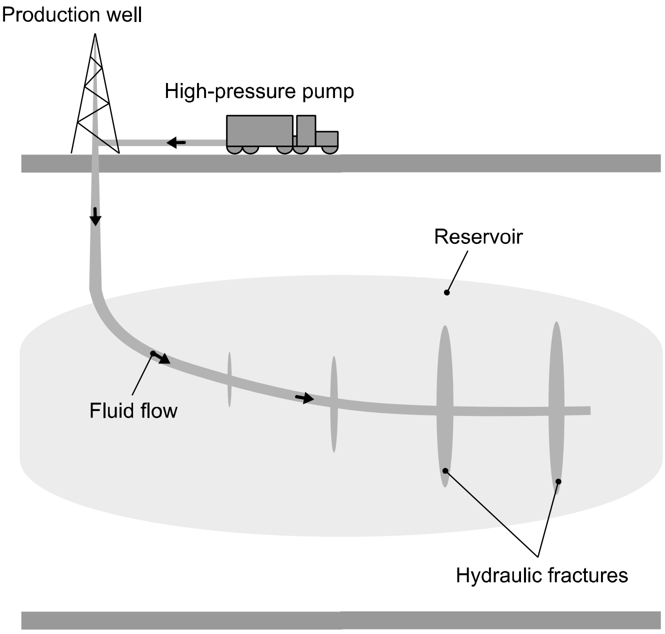

Figure 2.1: Fluid injection for initiating and propagating fractures in a reservoir.

Hydraulic fracturing is a highly complex phenomenon. The coupling of various physical processes complicates the modelling of hydraulic stimulation techniques. Hydraulic fracture modelling is often based on theories of linear elasticity (rock deformation), lubrication theory (fluid flow), Stokes' law (proppant transport and settling) and linear elastic fracture mechanics (crack initiation and propagation) (Adachi et al. 2007; Dayan, Stracener & Clark 2009). This chapter provides an overview of the most relevant work done thus far regarding:

- initiation and growth of the hydraulic fracture during the stimulation stage;
- flow and leak-off of the fracturing fluid;
- proppant transport, settlement and distribution inside the hydraulic fracture;
- physical properties of proppants and low-consolidated porous media;
- roughness-induced opening of natural fractures;
- residual opening and conductivity of hydraulic fractures;
- performance of hydraulically fractured wells;
- development of fracturing materials;
- environmental risks of hydraulic fracturing.

Other important aspects of hydraulic fracturing, such as formation damage (Rahman, Rahman & Rahman 2002), damage accumulation (Taylor, Chen & Kuszmaul 1986), secondary cracking (Aghighi & Rahman 2010) and proppant flow-back (Daneshy 2004), are very important and can be considered in the future work. Nevertheless, these aspects are beyond the scope of this thesis.

2.2 Fracture initiation and propagation

The first attempt to estimate the breakdown pressure, namely the fluid pressure required to initiate and drive a longitudinal fracture into the rock formation, was a model developed by Hubbert and Willis (1957). Later, this model was improved by the inclusion of Terzaghi's effective stress theory, enabling the modelling of porous permeable materials (Haimson & Fairhurst 1967).

For a long time, Haimson and Fairhurst's (1967) model was considered state of the art in the prediction methodology of the breakdown pressure. However, as this model was not capable of describing many observed phenomena (such as the unusual high breakdown pressures of some field treatments), more refined and adequate models were later suggested (Guo, Morgenstern & Scott 1993). A more generalised version of Haimson and Fairhurst's (1967) model was conceived by replacing Terzaghi's effective stress theory with Biot's theory of poro-elasticity (Schmitt & Zoback 1989). However, this change was not sufficient to foresee pressurisation rate and borehole size effects because it overlooked the first phase of fracture growth, that occurring between initiation and breakdown. To overcome this problem, a number of more sophisticated models were developed (Detournay & Carbonell 1997; Ito & Hayashi 1991). The need to distinguish three specific pressures (breakdown, initiation and unstable fracture propagation pressure) when considering the propagation of fluid driven fractures has also been discussed (Detournay & Carbonell 1997). This is an important issue to be considered, since there is evidence (Zoback et al. 1977) that fracture propagation may begin before the breakdown pressure is reached.

Analytical models to simulate fracture growth have been proposed by a number of researchers (Garagash 2006a, 2006b; Garagash & Detournay 2005; Germanovich et al. 1995). Many studies investigated the influence of micro-cracks (Dyskin & Germanovich 1993), stress concentration (Yang et al. 1997) and wellbore orientation (Hossain, Rahman & Rahman 2000) on hydraulic fracture initiation and growth. Considerable attention has also been devoted to development of numerical algorithms to simulate three-dimensional propagation of hydraulic fractures in layered strata characterised by different mechanical properties and/or *in situ* stresses (Advani, Lee & Lee 1990; Clifton & Abou-Sayed 1981; Hossain & Rahman 2008; Peirce & Siebrits 2001; Shah, Carter & Ingraffea 1997; Siebrits & Peirce 2002; Sousa, Carter & Ingraffea 1993).

2.3 Flow and leak-off of the stimulating fluid

The theory of fluid flow in fractures often assumes the stimulating fluid to be incompressible and Newtonian. This flow is usually described by lubrication equations (Adachi, Detournay & Peirce 2010; Chekhonin & Levonyan 2012; Zimmerman & Bodvarsson 1996), namely local and global continuity equations and the Poiseuille law (Batchelor 2000). These equations can be derived from the Navier–Stokes theory by supposing unidirectional flow, disregarding inertial effects (small Reynolds number) and accounting for the zero tangential velocity of fluid at the fracture surfaces (no-slip condition) (Chekhonin & Levonyan 2012). If fluid is injected at a constant flow rate and pressure, the dominant phenomenon affecting crack propagation is the fluid flow into the rock formation – a phenomenon commonly known as fluid leak-off.

Fluid leak-off is essentially a penetration process controlled by many factors, such as fluid composition, viscosity, flow rate and pressure and reservoir properties (e.g., permeability, pressure, temperature, fluid saturation, pore size, micro-fractures) (Constien et al. 2000). As it was pointed out by Adachi et al. (2007), up to 90 per cent of the fracturing fluid may flow into the reservoir during the injection stage. This fact highlights the importance of considering fluid leak-off when designing hydraulic stimulation developments to obtain accurate estimates of, for example, the total fluid volume to be injected, fluid flow and pressure inside the fracture, fracture propagation rate and initial geometry.

Carter's (1957) simple model of fluid diffusion through fracture walls is the standard approach for simulating fluid leak-off (Carter et al. 2000; Mathias & Reeuwijk 2009). This model assumes that flow is one-dimensional and normal to the fracture faces. In addition, it accounts for a negligible difference between the fluid pressure in the fracture and the pore pressure. Carter's (1957) model assumptions are acceptable as long as the fracture propagates sufficiently fast, rendering any non-orthogonal leak-off insignificant (Adachi et al. 2007). Otherwise, Carter's (1957) model might lead to overestimations of the fracture length (van den Hoek 2000). However, laboratory tests show that, in highly permeable reservoirs, the leak-off flow is not orthogonal (e.g., Khodaverdian & McElfresh 2000). Therefore, aiming to overcome the limitations of Carter's (1957) model, several new approaches accounting for transient flow (Gringarten, Ramey Jr. & Raghavan 1974), highly permeable formations (Ispas et al. 1998; Valkó & Economides 1997; van den Hoek 2000) and naturally fractured reservoirs (Li et al. 2007) were proposed. Two- and three-dimensional formulations were also used in past studies (Gordeyev & Entov 1997; Hagoort, Weatherill & Settari 1980; Mathias & Reeuwijk 2009; Settari 1980, 1985).

2.4 Fracture geometry during hydraulic stimulation

Early single planar fracture models for hydraulic fracturing have been developed to estimate measurable parameters assuming fracture geometry in a homogeneous setting. Such models are commonly employed because of their simplicity and have been thoroughly reviewed (Adachi et al. 2007; Economides & Nolte 2000; Mahrer 1999; Rahman & Rahman 2010; Warpinski et al. 1994). Two of the earliest and most popular approaches to predict fracture geometry (length, height, opening) are the PKN (Nordgren 1972; Perkins & Kern 1961) and KGD (Geertsma & de Klerk 1969; Khristianovic & Zheltov 1955) models. The PKN model provides acceptable predictions when fracture length is much larger than fracture height, whereas adequate estimates are given by the KGD model in situations when fracture length is much smaller than fracture height (Economides, Hill & Ehlig-Economides 1994). The main drawback of these models is that they rely on extremely idealised conditions, in which fracture height must be accurately estimated beforehand and is assumed to be constant along the fracture length (US EPA 2004).

To overcome the limitations of the PKN and KGD models, pseudo-three-dimensional (P3D) approaches were introduced. In these approaches the fracture height is a function of both time and position along the fracture. Modern P3D models are also capable of modelling fractures extended over various rock layers and under different stress conditions (Adachi et al. 2007).

Full three-dimensional (3D) approaches have also been proposed to model hydraulic fractures of arbitrary shape and orientation, since two-dimensional (2D) and P3D approaches do not have this capability. Although 3D models allow the full analysis of fracture propagation and geometry, a significant amount of data is required to justify their use. Besides, 3D analyses demand a considerable computational effort and even 3D models require the adoption of an idealised geometry to derive effective and useful solutions (Adachi et al. 2007).

A comprehensive comparative study of various 2D, P3D and 3D simulators was conducted by Warpinski et al. (1994), highlighting the major differences between these models.

2.5 Proppant transport, settlement and distribution

In their ground-breaking work, Kern, Perkins and Wyant (1959) reported findings regarding experiments on proppant (sand) transport and settling near the wellbore in vertical hydraulic fractures. This work presented convincing evidence of the development of a mound of settled sand on the bottom face of a vertical fracture. It was also observed that proppant particles built up until the injected fluid flow velocity was high enough and greater than some critical velocity. Subsequently, a number of other experimental works on proppant transport and settling have been conducted (see reviews by Clark & Güler 1983; Clark & Quadir 1981). A comprehensive summary of the proppant transport and settling phenomenon can be found in Woodworth and Miskimins (2007).

Analytical and numerical models followed the initial experiments on proppant transport and placement. Daneshy (1975, 1978) pioneered a simplified model for proppant transport and settling in hydraulic fractures. This model was further extended by Novotny (1977). These theoretical approaches assume that proppant transport and settling follow Stokes’ law. Novotny (1977) also incorporated a highly simplified equation to estimate the residual opening of hydraulic fractures. Following these early studies, more advanced and complex models for transport and settling of proppants have been proposed (Clifton & Wang 1988; Gadde et al. 2004; Smith & Klein 1995; Smith et al. 2001; Unwin & Hammond 1995).

In a thorough review, McLennan, Green and Bai (2008) noted that the proppant pack placed near the wellbore may act as a wedge, keeping the un-propped region open and working as a high conductive path for fluids to be extracted. This demonstrates the significance of accurately predicting proppant distribution inside hydraulic fractures. Further, it highlights the importance of considering proppant distribution when calculating residual opening and evaluating well performance.

2.6 Mechanical properties of proppants and low-consolidated porous media

Proppants are merely granular particles injected with the stimulating fluid. After these granular particles have settled, they are responsible for holding the fracture faces apart when the injection stage of the hydraulic stimulation concludes (see Section 2.8). The propped fracture then creates a highly conductive path between the reservoir and the wellbore. However, the conductivity of this path is strongly dependent on proppant composition, physical properties, packing, density, degradation, pack permeability and on the movement of fines in the fracture (Gulbis & Hodge 2000; Mader 1989).

In the beginning of loading, the pack of particles is highly compressible primarily because of pack rearrangement and densification mechanisms. Consequently, it is expected that the initial compressibility of the proppant pack inside the fracture has a significant effect on fracture residual opening and conductivity (Cutler et al. 1985; Montgomery & Steanson 1985).

Therefore, it is realistic to consider the proppant assembly's physical properties as analogous to that of packing of low/non-consolidated granular particles (Barree et al. 2003; Lee et al. 2010). There are many studies dedicated to packing, packing rearrangement and compressibility because of the universal practical importance of granular materials (see the reviews of Liu, Zhang & Yu 1999; Mesri & Vardhanabhuti 2009; Panayiotopoulos 1989; Vaisnys & Pilbeam 1975). In hydraulic fracturing stimulations, a standard gravel packing usually results in cubic aggregation of particles, which may then be densified and stabilised to a hexagonal packing configuration via liner vibration (Mader 1989). Although this technique increases pack stability, it negatively affects pack permeability (Barree et al. 2003) and the confining stresses may reduce pack permeability even further (Davies & Kuiper 1988).

Since sand is the most common type of proppant employed in hydraulic fracturing developments (Gulbis & Hodge 2000; McLennan, Green & Bai 2008; Daneshy 2010), models based on the classical consolidation theory of cohesionless soils (Terzaghi, Peck & Mesri 1996) are a viable option to simulate the compressive behaviour of proppant packs. Other more comprehensive approaches available in the literature could also be used to describe this compressive behaviour (e.g., Pestana & Whittle 1995). However, models are yet to be developed for predicting compaction of low consolidated granular particles accounting for the percolation threshold and packing initial configurations, namely grain properties and arrangement.

2.7 Roughness-induced opening of natural fractures

Recent applications of hydraulic stimulations have relied on the non-uniform stress state of rock formations capable of inducing shear slip opening of natural fractures (Mayerhofer et al. 1997; Walker Jr. et al. 1998). In these applications, the fluid pressure acting inside natural fractures forces the interlocked faces apart. At the same time, shear stresses shift the fracture faces, inducing a mismatch. After fluid injection ceases, previously interlocked fractures become open by the roughness (asperities) of the fracture faces. This technique is often very efficient in reservoirs with geological discontinuities such as shale gas, tight oil/gas, coal-bed methane and hot dry rocks (Hossain, Rahman & Rahman 2002).

The effects of the natural fracture opening on permeability, conductivity and heat exchange have been investigated by several researchers (e.g., Koh, Roshan & Rahman 2011; Matsuki et al. 2006; Nasseri, Mohanty & Young 2006; Nemoto et al. 2009; Plouraboué et al. 2000; Roux, Plouraboué & Hulin 1998; Vilarrasa et al. 2011). To estimate this so-called roughness-induced opening, a number of simplified semi-empirical equations were derived (Olsson & Brown 1993; Piggott & Elsworth 1991; Willis-Richards, Watanabe & Takahashi 1996). More sophisticated analytical and numerical mathematical models were developed by Dyskin & Galybin (2001), Dyskin & Germanovich (1995), Hossain, Rahman & Rahman (2002), Kotousov, Bortolan Neto and Rahman (2011), Naceur, Thiercelin and Touboul (1990), Nassir, Settari and Wan (2010), Rahman, Hossain and Rahman (2000, 2002) and Sousa, Carter and Ingraffea (1993). Fractal mechanics has also been utilised by some authors for modelling rough fractures (e.g., Borodich & Mosolov 1992; Brown 1989; Babadagli & Develi 2003; Cherepanov, Balankin & Ivanova 1995). Nevertheless, none of the models discussed above take into account deformations of fracture asperities because of confining stresses after the pressurised fluid is removed. This phenomenon is of practical relevance, as it determines the residual opening of stimulated natural fractures.

2.8 Fracture residual opening and conductivity

The product of the propped fracture permeability and opening is a common definition of fracture conductivity (Barree et al. 2003; Economides & Nolte 2000), which is a fundamental parameter for assessing how well the fracture will convey the fluids to be recovered (Montgomery & Steanson 1985). Hence, appropriate estimates of proppant pack characteristics and fracture residual opening are of paramount importance to evaluate accurately the performance of hydraulically stimulated wells.

The models briefly discussed above in Sections 2.4 and 2.7 are only appropriate for estimating fracture propagation and opening at the stimulation stage, when the fluid pressure is applied to fracture faces. Such models are very useful for evaluating crack propagation and volume of fluid to be injected, as well as proppant size/volume and its transport and distribution along the fracture. Therefore, unless comprehensive and complex 3D numerical analyses are performed, evaluation of residual fracture opening is limited to a number of simplified approaches, such as those of Economides, Hill and Ehlig-Economides (1994), Papanastasiou (2000) and Wood and Junkin (1970). These approaches are very limited because they do not consider confining stresses, fracture opening during the fluid injection stage or proppant compressibility and its distribution inside the fracture. Besides, the assessment of the efficiency of hydraulically stimulated wells is typically based on elliptical or constant-thickness shapes of fracture opening (e.g., Entov & Murzenko 1994; Kanevskaya & Kats 1996; Murzenko 1994; Valkó & Economides 1996, 1998; Zazovskii & Todua 1990), which in many cases cannot provide realistic representations of the fracture profile.

2.9 Well performance

The performance of producing wells is often estimated by the productivity index, which is defined as the ratio of fluid production rate to the pressure drawdown, that is, the difference between the reservoir pressure and the flowing bottom-hole pressure (Diyashev & Economides 2006).

Under ideal conditions, the reservoir and fracture may be taken as uniform, homogeneous and isotropic. For these conditions, fluid flow can be derived from Darcy's law for steady-state laminar flow, deeming the flow rate to be proportional to the pressure gradient (Economides & Nolte 2000). In addition to the pressure gradient, key parameters in Darcy's law are fluid viscosity, medium permeability and dimensions. The performance of hydraulically fractured wells is often evaluated using these simplifications and several approaches are available in the literature (e.g., Entov & Murzenko 1994; Kanevskaya & Kats 1996; Li, Jia & Wei 1996; Murzenko 1994; Zazovskii & Todua 1990).

Non-ideal conditions, such as non-linear fluid properties and heterogeneous reservoir structure, induce the fluid flow in the fracture and in the reservoir to non-Darcy (inertial-turbulent) behaviour (Poe Jr. & Economides 2000; Vincent, Pearson & John 1999). In this regime when the fluid velocity is greater than some critical velocity, the fluid flow is not laminar (Holditch & Morse 1976). The evaluation of non-laminar fluid flow is possible only with numerical methods such as the finite difference and finite element methods (Poe Jr. & Economides 2000).

Well productivity estimates often rely on the assumption that fractures are fully supported and make radical assumptions regarding proppant compressibility. Another common assumption is uniform permeability inside the fracture. The latter simplification is very unrealistic for cases of partially propped fractures, which essentially have different zones of distinct permeability.

2.10 Development of fracturing materials

Fluid and proppant are the two main components of fracturing materials (Daneshy 2010). The first fracture treatment in 1947 was conducted using gasoline gelled with napalm and since then a number of other fracturing fluids have been developed (Holditch 2007). Nowadays, fracturing fluids are usually based on water, brine, oil (kerosene, diesel fuel and propane), methanol, or a combination of water and methanol (US EPA 2004). Nitrogen or carbon-dioxide is sometimes mixed to water to form foamed fracturing fluids. The most common fracturing fluid is a mixture of water and additives (Daneshy 2010). Additives are included into the mixture to perform specific tasks, such as increase fluid viscosity to carry proppants inside the fracture, reduce fluid friction, clay stabilisation, surfactant and biocide (US EPA 2004; Daneshy 2010). Nevertheless, new fracturing fluids are still necessary for better hydraulic fracturing operations in low- to moderate-temperature reservoirs (Holditch 2007).

To create a high permeable path that extends deep into the rock formation, the proppant needs to be carried far into the fracture. Common proppants used in hydraulic fracturing developments include natural sand, ceramics, bauxite and walnut hulls (Holditch 2007). As most industrial proppants are much heavier than the fracturing fluid, the main mechanism for transporting proppants is the fluid viscosity (Daneshy 2010). The industry would greatly benefit, therefore, of new stronger, lightweight proppants.

2.11 Environmental risks

Hydraulic fracturing has raised environmental concerns and the adequacy of current regulatory policies has been challenged (Holahan & Arnold 2013). Such concerns include risks to air quality, contamination of shallow aquifers by gases (e.g., methane) and hydraulic fracturing chemicals, mishandling of wastewater disposal, and health effects of these factors on local community (Brown 2007; Dreyer & Stang 2013; Osborn et al. 2011; US EPA 2004; Warner et al. 2012). For these reasons, some countries have imposed strong regulations (e.g., the United Kingdom), interrupted issuing permits (e.g., Germany) or banned (e.g., France) hydraulic fracturing developments (Dreyer & Stang 2013; Holahan & Arnold 2013).

Chemical contamination of underground or surface sources of fresh water have originated exclusively from road transport of fracturing components and fluids, poor storage and surface mixing of fluids, or inadequate well construction methods (King 2012). A report by the United States Environmental Protection Agency (US EPA 2004) has concluded that the injection of hydraulic fracturing fluids into coalbed methane wells poses little or no threat to underground sources of drinking water. Recent studies have pointed out, however, that the oil and gas industry must disclose to the public the processes and chemicals utilised, as well as prevent inadequate well construction to demonstrate hydraulic fracturing environmental hazards can be minimised to safe levels (Holahan & Arnold 2013; King 2012; Nakhwa, Huggins & Sweatman 2013).

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Chapter 3

Fracture Mechanics

Chapter 3

Fracture Mechanics

3.1 Introduction

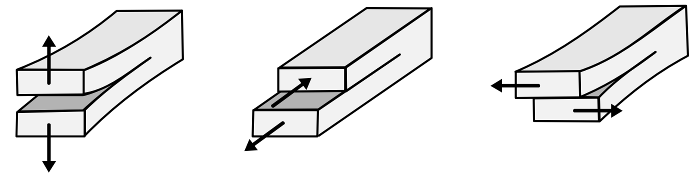
Griffith (1920) linked fracture stress to the flaw size of a given material and used a simple model of energy balance to formulate a fracture theory. This theory states that fracture occurs if the energy available for crack growth is sufficient to overcome the material resistance (Anderson 2005). Although Griffith's (1920) theory provided excellent estimates for brittle materials, the application of Griffith’s (1920) theory for ductile materials resulted in unrealistic assessments (Erdogan 2000; Irwin 1968). This shortcoming occurred because Griffith (1920) assumed material resistance to be equal to the surface energy of the material and disregarded other mechanisms of energy dissipation associated with crack propagation (Anderson 2005). To overcome this shortcoming, Irwin (1956) modified Griffith’s (1920) theory by including in the material resistance the work relating to plastic deformation. The modified theory proved to agree well with experimental data on brittle and ductile materials also. These pioneering works laid the cornerstone of contemporary fracture mechanics. Nowadays, the fracture mechanics theory is well established and many textbooks are available. These range from very basic works focusing on engineering principles to quite sophisticated discussions of mathematical aspects of the linear elastic fracture mechanics (LEFM) theory (e.g., Anderson 2005; Gdoutos 2005; Gross & Seelig 2005; Parker 1981; Recho 2012; Smith 1991; Tada, Paris & Irwin 2000).

Since the classical work of Perkins and Kern (1961) several models based on the theory of LEFM have been developed for predicting crack propagation under applied hydraulic pressure. The classical LEFM analytical models are based on solutions to crack problems obtained from the plane theory of elasticity, which is a two-dimensional (2D) theory. The dimension reduction greatly simplifies the solution procedure for actual three-dimensional (3D) problems. As in many practical situations the hydraulically stimulated fractures are long and wide, the strains perpendicular to the fracture plane are negligible. This fact often justifies the application of plane-strain solutions of linear elasticity to crack problems in geological reservoirs (Adachi et al. 2007; Economides & Nolte 2000).

Although more comprehensive analyses can be conducted using 3D numerical simulators such as Eclipse (GeoQuest Systems 2010) and Nexus (Landmark Software 2007), 2D LEFM models are still widely employed in reservoir engineering. These simplified models are very useful in providing benchmark solutions and a variety of numerical evaluations (Advani et al. 1987). As a basic knowledge of the LEFM is important for appreciating the thesis and research outcomes, a short background of this theory is presented in Section 3.2. This is followed by a brief introduction to the distributed dislocation technique (DDT) (Bilby & Eshelby 1968), which was implemented to derive the governing equations of the problems studied. The DDT strength lies in the fact that it gives detailed full-field solutions for crack problems at the expense of relatively little analytical demands as compared to the intricate technique of dual integral equations and, in addition, of relatively little computational efforts as compared to the Finite Element and Boundary Element methods (Gourgiotis & Georgiadis 2008). The Gauss–Chebyshev quadrature method, often employed for solving DDT integral equations (Hills et al. 1996), is also briefly outlined. The latter method was adopted to solve governing equations, analyse particular cases and obtain numerical results.

3.2 Linear elastic fracture mechanics

In fracture mechanics there are two alternative approaches for performing fracture analysis (Anderson 2005): the energy criterion (Griffith 1920; Irwin 1956), briefly discussed above, and the stress intensity approach (Irwin 1957, 1968). The energy criterion to fracture analysis was originally presented by Griffith (1920) and later improved by Irwin (1956) with the introduction of the energy release rate concept. The energy release rate, G, is defined as the rate of change in potential energy with crack area. Fracture initiation/propagation occurs when the energy release rate reaches a critical value, G_c. This critical value represents the material resistance to fracture (fracture toughness) and it is independent of the size and geometry of the cracked body (Anderson 2005).



a. Mode I b. Mode II c. Mode III

Figure 3.1: The three possible fracture modes: (a) opening, (b) in-plane shearing, and (c) out-of-plane shearing.

The stress intensity approach was developed by Irwin (1957, 1968) from Westergaard's (1939) technique for analysing stresses and displacements ahead of a sharp crack. Irwin (1957, 1968) used this technique to demonstrate that the stresses and displacements near the crack tip could be described by a single parameter related to the energy release rate. This parameter is known as the stress intensity factor and it depends on the fracture mode. Fracture mechanics distinguishes three basic fracture modes, which are illustrated in Figure 3.1. The most common fracture mode is the opening mode (or Mode I), which is the normal separation of crack faces because of the load applied normal to the crack plane (remote tension). The in-plane shearing mode (or Mode II) and the out-of-plane shearing mode (or Mode III) represent the other two classical fracture modes. These shearing modes correspond to the slide of one crack face with respect to the other because of an anti-symmetric loading (remote shear).

By defining a polar coordinate system with the origin set at the tip of the crack, as illustrated in Figure 3.2, the stress fields near the crack tip of an elastic isotropic body with Young's modulus E, shear modulus μ and Poisson's ratio ν can be written as (Anderson 2005; Gross & Seelig 2005)

$$\lim_{r\to 0} \sigma_{ij}^{(I)} = \frac{K_I}{\sqrt{2\pi r}} \mathcal{F}_{ij}^{(I)}(\theta), \tag{3.1a}$$

$$\lim_{r\to 0} \sigma_{ij}^{(II)} = \frac{K_{II}}{\sqrt{2\pi r}} \mathcal{F}_{ij}^{(II)}(\theta), \tag{3.1b}$$

$$\lim_{r\to 0} \sigma_{ij}^{(III)} = \frac{K_{III}}{\sqrt{2\pi r}} \mathcal{F}_{ij}^{(III)}(\theta), \tag{3.1c}$$

for the fracture modes I, II and III, respectively, where $i = x, y, z$ and $j = x, y, z$. In Equations 3.1a–c, σ_{ij} represent the components of the Cauchy stresses, K the stress intensity factor and r the distance from the crack tip. $\mathcal{F}_{ij}$ are dimensionless functions of the angular position, θ.

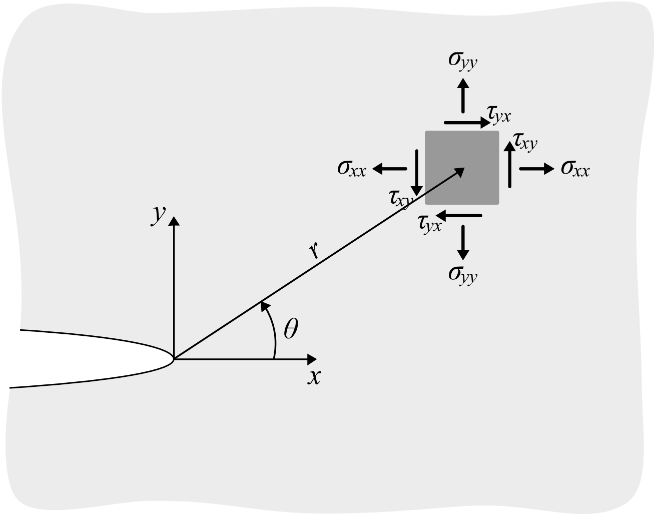


Figure 3.2: Vicinity of the crack tip.

A complete list of the stress and displacement fields at the crack tip for the three possible fracture modes is given in Tables 3.1 and 3.2. These fields are in Cartesian coordinates, with the origin set at the tip of the crack (see Figure 3.2). In a mixed-mode problem, the total stress component, $\sigma_{ij}^{(\text{total})}$, may be obtained by employing Bueckner's (1958) superposition principle:

$$\sigma_{ij}^{(\text{total})} = \sigma_{ij}^{(I)} + \sigma_{ij}^{(II)} + \sigma_{ij}^{(III)}. \tag{3.2}$$

Table 3.1: Stress fields at the crack tip for the three fracture modes in Cartesian coordinates (Anderson 2005; Gross & Seelig 2005).

Stress	Mode I (opening)	Mode II (in-plane shearing)	Mode III (out-of-plane shearing)
σ_{xx}	$\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$	$-\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$	0
σ_{yy}	$\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$	$\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$	0
τ_{xy}	$\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$	$\frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$	0
σ_{zz}	$\begin{cases}\nu(\sigma_{xx}+\sigma_{yy}), & \text{for plane strain}\\ 0, & \text{for plane stress}\end{cases}$	$\begin{cases}\nu(\sigma_{xx}+\sigma_{yy}), & \text{for plane strain}\\ 0, & \text{for plane stress}\end{cases}$	0
τ_{xz}	0	0	$-\frac{K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}$
τ_{yz}	0	0	$\frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}$

Table 3.2: Displacement fields at the crack tip for fracture modes I and II in Cartesian coordinates (Anderson 2005; Gross & Seelig 2005).

Displacement	Mode I (opening)	Mode II (in-plane shearing)
u_x	$\frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa-1+2\sin^2\frac{\theta}{2}\right)$	$\frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa+1+2\cos^2\frac{\theta}{2}\right)$
u_y	$\frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa+1-2\cos^2\frac{\theta}{2}\right)$	$-\frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa-1-2\sin^2\frac{\theta}{2}\right)$

$\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress.

The stress intensity factor represents the magnitude of the dominant singular term in a series expansion of the stress state and it characterises the stress, strain and displacement fields in the vicinity of the crack tip (Hills et al. 1996). For this reason, the stress intensity factor is one of the most important parameters in fracture mechanics (Anderson 2005). However, the determination of this parameter from remote loads and body geometry is not straightforward and closed-form solutions of the stress intensity factor are available only for a number of simple configurations (e.g., Tada, Paris & Irwin 2000). One configuration particularly interesting for hydraulic fracturing problems and for which a closed-form solution exists is that of an infinite elastic isotropic plate with a through-the-thickness crack subjected to constant tensile/compressive loading. This configuration was used by Khristianovic and Zheltov (1955) and Geertsma and de Klerk (1969) to derive the so-called KGD model for evaluating the opening of hydraulic fractures at the stimulation stage assuming plane-strain conditions. Westergaard's (1939) solution approach for this configuration is presented next.

3.2.1 Westergaard's solution for a centre crack with constant tensile loading

Consider a through-the-thickness crack of length $2a$ in an infinite elastic isotropic plate with a constant internal pressure, p, and subjected to constant remote stresses, σ^{∞}, as illustrated in Figure 3.3. The stresses are supposed to be negative in compression and positive in tension. As the loadings are normal to the crack plane, only the fracture opening mode (Mode I) is of practical relevance.

With the origin of the Cartesian coordinates set at the centre of the crack, Westergaard's (1939) stress function, $\mathcal{W}(x,y)$, is given by (Anderson 2005):

$$\mathcal{W}(x,y) = \frac{(p+\sigma^{\infty})(x+\imath y)}{\sqrt{(x+\imath y)^2 - a^2}}, \tag{3.3}$$

being $p = 0$ when $|x| > |a|$ and $\imath = \sqrt{-1}$.

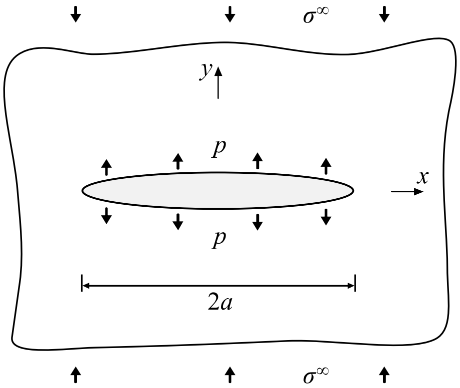


Figure 3.3: Through-the-thickness crack in an infinite elastic plate with constant internal pressure, p, and subjected to remote stresses, σ^∞.

At the plane $y = 0$, the function $\mathcal{W}$ is real for $|x| > |a|$ and imaginary for $-a < x < a$. The normal stresses on the crack plane, σ_{yy}, are thus expressed as (Anderson 2005; Hills et al. 1996):

$$\sigma_{yy} = \mathrm{Re}[\mathcal{W}(x,0)] = \frac{(p + \sigma^\infty)x}{\sqrt{x^2 - a^2}}. \tag{3.4}$$

Consider now the distance from the crack tip $r = x - a$ ($y = 0$), which yields

$$\sigma_{yy} = \frac{(p + \sigma^\infty)(r + a)}{\sqrt{r(r + 2a)}}. \tag{3.5}$$

An analysis of the stress field near the crack tips shows that Westergaard's (1939) approach leads to the expected inverse square root singularity, that is, the stresses tend to $r^{-1/2}$ as $r \to 0$.

For points very close to the crack tip, such that $r \ll a$, Westergaard's (1939) stress function can be written, in polar coordinates, as (Hills et al. 1996):

$$\mathcal{W}(r,\theta) = \frac{(p + \sigma^\infty)\sqrt{a}}{\sqrt{2r}} \exp\left(-\frac{i\theta}{2}\right). \tag{3.6}$$

Hence, the normal stresses on the crack plane ($\theta = 0$), σ_{yy}, are given by (Anderson 2005):

$$\sigma_{yy} = \mathrm{Re}[\mathcal{W}(r,0)] = \frac{(p + \sigma^\infty)\sqrt{a}}{\sqrt{2r}}. \tag{3.7}$$

From the equations in Table 3.1, the normal stresses on the crack plane ($\theta = y = 0$) read:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}. \tag{3.8}$$

Comparing Equations (3.7) and (3.8) yields the stress intensity factor for the opening mode, K_I, of the through-the-thickness crack depicted in Figure 3.3:

$$K_I = (p + \sigma^{\infty})\sqrt{\pi a}. \tag{3.9}$$

The opening between the crack faces, δ_y, at a position x can be described as (Anderson 2005; Tada, Paris & Irwin 2000; Wang 1996):

$$\delta_y = 2u_y = \frac{4(p + \sigma^{\infty})}{\bar{E}}\sqrt{a^2 - x^2}. \tag{3.10}$$

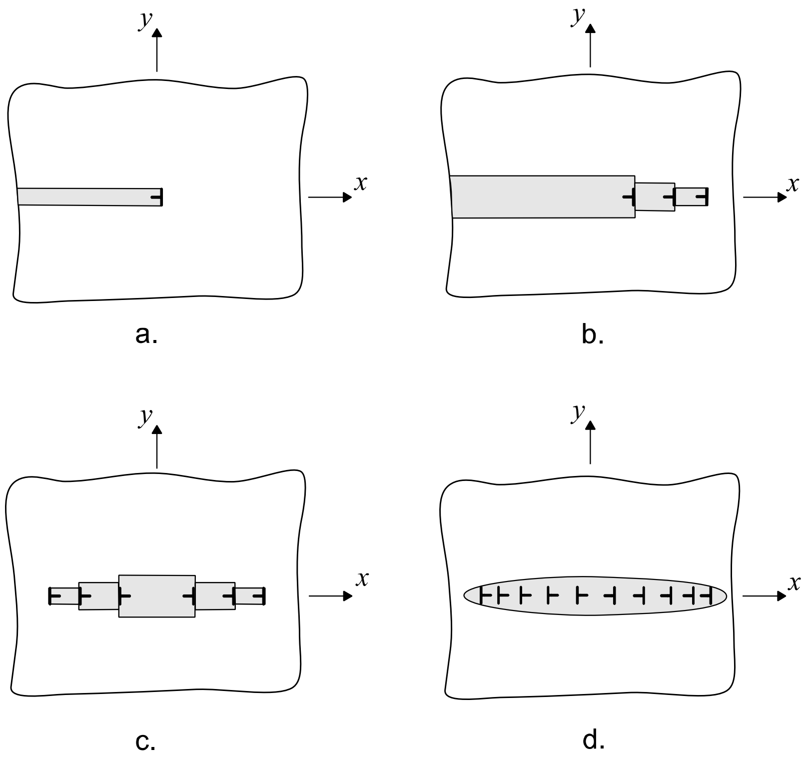
The reduced or generalised Young's modulus, $\bar{E}$, is defined as $\bar{E} = E$ for plane stress and as $\bar{E} = E/(1 - \nu^2)$ for plane strain conditions.

Most configurations for which there are closed-form solutions of the stress intensity factor consist of a crack with a simple shape (e.g., rectangle, ellipse) in an infinite plate (Anderson 2005). For bodies with more complex loading and geometry configurations, the determination of the stress intensity factor from remote loading and body geometry is much more complicated. An example of such a case is that of a hydraulic fracture maintained open by a pack of compressible proppants (see Chapters 7 to 9). Nonetheless, an effective and useful technique that can be employed to determine the stress intensity factor is the DDT, which will be discussed next.

3.3 The distributed dislocation technique

Frequently employed in fracture mechanics analyses and widely used in this work, the DDT is based on the classical work of Eshelby (1957). After this work, many authors analysed linear and non-linear crack problems with the DDT. Hills et al. (1996) thoroughly reviewed this technique, covering its application to a number of crack problems.

The DDT basic hypothesis comes from the rendering of a crack by an unknown distribution of strain nuclei. These strain nuclei represent the inclusion of fictional thin strips of material inside the crack. This inclusion of fictional strips is used as a mathematical tool for modelling the crack face separation, as well as for generating the resultant stress field of a fractured body (Codrington 2008). The stresses induced by the strain nuclei together with the stress field that would be present in an unfractured body under the action of external forces are represented in accordance with Bueckner's (1958) superposition principle. Ensuring the boundary condition requirement that the crack faces must remain traction free, the unknown distribution of nuclei can thus be obtained. The strain nucleus used in this thesis for modelling the crack is the edge dislocation illustrated in Figure 3.4a.



a. b. c. d.

Figure 3.4: Edge dislocations being employed to create a centre crack: (a) single dislocation, (b) addition of more dislocations, (c) removal of dislocations, and (d) final crack geometry (Codrington 2008).

The edge dislocation might be considered as a straight cut made along the negative x-axis followed by the insertion of a thin incompressible strip of thickness b_y before rejoining the fracture surfaces (Hills et al. 1996). By the repetition of this process (Figure 3.4b), together with the continuous removal of some other strips (Figure 3.4c), the emulation of the crack geometry can be achieved (Figure 3.4d). The problem of an infinite elastic isotropic plate with a through-the-thickness crack subjected to arbitrary tensile and compressive loadings is derived in Section 3.3.1 to help illustrate the DDT formulation and application.

3.3.1 DDT formulation for a centre crack with arbitrary loading

Consider a centre crack of length $2a$ in an infinite elastic isotropic medium of Young's modulus E and Poisson's ratio v. Let this crack be subjected to remote stresses, σ^{∞}, and pressure, $p(x)$, acting inside the crack, as shown in Figure 3.5. The stresses are assumed positive in tension and negative in compression.

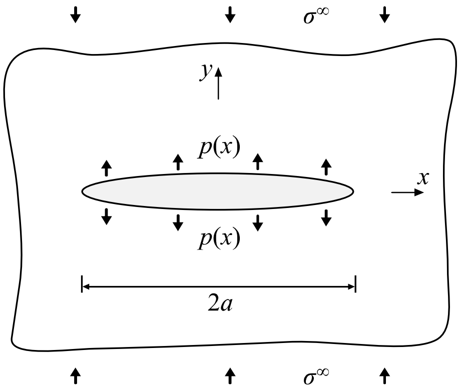


Figure 3.5: Through-the-thickness crack in an infinite elastic isotropic plate subjected to remote stresses, σ^{∞}, and crack-face tensile loading, $p(x)$.

With the origin of the Cartesian coordinates defined at the centre of the crack, the set of boundary conditions for this problem can be written as:

$$\sigma_{yy}(x^2 + y^2 \to \infty) = \sigma^{\infty}, \tag{3.11a}$$

$$\sigma_{yy}(0 \leq |x| \leq a) = \sigma^{\infty} + p(x), \tag{3.11b}$$

$$\delta_y(|x| > a) = 0; \tag{3.11c}$$

where σ_{yy} is the normal stress along the fracture and δ_y is the fracture opening.

Employing the DDT theory for this case, the fracture can be represented as a continuous distribution of dislocations placed along the crack length, namely the interval $-a \leq x \leq a$. By the integration of a single edge dislocation over the fracture length it is possible to obtain the resultant stress and strain fields because of the continuous distribution of dislocations. The normal stress along the fracture line, $\sigma_{yy}(x, 0)$, can thus be found from the corresponding Airy stress function based on the unknown dislocation density $\rho(\xi)$, such that (Hills et al. 1996):

$$\sigma_{yy}(x, 0) = -\frac{\bar{E}}{4\pi} \int_{-a}^{a} \frac{\rho(\xi)}{x - \xi} \mathrm{d}\xi, \tag{3.12}$$

with ξ representing an auxiliary coordinate over the x-axis. The reduced or generalised Young's modulus, $\bar{E}$, is defined as $\bar{E} = E$ for plane stress and as $\bar{E} = E/(1 - \nu^2)$ for plane strain conditions.

Using the boundary conditions of the problem and Equation 3.12, the following expression can be obtained:

$$\frac{\bar{E}}{4\pi} \int_{-a}^{a} \frac{\rho(\xi)}{x - \xi} \mathrm{d}\xi = -\sigma^{\infty} - p(x), \qquad 0 \leq |x| \leq a. \tag{3.13}$$

Expression 3.13 must be solved for the dislocation density, $\rho(\xi)$. After the solution of the dislocation density is obtained, the fracture opening, $\delta_y(x)$, can be found from (Hills et al. 1996):

$$\delta_y(x) = -\int_{-a}^{x} \rho(\xi) \mathrm{d}\xi. \tag{3.14}$$

Equation 3.14 is of extreme importance and provides the means for finding the residual opening of hydraulic fractures, as will be discussed in Chapters 6 to 9.

The physical requirement that the dislocation representation of the fracture should have no net content yields the following single-valued condition for the dislocation density function:

$$\int_{-a}^{a} \rho(\xi)\,\mathrm{d}\xi = 0. \tag{3.15}$$

The exact solution of the singular integral Equation 3.13 with the additional condition Equation 3.15 is not straightforward, as the inversion of the non-linear term of the integral is complicated and $p(x)$ may have a complex non-linear behaviour. For these reasons, closed-form solutions are not possible for most practical cases. To overcome this issue, a number of effective numerical methods are normally employed. One such scheme is the Gauss–Chebyshev quadrature method. This method is very simple, effective and accurate when applied to singular integral equations with Cauchy kernels (Hills et al. 1996). Therefore, the Gauss–Chebyshev quadrature was extensively used in the current research (see Chapters 6 to 10) and is introduced next.

3.3.2 Numerical solution of integral equations with simple Cauchy kernels

The Gauss–Chebyshev quadrature is an extension of the Gauss quadrature that can be used for obtaining the numerical solution of integrals with simple Cauchy kernels of the following kind:

$$\int_{-1}^{+1} \frac{f(s)}{\sqrt{1-s^2}}\,\mathrm{d}s, \tag{3.16}$$

where $f(s)$ is a given function. By employing the Gauss–Chebyshev quadrature rule for N integration points Equation 3.16 reads (Abramowitz & Stegun 1972):

$$\int_{-1}^{+1} \frac{f(s)}{\sqrt{1-s^2}}\,\mathrm{d}s \approx \sum_{i=1}^{N} w_i f(s_i). \tag{3.17}$$

The weight functions, w_i, and the discrete integration points, s_i, are given respectively by:

$$w_i = \pi/N \tag{3.18}$$

and

$$s_i = \cos\left(\pi\frac{2i-1}{2N}\right) \tag{3.19}$$

for $i = 1, 2, \dots N$.

Some mathematical manipulations are required to use the Gauss–Chebyshev quadrature (Equation 3.17) to solve the integral Equation 3.13. First, the following scale transformations over the interval $[-a, +a]$ are introduced:

$$s = \xi/a, \tag{3.20a}$$

$$t = x/a. \tag{3.20b}$$

The dislocation density, $\rho(s)$, tends to infinity in a square root singular manner as $|s|$ approaches unity (Hills et al. 1996). Hence, the dislocation density can be expressed as a product of the fundamental solution, $1/\sqrt{1-s^2}$, and an unknown regular function, $\psi(s)$, such that (Kotousov 2007):

$$\rho(s) = \frac{\psi(s)}{\sqrt{1-s^2}}. \tag{3.21}$$

Employing the scale transformations in Equation 3.20 and the Gauss–Chebyshev quadrature for N sampling points (Equation 3.17) in Equations 3.13, 3.14 and 3.15 yields the following numerical equations:

$$\frac{\bar{E}}{4N}\sum_{i=1}^{N}\frac{\psi(s_i)}{t_j - s_i} = -\sigma^{\infty} - p(t_j), \tag{3.22}$$

$$\frac{\pi a}{N}\sum_{i=1}^{N}\psi(s_i) = 0, \tag{3.23}$$

$$\delta_y(t_j) = -\frac{\pi a}{N}\sum_{i=1}^{j}\psi(s_i); \tag{3.24}$$

where $i = 1, 2 \dots N$, $j = 1, 2, \dots N-1$, and the discrete collocation points, t_j, are given by (Hills et al. 1996):

$$t_j = \cos\left(\pi \frac{j}{N}\right). \tag{3.25}$$

Equations 3.22 and 3.23 form a system of non-linear algebraic equations. This system can be solved computationally for the unknown function $\psi(s_i)$ without difficulty by employing standard numerical iterative procedures.

After solving the system of non-linear algebraic equations, it is possible to obtain the opening of the fracture by substituting the discrete values of the unknown function, $\psi(s_i)$, into Equation 3.24. Additionally, by employing the Gauss–Chebyshev quadrature into Equation 3.12, the net normal stresses along the crack can be calculated numerically as:

$$\sigma_{yy}(x, 0) = -\frac{\bar{E}}{4N} \sum_{i=1}^{N} \frac{\psi(s_i)}{t_j - s_i}. \tag{3.26}$$

Finally, an asymptotic analysis of the crack tip opening displacement for the singular points (tips of the crack) provides the following expression for the Mode I stress intensity factor, K_I (Kotousov & Codrington 2010):

$$K_I = \frac{4}{\bar{E}} \psi(\pm 1). \tag{3.27}$$

The values of $\psi(+1)$ and $\psi(-1)$ may be calculated from Krenk's interpolation formulae (Hills et al. 1996):

$$\psi(+1) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sin[(2N-1)(2i-1)\,\pi/(4N)]\psi(s_i)}{\sin[(2i-1)\,\pi/(4N)]}, \tag{3.28a}$$

$$\psi(-1) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sin[(2N-1)(2i-1)\,\pi/(4N)]\psi\big(s_{(N+1-i)}\big)}{\sin[(2i-1)\,\pi/(4N)]}. \tag{3.28b}$$

The models presented in Chapters 6 to 9 for evaluating the residual opening of hydraulic fractures are based on the formulation presented above. In these chapters, $p(x)$ is substituted by an appropriate function to model the compressive behaviour of the mechanism that keeps the fracture open during the resource recovery stage. For the cases where $p(x)$ exhibits a strong non-linear behaviour, the solution of the system of non-linear algebraic equations formed by Equations 3.22 and 3.23 requires the use of an iterative procedure. One of the various iterative procedures available is the Newton–Raphson method, which was employed for solving the formulae presented in Chapters 6 to 9. This method was chosen for its reliability and high convergence rate (Bathe 1996) and is discussed in detail in Chapter 8. The Gauss–Chebyshev quadrature is also used in Chapter 10 to solve the governing equation of the fluid flow towards the hydraulic fracture.

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Chapter 4

Application of the Contact Theory to Evaluation of Elastic Properties of Low Consolidated Porous Media

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Chapter 6

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Chapter 7

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Chapter 8

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Chapter 9

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Chapter 10

The Performance of Hydraulic Fractures Partially Filled with Compressible Proppant

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Chapter 11

Summary, Recommendations and Conclusions

Chapter 11

Summary, Recommendations and Conclusions

11.1 Introduction

From the thorough literature review presented in Chapter 2, it was apparent that past analytical models available for evaluating well productivity typically use simple elliptical or constant thickness shapes of fracture opening (e.g., Economides & Nolte 2000; Entov & Murzenko 1994; Kanevskaya & Kats 1996; Murzenko 1994; Valkó & Economides 1996, 1998; Zazovskii & Todua 1990). As proppant transport, settling and distribution inside the fracture is non-uniform (Kern, Perkins & Wyant 1959; Novotny 1977; Unwin & Hammond 1995), these shapes are unlikely to be realistic approximations of the actual fracture opening profile.

To address this shortcoming, the current research developed mathematical models and computational tools to more adequately predict the effects of proppant pack compressibility and distribution along the fracture on well productivity. These new models and computational tools provide a new framework for the design and evaluation of the efficiency of hydraulic stimulating techniques. The models are described in the compendium of published papers presented in Chapters 4 to 10.

The study was divided into three cohesive and interconnected parts:

1. theory of low consolidated porous media (Chapters 4 and 5),
2. evaluation of the residual opening of hydraulic fractures (Chapters 6 to 9) and
3. investigation of the fracture residual opening influence on well productivity (Chapter 10).

The compressive behaviour of the proppant pack (low consolidated particles) was studied in Part 1. As demonstrated in Part 2, the compressibility of the proppant pack influences the residual opening of hydraulic fractures. The latter affects fracture conductivity (Cooke Jr. 1973; Davies & Kuiper 1988) and well productivity, as verified in Part 3. The purpose of Sections 11.2 to 11.4 is thus to provide a brief summary of the major outcomes and conclusions obtained from each part. Recommendations for future work and for further developments of the models presented in this thesis are made in Section 11.5. The thesis concludes (Section 11.6) with a final discussion of the outcomes of this work.

11.2 Theory of low consolidated porous media

The calculation of the average mechanical properties of mixers, composites and graded materials is largely based on rule-of-mixtures (Kim 2000) and micromechanics models (Li & Wang 2005). The compressive behaviour of cohesionless soils has also received attention and many models have been proposed (Pestana & Whittle 1995). These models usually disregard some important characteristics of the porous medium, such as the mechanisms of particle deformation, particle arrangement, the percolation threshold of the medium, or the compressibility of the saturating fluid. However, these characteristics have a significant effect on the physical properties of porous media (Nur et al. 1998; Panayiotopoulos 1989). Hence, the first part of this thesis (Chapters 4 and 5) focused on the development of a new analytical approach to evaluate the elastic properties of low consolidated porous media. The basic underpinning conjecture of this new analytical approach rests on a physical consolidation model of uniform spherical particles, packing theory (particle arrangement, porosity, percolation limit) and classical Hertz contact theory.

The initial development of this analytical approach focused on dry conditions and was presented in Chapter 4. Preliminary results demonstrated considerable discrepancies in the evaluation of the elastic properties of the current and classical micromechanical theories. The discrepancies observed can be explained on the basis that the classical micromechanical theories disregard some packing characteristics, such as the particle arrangement. Unlike past theories, the developed approach effectively describes the tendency and experimental results near the percolation limit, which correspond to a low consolidated state of particles pack.

In Chapter 5 the approach was further extended to account for saturated conditions. This extended approach describes the results from experimental studies near the percolation threshold well. In addition, it was observed that significant discrepancies between experimental and classical micromechanical theories results for the elastic properties of low consolidated porous media may occur.

The analytical models developed in Chapters 4 and 5 provide an adequate evaluation of the elastic properties of low consolidated porous media for dry and saturated conditions without introducing empirical parameters. Its importance to the main goal of this project lies in the fact that many natural resources are located in unconsolidated reservoirs (Hagin & Zoback 2007). In addition, the developed analytical models were used for estimating the compressive response of a low consolidated pack of particles injected inside a hydraulic fracture (Chapter 9).

11.3 Evaluation of the residual opening of hydraulic fractures

To date, much of the research and development of hydraulic fracturing models has bypassed the residual opening of hydraulic fractures, despite its considerable influence on fracture conductivity and well performance. As discussed in Chapter 2, only a few extremely simplified models are available in the literature. Therefore, the main objective of Part 2 (Chapters 6 to 9) was the development of new mathematical models and methods for evaluating the residual aperture of hydraulic stimulated fractures, considering the compressibility and distribution of the mechanism that keeps the hydraulic fracture open during the resource recovery stage. Four different cases where analysed:

- a fracture subjected to compression and shear slip sustained open by the roughness (asperities) on the crack faces (Chapter 6). Goodman's (1976) equation was employed to model the deformation of the asperities;
- a hydraulic fracture fully filled with compressible proppant (Chapter 7). The proppant compressive response was modelled by Terzaghi's soil consolidation theory (Terzaghi, Peck & Mesri 1996);
- a hydraulic fracture partially filled with a proppant agent (Chapter 8). The influence of proppant compressibility and distribution on residual fracture openings is investigated;
- a hydraulic fracture partially filled with a compressible pack of low consolidated particles (Chapter 9). The new theory of low consolidated porous medium developed in Part 1 (Chapters 5 and 6) was used to model the compressive behaviour of the pack.

The cases analysed, as well as the main conclusions obtained from each case, are discussed in the following paragraphs.

In Chapter 6 a simplified non-linear model for a crack in an infinite linear elastic medium subjected to both remote compressive and tensile stresses was developed. The governing equations are based on the distributed dislocation technique (DDT) and incorporate a number of reasonable simplifications widely used in past analytical models. The new model allows the evaluation of the residual aperture of fractures with rough surfaces subjected to shear slippage. Goodman's (1976) asperity model was included in the DDT formulation. The discussion provided at the end of Chapter 6 showed that the developed model agrees with the physical limitations of the problem.

In Chapter 7 the hydraulic fracture is assumed to be fully filled with a pack of compressible proppants after the well stimulation stage. The classical soil consolidation theory (Terzaghi, Peck & Mesri 1996) was used to model the proppant pack compressive behaviour because sand is often used as a propping agent (Woodworth & Miskimins 2007). It was verified that the model reflects the physically expected behaviour:

- The fracture residual opening is insignificant when the compressibility of the proppant is extremely high;
- The initial and residual openings are similar if the fracture is filled with incompressible proppant.

The approach presented in Chapter 7 was extended in Chapter 8 to account for the proppant distribution inside the hydraulic fracture. Several cases of proppant distribution were analysed. In addition to the conclusions obtained in Chapter 7, it was verified that the proppant distribution has a large influence on the residual opening of hydraulic fractures. The results revealed that the residual fracture opening profile is significantly lower in the proppant-free region.

In Chapter 9 the approach developed in Part 1 (Chapters 4 and 5) was incorporated into the DDT formulation to address situations where the fracture is partially filled with low consolidated granular particles saturated with a compressible fluid. It was observed that the lower the propped length the higher the stress concentrations over the proppant pack. This observation suggests that the partial distribution of proppants inside the fracture may be very beneficial for preventing proppant flow-back. Moreover, the increased stress concentrations over the proppant pack can lead to a significant reduction of the proppant pack permeability and may jeopardise the fracture conductivity.

The solution approach for all cases studied in this part of the thesis employed the Gauss–Chebyshev quadrature method to convert the integral equations into a system of algebraic equations. A brief description of the numerical method was given in Chapters 6, 7 and 9, with a more detailed portrayal of the derived numerical and computational formulations provided in Chapter 8. It was found that Cox and Rose's (1996) mathematical model for simulating the composite patching repair of fatigue cracks bears some similarities to the model developed in this work. This fact allowed the proper validation of the general numerical model developed (see Chapters 6 to 9).

Overall, it was confirmed that the mechanical properties of proppant and fracture roughness strongly influence the residual opening of the hydraulic fracture. Some of the results also suggested that partially propped fractures may be less prone to proppant flow-back because of the higher stresses found on the proppant pack (Chapter 9). The conducted numerical analyses effectively demonstrated the importance of accounting the residual opening of hydraulic fractures for guiding the design of hydraulic stimulating techniques (the main goal of this thesis). However, direct experimental evidence of the effect of the fracture residual aperture and proppant distribution on the performance of hydraulic stimulated wells is yet to be provided.

11.4 Investigation of the fracture residual opening influence on well productivity

The literature review in Chapter 2 revealed that the available models to calculate well productivity employ extremely simplified fracture opening shapes and disregard proppant distribution. The third and final part of this thesis (Chapter 10) focused on the performance evaluation of hydraulically fractured wells considering more realistic fracture profiles, as well as proppant distribution.

The productivity index of the hydraulically stimulated well was calculated from an extended analytical model originally developed by Zazovskii and Todua

(1990). This new model accounts for two fracture regions of distinct permeability. In addition, it can incorporate any mechanical behaviour of low consolidated granular particles including the one developed in the first part of this thesis (Chapters 4 to 5) for predicting the fracture residual opening. The formulation developed in Chapter 8 was used in two case studies conducted in Chapter 10. The first case study analysed the influence of proppant distribution on well productivity, and the second investigated the combined effects of proppant distribution and compressibility on well performance. Both case studies considered various scenarios of proppant distribution and compressibility. To concentrate the analysis exclusively on proppant pack distribution and compressibility effects on well performance, constant pack permeability was considered throughout the case studies.

It was found that proppant distribution significantly influences well output. The increase in propped length reduces the fracture conductivity required to achieve an optimum productivity. This fact indicates that maximum proppant distribution (i.e., a fully filled fracture) is very beneficial for fractures with low penetration ratio but not essential for the opposite case. This study also suggested that, in certain conditions, partially propped fractures might be more beneficial for increasing well productivity than fractures fully filled with proppant. The relatively lower permeability of the proppant pack may hinder the fluid flow coming from the un-propped region of the fracture. Indeed, this phenomenon has already been observed experimentally (McLennan, Green & Bai 2008).

The sensitivity study performed in Appendix D further emphasises the conclusions in Chapter 10 by investigating the parameters for which the residual opening of the fracture leads to significant production enhancement. It was found that at relatively low confining stresses, the residual opening can lead to an increase of up to 50 per cent on the predicted well productivity. For typical conditions of confining stresses, the effect of the residual opening on the productivity index varies between five and 10 per cent.

Finally, the combined effect of proppant compressibility and distribution was analysed. A linear decrease in fracture conductivity with increase of proppant pack compressibility was observed. Changes in proppant pack permeability and fracture height only affect the slope of this linear relationship. A more thorough analysis of the results reported in Chapter 10 suggests that an increase from 25 to 100 per cent in fracture propped length can enhance the predicted well productivity by up to 87 per cent for fractures filled with highly compressible proppants. For fractures filled with slightly or moderately compressible proppants, an increase in the predicted well productivity of up to eight and 20 per cent, respectively, may be achieved by increasing the proppant coverage from 25 to 100 per cent of the crack length. This analysis effectively demonstrates that both proppant pack compressibility and distribution play major roles in the performance of hydraulically stimulated fractures. Nevertheless, further investigation is needed to evaluate the combined effect of proppant pack compressibility, distribution and permeability reduction with compaction.

Overall, the study reported in Chapter 10 provided a new computationally effective tool for evaluating the productivity of hydraulically stimulated wells. Further, it supplied insightful views on how proppant pack compressibility and distribution affect well production. These outcomes demonstrate the benefits of the modelling framework developed in this work for guiding the design and application of hydraulic stimulating processes.

11.5 Recommendations for future research

Although the non-linear models developed in this thesis encompass some phenomena that have been overlooked in previous studies, there is much room for further improvement. For example, many hydraulic fracturing operations are carried out in weakly consolidated rock formations that exhibit non-linear mechanical behaviour. This fact challenges the application of the current models based on the linear elastic fracture mechanics (Adachi et al. 2007). In addition, the heterogeneity and layering of rock formations, as well as the secondary cracking (branching) of hydraulic fractures, could be contemplated in future models. More specific recommendations for future improvements are discussed below.

The model developed for predicting the elastic properties of low consolidated media can be enhanced by accounting for different sizes and random packing of granular particles. Consideration of particle roughness, shape, cementation, material non-uniformity and crumbling, for example, would increase the model accuracy even further at the expense of complexity (Chang, Chao & Chang 1995; Dvorkin, Nur & Yin 1994; Guises et al. 2009; Hatcher, Chilingarian & Solum 1989; Tiab & Donaldson 2004).

Further developments of the formulation for evaluating the production of hydraulically stimulated wells may consider the transient and turbulent fluid flow in the fracture, as well as the pressure drawdown decrease over time. This formulation can also be improved to provide the productivity index of wells located in naturally fractured formations (Cinco, Samaniego & Dominguez 1978; Cinco-Ley & Samaniego-V. 1981; Du, Guan & Liang 2005).

11.6 Conclusions

The main objective of this research was to gain a thorough understanding of hydraulic fracturing and develop a new framework capable of evaluating the performance of hydraulically stimulated wells. The background knowledge acquired was then used to formulate new non-linear analytical and semi-analytical models. These models facilitate the evaluation of the elastic properties of low consolidated porous media, estimation of the residual opening of hydraulic fractures, and calculation of the performance of hydraulic stimulated wells. The developed models were partially validated against experimental data and analytical studies available in the literature.

The analytical approach developed to predict the elastic properties of low consolidated porous media takes into account particle mechanical properties and packing aspects. It may be useful for modelling low consolidated porous rocks, cohesionless soils and general granular packs, for instance. As it was shown later in this thesis (Chapter 9), this analytical approach may successfully be used for modelling the compressive behaviour of proppant packs saturated with a compressible fluid.

Studies on the evaluation of the residual opening of hydraulically stimulated fractures were conducted and a new semi-analytical model, based on the DDT, was developed. As expected, these studies demonstrated that the compressibility and distribution of the propping mechanism (proppants, crack roughness) have a considerable effect on the fracture residual opening profile. The increased stress over the proppant pack of partially propped fractures suggests that these fractures are less likely to experience proppant flow-back.

It was demonstrated that the partial filling of a hydraulic fracture may be more beneficial for well productivity than a fully filled fracture is. Additionally, the loss of fracture opening because of high proppant pack compressibility or relatively high confining stresses can significantly affect well productivity.

The compilation of peer-reviewed articles in this thesis presents the development, validation and application of a new framework conceived for evaluating the performance of hydraulically stimulated wells. This framework is comprised of non-linear models based on the classical theories of solid, fluid, rock, soil, fracture and contact mechanics. The partial validation against experimental data and/or analytical approaches demonstrated the effectiveness of these models. Overall, the non-linear approaches developed in this thesis provide the alternative for evaluation of the productivity of hydraulically fractured wells without requiring an excessive amount of input data. Nevertheless, the non-linear models presented may be improved in future developments as discussed earlier in this chapter.

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Appendix A

Theoretical Model for Roughness Induced Opening of Cracks Subjected to Compression and Shear Loading

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Appendix B

A New Approach to Evaluate the Performance of Partially Propped Hydraulic Fractures

Bortolan Neto, L., Khanna, A. & Kotousov, A. (2013) A new approach to evaluate the performance of partially propped hydraulic fractures.
APPEA Journal, 2013, pp. 355-362

NOTE:
This publication is included on pages 195-202 in the print copy
of the thesis held in the University of Adelaide Library.

Appendix C

On a Rigid Inclusion Pressed Between Two Elastic Half Spaces

Kotousov, A., Bortolan Neto, L. & Khanna, A. (2014) On a rigid inclusion pressed between two elastic half spaces.
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Appendix D

Effect of Residual Opening on the Inflow Performance of a Hydraulic Fracture

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