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**NON-LINEAR MODELS FOR EVALUATING THE RESIDUAL  
OPENING OF HYDRAULICALLY STIMULATED FRACTURES  
AND ITS IMPACT ON WELL PERFORMANCE**

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by

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## **Abstract**

Hydraulic stimulation techniques have been employed successfully over the past 60 years to enhance the productivity of oil and gas reservoirs. These techniques work by injecting a pressurised fluid into the wellbore to initiate and propagate an artificial fracture or to open a network of existing fractures. These techniques are also commonly known as hydraulic fracturing or fracking. The main objective of hydraulic stimulation is to create highly conductive pathways, which can significantly increase the permeability of the reservoir and, subsequently, improve the well productivity. An injection of small particles (usually known as propping agents or proppants) with the fracturing fluid is the most common method to prevent the stimulated fractures from full closure during the production stage because of confining stresses.

To date, research has largely focused on the assessment of conditions and characteristics of fluid-driven fractures, as well as proppant transport and settlement mechanisms. The modern theory of hydraulic fractures is based on linear elastic fracture mechanics and theories of poro-elasticity, fluid flow in narrow openings and suspension flow in porous media. Despite numerous studies being carried out, few are devoted to the residual opening of hydraulic fractures, which has a significant effect on well productivity. There are many exciting potential applications and developments of hydraulic stimulation techniques for geothermal reservoirs and coal seam gas production. These all require new and more comprehensive theories, supported by analytical and numerical solutions capable of describing the non-linear effects of proppant placement and

compressibility on the fracture residual opening profile and, ultimately, on the reservoir permeability and well performance.

In order to address these needs and gaps, this thesis aims to develop:

- a new mechanical model for predicting the mechanical response of saturated and unsaturated low-consolidated granular particles to compressive loading;
- a new mathematical method and non-linear solutions for evaluating the residual aperture of fractures partially filled with unconsolidated compressible particles (proppant) and subjected to compressive loading;
- a new mathematical model for evaluating the production rate of hydraulically stimulated wells taking into account the residual closure and various regions of distinct permeability along the fracture.

These new models are all based on the classical theories of solid, fluid, contact, fracture, rock and soil mechanics, which provide a framework for evaluating the residual opening profiles (aperture) of hydraulically stimulated fractures, as well as the influence of the fracture residual aperture on the well performance.

A number of simplifications are used to formulate the mathematical models and develop non-linear solutions. Many of these simplifications, such as two-dimensional problem geometry, plane strain conditions and linear elastic behaviour of the medium, represent a well-established foundation for analytical and numerical modelling in reservoir engineering. Accounting for other important phenomena, such as proppant flow-back and secondary cracking, is beyond the scope of this thesis but may be included in future work. The numerical results obtained within the developed models indicate that the residual openings and distribution of proppant along the fracture have a significant effect on well productivity (up to 50 per cent in the case of a relatively low level of confining stresses in the reservoir) and must be incorporated into the evaluation of the efficiency of hydraulic stimulation techniques and assessment of well productivity.

## **Declaration**

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or tertiary institution without the prior approval of The University of Adelaide.

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Date



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*Luiz Bortolan Neto*



## Thesis by Publication

This thesis is comprised of a combination of peer-reviewed publications and submitted journal articles in accordance with the Academic Program Rules 2013 of The University of Adelaide. The journals involved all deal with subject matter closely related to the research field of this thesis.

This thesis is based on the following publications:

1. Bortolan Neto, L, Kotousov, A & Bedrikovetsky, P 2011, ‘Application of contact theory to evaluation of elastic properties of low consolidated porous media’, *International Journal of Fracture*, vol. 168, no. 2, pp. 267–276. doi:10.1007/s10704-010-9574-6
2. Bortolan Neto, L, Kotousov, A & Bedrikovetsky, P 2011, ‘Elastic properties of porous media in the vicinity of the percolation limit’, *Journal of Petroleum Science and Engineering*, vol. 78, no. 2, pp. 328–333. doi:10.1016/j.petrol.2011.06.026
3. Bortolan Neto, L. & Kotousov, A 2012, ‘On the residual opening of cracks with rough faces stimulated by shear slip’, in A Kotousov, R Das & S Wildy (eds), *7<sup>th</sup> Australasian Congress on Applied Mechanics (ACAM 7)*, Engineers Australia, Adelaide, pp. 867–876.
4. Bortolan Neto, L & Kotousov, A 2012, ‘Residual opening of hydraulically stimulated fractures filled with granular particles’, *Journal of Petroleum Science and Engineering*, vol. 100, pp. 24–29. doi:10.1016/j.petrol.2012.11.014

5. Bortolan Neto, L & Kotousov, A 2013, 'Residual opening of hydraulic fractures filled with compressible proppant', *International Journal of Rock Mechanics and Mining Sciences*, vol. 61, pp. 223–230. doi:10.1016/j.ijrmms.2013.02.012
6. Bortolan Neto, L & Kotousov, A 2013, 'On the residual opening of hydraulic fractures', *International Journal of Fracture*, vol. 181, no. 1, pp. 127–137. doi:10.1007/s10704-013-9828-1
7. Bortolan Neto, L & Khanna, A 2013, 'The performance of hydraulic fractures partially filled with compressible proppant', *Australian Journal of Multi-disciplinary Engineering*, vol. 10, no. 2, pp. 185–197. doi:10.7158/N13-AC08.2013.10.2

The following articles are relevant to the present work and are included as appendices:

8. Kotousov, A, Bortolan Neto, L & Rahman, SS 2011, 'Theoretical model for roughness-induced opening of cracks subjected to compression and shear loading', *International Journal of Fracture*, vol. 172, no. 1, pp. 9–18. doi:10.1007/s10704-011-9642-6
9. Bortolan Neto, L, Khanna, A & Kotousov, A 2013, 'A new approach to evaluate the performance of partially propped hydraulic fractures' in *APPEA 2013 Journal and Conference Proceedings*, vol. 53, pp. 355–362. APPEA 2013 Conference and Exhibition, Brisbane, Australia, APPEA and Media Dynamics.
10. Kotousov, A, Bortolan Neto, L & Khanna, A 2014, 'On a rigid inclusion pressed between two elastic half spaces', *Mechanics of Materials*, vol. 68, pp. 38–44. doi:10.1016/j.mechmat.2013.08.004
11. Khanna, A, Bortolan Neto, L & Kotousov, A 2014, 'Effect of residual opening on the inflow performance of a hydraulic fracture'. *International Journal of Engineering Science*, vol. 74, pp. 80–90. doi:10.1016/j.ijengsci.2013.08.012

# Table of Contents

<b>Abstract .....</b>	<b>i</b>
<b>Declaration .....</b>	<b>iii</b>
<b>Acknowledgements .....</b>	<b>v</b>
<b>Thesis by Publication .....</b>	<b>vii</b>
<b>Table of Contents .....</b>	<b>ix</b>
<b>Glossary.....</b>	<b>xvii</b>
<b>1. Introduction .....</b>	<b>1</b>
1.1 Overview .....	3
1.2 Summary of Gaps .....	5
1.3 Main Objective and Methodology .....	6
1.4 Specific Aims and Details of the Publications .....	7
1.4.1 Theory of low consolidated porous media .....	8
1.4.2 Evaluation of the residual opening of hydraulic fractures .....	10
1.4.3 Investigation of the fracture residual opening influence on well productivity .....	14
1.5 Organisation of the Thesis .....	15
References .....	16

<b>2. Basic Aspects of Design and Efficiency Evaluation of Hydraulic Stimulations .....</b>	<b>23</b>
2.1 Introduction .....	25
2.2 Fracture Initiation and Propagation .....	27
2.3 Flow and Leak-off of the Stimulating Fluid .....	29
2.4 Fracture Geometry during Hydraulic Stimulation .....	30
2.5 Proppant Transport, Settlement and Distribution .....	31
2.6 Mechanical Properties of Proppants and Low-Consolidated Porous Media .....	32
2.7 Roughness-induced Opening of Natural Fractures .....	33
2.8 Fracture Residual Opening and Conductivity.....	34
2.9 Well Performance .....	35
2.10 Development of Fracturing Materials.....	36
2.10 Environmental risks .....	37
References .....	38
<b>3. Fracture Mechanics .....</b>	<b>51</b>
3.1 Introduction.....	53
3.2 Linear Elastic Fracture Mechanics.....	55
3.2.1 Westergaard’s solution for a centre crack with constant tensile loading .....	58
3.3 The Distributed Dislocation Technique .....	60
3.3.1 DDT formulation for a centre crack with arbitrary loading.....	62
3.3.2 Numerical solution of integral equations with simple Cauchy kernels.....	64
References .....	67

<b>4. Application of the Contact Theory to Evaluation of Elastic Properties of Low Consolidated Porous Media .....</b>	<b>71</b>
Statement of Authorship .....	73
Abstract .....	75
Introduction .....	75
Diagenesis Process and Packing of Spherical Particles .....	77
Bulk Modulus of Low Consolidated Medium .....	79
Discussion .....	81
References .....	83
<b>5. Elastic Properties of Porous Media in the Vicinity of the Percolation Limit .....</b>	<b>85</b>
Statement of Authorship .....	87
Abstract .....	89
1. Introduction .....	89
2. Diagenesis Process and Packing of Spherical Particles.....	90
3. Bulk Modulus of Low Consolidated Medium.....	91
4. Discussion and Conclusion.....	92
References .....	94
<b>6. On the Residual Opening of Cracks with Rough Faces Stimulated by Shear Slip .....</b>	<b>95</b>
Statement of Authorship .....	97
Abstract .....	99
1 Introduction .....	99
2 Mechanical Model .....	100

3	System of Integral Equations .....	102
4	Solution Procedure .....	102
5	Stress Analysis .....	104
6	Validation.....	104
7	Physical Remarks .....	106
8	Conclusions.....	106
	References .....	107
<b>7.</b>	<b>Residual Opening of Hydraulically Stimulated Fractures Filled with Granular Particles .....</b>	<b>109</b>
	Statement of Authorship.....	111
	Abstract.....	113
	Nomenclature .....	114
1.	Introduction.....	113
2.	Problem Formulation .....	115
2.1.	Soil compressibility and settlement .....	115
2.2	DDT formulation .....	115
2.3	Numerical and computational formulations.....	116
3.	Discussion .....	116
3.1.	Validation of the proposed approach .....	116
3.2.	Residual stress intensity factor: physical remarks .....	117
4.	Conclusions.....	118
	References .....	118

<b>8. Residual Opening of Hydraulic Fractures Filled with Compressible Proppant .....</b>	<b>119</b>
Statement of Authorship .....	121
Abstract .....	123
1. Introduction .....	123
2. Mechanical Model .....	124
3. Mathematical Model.....	125
3.1. Governing equations.....	125
3.2. Proppant response.....	125
4. Solution Procedure .....	126
4.1. Numerical formulation .....	126
4.2. Computational formulation .....	126
4.3. Analysis of stresses.....	127
5. Results and Discussion .....	127
5.1. Validation of the proposed method .....	127
5.2. Stress intensity factor .....	128
5.3. Fracture profiles.....	128
6. Conclusions .....	129
References.....	129
<b>9. On the Residual Opening of Hydraulic Fractures .....</b>	<b>131</b>
Statement of Authorship .....	133
Abstract .....	135
1 Introduction .....	135
2 Mechanical Model and Boundary Conditions.....	137

3	Governing Equations.....	138
4	Mechanical Behaviour of a Pack of Compressible Particles .....	139
5	Solution Procedure.....	139
5.1	Numerical formulation.....	140
5.2	Computational formulation.....	140
5.3	Stress analysis .....	140
6	Physical Remarks.....	141
6.1	Hydraulic fracture residual opening and stress response.....	141
6.2	Stress intensity factor.....	142
7	Conclusions.....	142
	Appendix: Validation of the Computational Approach.....	143
	References .....	144

<b>10.</b>	<b>The Performance of Hydraulic Fractures Partially Filled with Compressible Proppant .....</b>	<b>147</b>
	Statement of Authorship.....	149
	Abstract.....	151
1.	Introduction.....	151
2.	Problem Formulation .....	152
3.	Mechanical Model for Residual Opening.....	154
4.	Model for Fluid Flow towards the Hydraulic Fracture .....	154
5.	The Productivity Index .....	155
6.	Results and Discussion .....	156
6.1	Proppant distribution influence on well productivity .....	156
6.2	Proppant pack compressibility effect on well productivity .....	158



7. Conclusion .....	159
References .....	160
Appendix A: Proppant Response Model .....	161
Appendix B: Discretisation of Governing Equations .....	162
<b>11. Summary, Recommendations and Conclusions .....</b>	<b>165</b>
11.1 Introduction .....	167
11.2 Theory of Low Consolidated Porous Media .....	168
11.3 Evaluation of the Residual Opening of Hydraulic Fractures .....	170
11.4 Investigation of the Fracture Residual Opening Influence on Well Productivity .....	172
11.5 Recommendations for Future Research .....	174
11.6 Conclusions .....	175
References .....	177
<b>Appendix A. Theoretical model for roughness induced opening of cracks subjected to compression and shear loading.....</b>	<b>181</b>
<b>Appendix B. A new approach to evaluate the performance of partially propped hydraulic fractures.....</b>	<b>193</b>
<b>Appendix C. On a rigid inclusion pressed between two elastic half spaces .....</b>	<b>203</b>
<b>Appendix D. Effect of residual opening on the inflow performance of a hydraulic fracture.....</b>	<b>213</b>



## Glossary

*Bottomhole pressure:* The pressure at the depth of the producing layer.

*Breakdown pressure:* The pressure at which a formation matrix fractures. The breakdown pressure is determined before establishing feasible reservoir stimulation techniques. Hydraulic fracturing treatments are performed above the breakdown pressure, whilst matrix treatments are carried out with the treatment pressure safely below the breakdown pressure.

*Breaker:* Any substance used to degrade the viscosity of polymer-based fracturing fluids, helping to enhance post-fracturing fluid recovery.

*Damage zone:* The area surrounding the wellbore that has been harmed by the drilling process.

*Fluid leak-off:* See *fluid loss*.

*Fluid loss:* The leakage of the stimulating fluid into the formation matrix.

*Formation matrix:* The rock mass around the borehole.

*Fracture conductivity:* The fracture capability to transmit fluids from the reservoir to the wellbore at the production stage.

*Pay zone:* A reservoir or portion of a reservoir that contains economically producible natural resources.

*Penetration ratio:* The ratio of the hydraulic fracture length to the equivalent reservoir length.

*Percolation:* Movement and filtering of fluids through porous materials.

*Percolation limit:* See *percolation threshold*.

*Percolation threshold:* The limit that defines the medium as frame supported or fluid supported. Below this threshold, no connections between solid particles exist and the medium is fluid supported. Above the percolation limit, the medium is frame supported because of the connections between solid particles.

*Permeability:* The ability of a porous medium to convey fluids.

*Porosity:* The percentage of pore volume or void space of a porous medium.

*Pressure drawdown:* The difference between the average reservoir pressure and the flowing bottomhole pressure.

*Proppant:* Portmanteau of the words ‘propping’ and ‘agent’. See *propping agent* for further details.

*Proppant flow-back:* Proppant being produced out of a hydraulically fractured well during the resource recovery phase.

*Propping agent:* Granular particles mixed with fracturing fluid to hold fractures open after a hydraulic fracturing treatment. In addition to naturally occurring sand grains, man-made or specially engineered proppants may also be used. Proppant materials are sorted for size and sphericity to provide an efficient conduit for production of fluid from the reservoir to the wellbore.

*Reservoir:* A subsurface body of rock with sufficient porosity and permeability to store and convey fluids.

*Stimulation:* A treatment carried out to reinstate or enhance well productivity. Stimulation treatments fall into two main groups: hydraulic fracturing treatments and matrix treatments. Fracturing treatments induce highly conductive flow paths between the reservoir and the wellbore. Matrix treatments are designed to restore the natural permeability of the reservoir following damage to the near-wellbore area.

*Unconventional reservoir:* Any reservoir that requires special recovery operations (e.g., stimulation treatments) outside the conventional operating practices. These operations must overcome economic constraints in order to make production from unconventional reservoirs monetarily viable.

*Wellbore:* The drilled hole or borehole, including the open-hole or uncased portion of the well.