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Modelling and design of magnetic levitation systems for vibration isolation

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DECLARATION

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Acknowledgements & Dedication

Thanks to my supervisors Ben and Anthony, who gave me the freedom to be myself and who inspired me beyond my work. My appreciation couldn't be greater for the long leash they allowed me. With stricter guidance I may have finished sooner, but I believe neither I nor this thesis would have been the better for it.

This thesis is dedicated to those many with whom I've shared this happy life, especially to those in this time who have come into this strange world and to those who have departed from it.

ABSTRACT

Vibration disturbance has a consistent negative impact on equipment and processes. The central theme of this thesis is the investigation of using permanent magnets in the design of a system for vibration isolation.

The thesis begins with a comprehensive literature review on the subjects of passive and active vibration isolation, permanent magnetic systems, and the common area between these on nonlinear vibration systems using magnetic forces. The use of cylindrical and cuboid magnets is the primary focus of this work for which analytical solutions are known for calculating forces and torques. Subsequently, the state of the art in analytical modelling of permanent magnet systems is covered, including a contribution in this area for calculating the forces between cylindrical magnets.

A range of load bearing designs using simple permanent magnet arrangements are examined, with multiple designs suitable for a variety of objectives. A particular emphasis is placed on a system using inclined magnets, which can exhibit a load-independent resonance frequency. Load bearing using multipole magnet arrays is also discussed, in which a large number of magnets are used to generate more complex magnetic fields. A variety of multipole arrays are compared against each other, including linear and planar magnetisation patterns, and an optimisation is performed on a linear array with some resulting guidelines for designing such systems for load bearing.

Permanent magnet levitation requires either passive or active stabilisation; therefore, the design of electromagnetic actuators for active control is covered with a new efficient method for calculating the forces between a cylindrical magnet and a solenoid. The optimisation of a solenoid actuator is performed and geometric parameters are found which are near-optimal for a range of operating conditions.

Two quasi–zero stiffness systems are introduced and analysed next. These systems are designed with a nonlinearity such that low stiffnesses are achieved while bearing large loads. The first system analysed is a purely mechanical device using linear springs; unlike most analyses of this design, the horizontal forces are also considered and it is shown that quasi–zero stiffness is capable in all translational directions simultaneously. However, a notable disadvantage of such spring systems is their difficulty in online tuning to adapt to changing operating conditions. A magnetic quasi–zero stiffness system is then analysed in detail and design criteria are introduced, providing a design framework for such systems and showing how the complex interaction of variables affects the resulting dynamic behaviour. Although the system is nonlinear, the effects of the nonlinearities on the vibration response are shown to be generally negligible.

The thesis concludes with some experimental results of the same quasi–zero stiffness system, constructed as a single degree of freedom prototype. The quasi-static and dynamic behaviour of the system matches the theory well, and active vibration control is performed to improve the vibration isolation characteristics of the device.

TABLE OF CONTENTS

List of Tables

LIST OF FIGURES

4 MULTIPOLE ARRAYS FOR IMPROVED LEVITATION 129

NOMENCLATURE

Most mathematical symbols used in this document are listed herein with page number cross-referencing. Page numbers listed will not include where these symbols are used in graphs or some figures. There are some cases where the same symbol has been used for multiple purposes; these should be unambiguous based on the context of the work.

In mathematical contexts, care has been take to be consistent in the use of parentheses for function arguments (such as $f(x) = x^2$) and square brackets for both grouping ($A \times \left[B \times C\right]$) and vector/matrix notation ($\mathbf{F} = \left[F_x, F_y, F_z\right]^\mathsf{T}$).

Symbol	Description	Page
$\mathbf{A}$	Vector area (direction normal to plane)	48, 52
A	Area	58, 59
A_e	Amplitude of sinusoidal base excitation	40, 208–211
a	Half-length of cuboid magnet, $\hat{x}$ direction	60, 62, 65, 67, 68
a	Cuboid magnet depth	80, 84, 109, 110
a	Cube magnet side length	xii, xiii, 64, 65, 192–194, 196–209
a_w	Cross-sectional area of the wire in the coil	166
$\mathbf{B}$	Magnetic flux density vector	28, 33, 48–52, 57, 59, 158
B_r	Remanence magnetisation	50–52, 55, 58, 61–68, 73, 75, 76, 120, 144, 146, 155, 157–159, 161, 164, 167, 195–197, 215
b	Cuboid magnet face side length	80, 84, 108–110, 133–136, 138, 139, 141
b	Half-length of cuboid magnet, $\hat{y}$ direction	60, 62, 65, 67–69
$\left[BH\right]_{\max}$	Maximum energy product	51, 55
C_i	Compression of the inclined springs at quasi–zero stiffness	189
C_v	Compression of the vertical spring at quasi–zero stiffness	189
c	Relative velocity damping coefficient	xxiii, 5, 7, 8, 16, 119, 120
c	Damping coefficient	5, 7–10, 18, 41, 43, 85, 206, 208, 226, 231–233
c	Half-length of cuboid magnet, $\hat{z}$ direction	60, 62, 65, 67, 68
c	Cube magnet side length	91, 103, 106

Symbol	Description	Page
c_a	Damping coefficient of the vibration neutraliser	16, 18
$\mathbf{d}$	Distance vector between the centres of two magnets, $\mathbf{d} = \left[d_x, d_y, d_z\right]^{\mathsf{T}}$	xv, 60, 62, 63, 65–67, 73
$\mathbf{d}_1$	Position vector for differential magnet volume in cylindrical coordinates $\mathbf{d}_1 = \left[r_1, \phi_1, z_1\right]^{\mathsf{T}}$	xv, 158
$\mathbf{d}_2$	Position vector for differential coil volume in cylindrical coordinates, $\mathbf{d}_2 = \left[r_2, \phi_2, z_2\right]^{\mathsf{T}}$	xv, 158
d	Inclined magnets horizontal offset at the nominal position	108–112, 114, 117–120
d	Normalised magnet gap	xii, xiii, xix, 192–195, 197–211
d	Magnetic spring depth	103, 105, 106, 133
d_w	Diameter of wire	166–171, 173
e_r	Radial distance between two cylindrical objects	164, 165
$\mathbf{F}$	Force vector	57, 58, 68, 99, 109, 110, 116, 118, 119, 158
$\mathbf{F}_{z,x}$	Force between two cuboid magnets; the first magnetised in the $\hat{z}$ direction, the second in $\hat{x}$	66
$\mathbf{F}_{z,y}$	Force between two cuboid magnets; the first magnetised in the $\hat{z}$ direction, the second in $\hat{y}$	xxiii, 63, 66, 78
$\mathbf{F}_{z,z}$	Force between two cuboid magnets both magnetised in the $\hat{z}$ direction	xxiv, 61, 65, 66, 78, 80, 81, 109, 118
$\mathbf{F}_{\text{eddy}}$	Eddy current force vector	33
F	Magnitude of force	41, 68, 78, 80, 81, 85, 93, 102, 107, 110–113, 119, 120, 169–171, 173, 224
$F(s)$	Force input response, Laplace domain	231, 232
F_c	Axial force between a thick coil and a magnet	156–159, 164, 167–169, 175, 176
F_f	Axial force between two circular current loops	156, 164
F_g	Gravity force	102
F_r	Radial force between two cylindrical thin coils/magnets	78
F_T	Quasi–zero stiffness vertical magnet force function	194, 206
F_z	Axial force between two cylindrical thin coils/magnets	76, 78, 157
F_{attr}	Magnet force (attraction magnet)	194
F_{repl}	Magnet force (repulsion magnet)	194

Symbol	**Description**	**Page**
F_i	Force characteristic of the inclined spring, with vertical component F_{i_v} and horizontal component F_{i_h}	xvi, 182, 186
F_v	Force characteristic of the vertical spring, with vertical component F_{v_v} and horizontal component F_{v_h}	xvi, 182, 186
F_t	Force characteristic of the overall inclined spring quasi–zero stiffness mechanism, with vertical component F_{t_v} and horizontal component F_{t_h}	xvi, 182–186
F_s	Magnet force for coaxial cube magnets	64, 81, 193, 194, 196, 197
f	Force input	5–7, 231, 233
f_a	Active force input for an inertial mass or vibration neutraliser	viii, 16, 17
f_d	Force disturbance	5
f_q	Normalised force at equilibrium	198, 199
f_T	Normalised Quasi–zero stiffness vertical magnet force	194, 197–200, 206
f_s	Normalised magnet force for coaxial cube magnets	64, 193–195
$\mathbf{G}$	Force vector between inclined magnets in the local coordinate systems of the fixed magnet	109
G	Axial gap between the dual coils	175, 176, 219
G	Gap between magnets in various magnetic spring designs	91, 99, 100, 103, 133, 141, 142
$G(s)$	Transfer function	231–233
g	Acceleration due to gravity	85, 107, 111, 119, 120, 179, 198, 200, 206, 224
g	Normalised magnet gap	222, 226
g_a	Feedback gain on relative acceleration	7–9, 11
g_c	Feedback gain on relative velocity	7, 8, 10, 11
g_d	Feedback gain on absolute acceleration	7–9, 11
g_k	Feedback gain on relative displacement	7–9, 11, 85
g_m	Feedback gain on absolute displacement	7–9, 11
g_v	Feedback gain on absolute velocity	7, 8, 10, 11, 232, 233
$\mathbf{H}$	Magnetic field strength vector	48–51
H	Height of the apparatus	214, 215, 222–224
H	Length of vertical spring under load	181, 183, 186, 189
H_0	Undeflected vertical spring length	180, 181, 183, 189
H_c	Coercivity	51, 52
h	Inclined spring vertical dimension	180–183, 186

Symbol	Description	Page
h	Multipole array height	133, 136–138, 140, 141
h	Normalised nominal magnet displacement	192, 194, 195, 197–199, 201, 206, 210
h_b	Beam height	215, 223
h_c	Height of the coil	219
h_m	Height of the magnets support	214, 215, 222–224
h_q	Normalised displacement at equilibrium	198–201, 205, 206, 208
h_s	Height of the laser sensor	214, 215, 223
h_ϵ	Height 'buffer' to account for additional thicknesses	222, 224
I	Current	48, 75, 155–159, 161, 164, 167, 170, 171, 175
I_m	Inclined magnet system moment of inertia	119, 120
i	The imaginary number $\sqrt{-1}$	7, 8, 10, 226
i	Magnet index in the $\hat{x}$ direction.	144–147
$\mathbf{J}$	Current density vector	48, 158
$\mathbf{J}_m$	Equivalent 'surface current' vector due to magnetisation	48
$\mathbf{J}_{\text{eddy}}$	Eddy current density vector	33
j	Magnet index in the $\hat{y}$ direction.	144–147
$\mathbf{K}_{z,y}$	Stiffness between two cuboid magnets; the first magnetised in the $\hat{z}$ direction, the second in $\hat{x}$	63
$\mathbf{K}_{z,z}$	Stiffness between two cuboid magnets magnetised in the $\hat{z}$ direction	xxiii, 62
K	Magnitude of stiffness characteristic (derivative of force with respect to displacement)	87–90, 92
$K_c(s)$	Controller transfer function	232, 233
K_h	Stiffness characteristic in the horizontal direction	186, 187
K_q	Stiffness at equilibrium	200, 205, 208, 210
K_T	Quasi–zero stiffness vertical magnet stiffness function	194, 200, 206, 208, 210
K_v	Stiffness characteristic in the vertical direction	184
K_{attr}	Magnet stiffness (attraction magnet)	205
K_{repl}	Magnet stiffness (repulsion magnet)	205
K_s	Magnet stiffness characteristic for coaxial cube magnets	64, 193

Symbol	**Description**	**Page**
k	Stiffness coefficient	xxiii, xxiv, 5, 7–10, 16, 18, 39, 41, 43, 107, 110, 111, 113, 120, 179, 224, 226, 231–233
k_i	Inclined spring stiffness	180–184, 187
k_v	Vertical spring stiffness	180–183, 186
k_a	Stiffness of the vibration neutraliser	16, 18
k_T	Normalised Quasi–zero stiffness vertical magnet stiffness	194, 197, 206
k_{lin}	Linearised stiffness at a certain point	43
k_s	Normalised magnet stiffness characteristic for coaxial cube magnets	64, 65, 193, 194
L	Length of inclined spring under load	181, 182, 186, 189
L_0	Undeflected inclined spring length	180–182, 186, 189
L_b	Beam length	215
L_c	Coil length	75, 154, 155, 157–159, 161, 164, 166, 167
L_m	Magnet length	82–84, 154, 155, 157–159, 161, 164, 167, 172, 215, 219, 222–224
$\mathbf{l}$	Displacement vector between magnet centre and centre of mass (lever arm)	118
l	Lever arm	103, 105, 117–120
l	Multipole array length	133–136, 138
l	Normalised nominal magnet displacement	64, 65, 193, 195
l_m	Horizontal offset of the magnets support	214, 215, 223
l_s	Horizontal offset of the laser sensor	214, 215, 223
l_w	Length of the wire in the coil	166, 167
$\mathbf{M}$	Magnetisation vector	28, 48–50, 57, 144, 158
$\hat{\mathbf{M}}$	Unit magnetisation vector	144–147
M_{sat}	Magnetisation at saturation	50, 51
$\mathbf{m}$	Magnetic dipole	48
m	Mass	xxiii, xxiv, 5, 7–10, 16, 18, 41, 43, 85, 107, 119, 120, 179, 198–202, 204–209, 226, 231–233
m_a	Vibration neutraliser mass	16, 18

Symbol	Description	Page
m_{eq}	Equivalent mass	111, 224
N	Magnet grade, units MG Oe	55
N	Multipole array number of magnets	133–136, 144, 146
N	Number of turns in the coil	75, 155, 158, 159, 164, 175
N_m	Magnet equivalent ‘turns’ for filament current model	156, 157, 161
N_r	Number of turns in the radial direction	155–157, 160, 161, 166, 167, 175
N_z	Number of turns in the axial direction	155–157, 161, 166, 167
$\hat{\mathbf{n}}$	Surface normal vector	57, 158
n	Exponential for empirical magnet force equation	195–197, 200
$P_{bb}(\omega)$	Power spectrum accelerometer measurements of the base disturbance	39, 40, 228
$P_{mb}(\omega)$	Cross spectrum accelerometer measurements between the moving magnet and base disturbance	39, 40
$P_{mm}(\omega)$	Power spectrum accelerometer measurements of the moving magnet	228
$\mathbf{p}$	Displacement vector due to rotation around centre of mass	118
p	Disturbance	41, 43, 85, 86
p	Dual-multipole array horizontal offset	150, 151
p_b	Beam pin origin height	214, 215, 223
p_g	Magnet gap	222, 223
p_m	Low magnet height	214, 215, 222, 224
p_n	High magnet height	214, 215, 222–224
p_q	Quasi–zero stiffness position	222, 223
Q_0	Coefficient for empirical magnet force equation	195–197, 200
Q_1	Coefficient for empirical magnet force equation	195–197, 200
Q_2	Coefficient for empirical magnet force equation	195
Q_3	Coefficient for empirical magnet force equation	195
$q_0(d)$	Polynomial coefficient for modelling magnet force	194, 195
$q_2(d)$	Polynomial coefficient for modelling magnet force	194, 195
$q_4(d)$	Polynomial coefficient for modelling magnet force	194, 195
$\mathbf{R}_x$	Planar rotation matrix around the $\hat{x}$ axis	66, 67
$\mathbf{R}_y$	Planar rotation matrix around the $\hat{y}$ axis	66
$\mathbf{R}_z$	Planar rotation matrix around the $\hat{z}$ axis	66, 109, 118
$\mathbf{R}$	Distance vector between a magnet’s centre and one of its corners/nodes (floating magnet)	62, 63, 73

Symbol	Description	Page
R	Multipole array number of magnets per wavelength	133–139
R	Coil resistance	166–171, 173, 218, 219
R_c	Thick coil outer radius	155, 157–159, 161, 164, 166, 167, 175, 219
R_m	Magnet outer radius	82–84, 154–159, 161, 164, 166, 167, 172, 215, 219
$\mathbf{r}$	Distance vector between a magnet's centre and one of its corners/nodes (fixed magnet)	62, 63
r	Euclidean distance of δ, $\sqrt{\delta_x^2 + \delta_y^2 + \delta_z^2}$	62–64
r	Radial component of distance vector in cylindrical coordinates	xv, 158, 163, 164
r	Radius	156, 164, 165
r_c	Thick coil inner radius	154, 155, 157–159, 161, 164, 166, 167, 175, 219
r_g	Clearance between magnet and inner coil radii	154, 167
$\mathbf{S}$	Vector of magnet side lengths (floating)	65–67
S	Integration surface	57, 158
S_f	Factor of safety	171, 173
S_w	White noise variance	41–43
$\mathbf{s}$	Position vector	109, 118
$\mathbf{s}$	Vector of magnet side lengths (fixed)	65–67
s	Differential region of the integration surface	57, 158
s	Laplace variable	xv–xvii, xx, xxi, 7, 8, 231–233
$\mathbf{T}_{z,y}$	Torque between two cuboid magnets; the first magnetised in the $\hat{z}$ direction, the second in $\hat{y}$	78
$\mathbf{T}_{z,z}$	Torque between two cuboid magnets magnetised in the $\hat{z}$ direction	xxiv, 73, 78
T	Period	40
$T(s), T(\omega)$	Transmissibility	xxi, 8, 10, 39, 226, 228, 230, 231
T_{RSS}	Root-sum-square of the transmissibility magnitude	10, 230, 231
T_z	Inclined magnets torque	118, 119
$\mathbf{t}$	Displacement vectors from the spring magnet centres to the centre of rotation in the coordinate system of the magnets	118

Symbol	**Description**	**Page**
t	Time	7, 40, 41, 43, 208, 231
t_b	Beam shell thickness	215, 223
U	Potential energy	28
u	Cuboid magnet unit length, cube root of volume	110–112, 114, 117, 119, 120
V	Volume	33, 48, 80, 81, 83, 110, 114, 158, 167–171, 173
$V(\omega)$	Variance gain, alternative of transmissibility T	40
$\mathbf{v}$	Velocity vector	33
v	Differential region of the integration volume	33, 158
W	Multipole array number of wavelengths	133–140
w	Inclined spring horizontal dimension	180–182, 186
w_b	Beam width	215
$X_1(s)$	Base response, Laplace domain	7, 8, 18, 231–233
$X_2(s)$	Vibration mass response, Laplace domain	7, 8, 18, 231–233
$\mathbf{x}$	Displacement vector	57–59
x	Horizontal displacement of the inclined spring	180–187, 189
$\hat{x}$	Cartesian unit vector	x, xiv, xvi–xviii, xx, xxiii, 60, 61, 66, 67, 69, 78, 88, 91, 92, 94–99, 101–103, 105, 106, 109, 118, 121, 124, 144–146, 149, 150
x	Displacement	41, 43, 80, 81, 179, 222, 223
x	Inclined magnets horizontal displacement	109, 110, 112, 118, 119, 121–123
x_1	Base displacement	viii, 5, 7, 8, 16, 40, 192, 206, 208, 220, 221, 231, 233
x_2	Displacement of the vibration mass	viii, 5, 7, 8, 16, 40, 192, 194, 206, 220, 221, 231, 233
x_a	Vibration neutraliser displacement	16
x_b	Displacement of the beam at the position of the laser sensor from 'zero'	223
x_m	Displacement of the magnets support	223, 224

Symbol	Description	Page
x_p	Projected displacement of the beam to the magnets support	223
x_s	Displacement measured by the laser sensor	214, 215, 222, 223
x_m	Magnet centre position	224
$\hat{y}$	Cartesian unit vector	x, xiv, xvi, xvii, xx, xxiii, 60, 61, 63, 66, 68, 70, 71, 86, 90, 94–98, 103–105, 107, 109, 121, 123, 124, 144–146, 149–151
y	Horizontal displacement	85, 86, 149, 150
y	Inclined magnets vertical displacement	108–111, 113, 118–123
$\hat{z}$	Cartesian unit vector	x, xiv, xvi, xviii, xx, xxiii, xxiv, 60, 61, 63, 66, 68, 70, 71, 86, 88, 91–99, 101–107, 109, 117, 118, 144–146, 149, 150
z	Axial displacement	155–159, 164, 165, 167–169, 175, 176
z	Vertical displacement	85, 86, 99, 100, 103, 136, 140, 141, 149, 150, 180–186, 189, 217
z	Axial component of distance vector in cylindrical coordinates	xv, 158, 164
z	Inclined magnets out-of-plane displacement	109, 110, 118
$z_{\min}$	Maximum deflection of the vertical spring	183
α	Ratio between the inclined and vertical spring stiffnesses	180, 181, 183, 184, 187–190
α	Cylindrical magnet aspect ratio	82–84, 154, 155, 167–170, 173
β	Coil aspect ratio	154, 155, 167–170, 173
γ	Ratio between the inclined spring width and height	xxii, 182–187, 189
γ	Ratio between magnet lengths in a quasi-Halbach array	138, 140
γ	Square-face cuboid magnet aspect ratio	80, 81, 84, 110, 120
δ	Distance vector between a pair of corners/nodes of two magnets, $\delta = \left[\delta_x, \delta_y, \delta_z\right]^{\mathsf{T}}$	xx, xxii, 61–63, 73

Symbol	Description	Page
δ	Maximum displacement bound	199, 201–203, 205–207
δ	Displacement increment	110, 111, 113
ϵ	Percentage difference between γ and γ_{QZS}	185, 186
ϵ	Closest (normalised) allowable displacement from quasi–zero stiffness to avoid instability	199, 201–203, 205, 207
ζ	Damping ratio, $0.5c/\sqrt{km}$.	xxiv, 120, 208, 226, 232, 236
η	Ratio between inclined and vertical spring lengths	180, 181, 183, 184, 186–190
η_k	Nonlinearity measure	206–208, 210
θ	Magnet rotation/inclination	67–70, 72, 108–112, 114, 117–120
ϑ	Multipole array magnetisation rotation between successive magnets	133–135, 146
ϑ_0	Magnetisation direction of the first magnet in a multipole array	133, 146
ϑ_{xz}	Magnetisation direction in the $\hat{x}$–$\hat{z}$ plane of the first magnet in a multipole array	146
ϑ_{yz}	Magnetisation direction in the $\hat{y}$–$\hat{z}$ plane of the first magnet in a multipole array	146
$\boldsymbol{\kappa}_{z,y}$	'Stiffness' between two magnetic nodes for cuboid magnets magnetised in the $\hat{z}$ and $\hat{y}$ directions respectively, used to calculate $\mathbf{K}_{z,z}$	63, 64
$\boldsymbol{\kappa}_{z,z}$	'Stiffness' between two magnetic nodes for cuboid magnets magnetised in the $\hat{z}$ direction, used to calculate $\mathbf{K}_{z,z}$	62, 63
κ	Stiffness ratio	205
λ	Multipole array wavelength	133–136, 139
μ	Magnetic permeability of a material	30, 31, 49, 52
μ_0	Magnetic permeability of the vacuum	48–52, 55, 57, 58, 61–65, 68, 73, 75, 76, 156–158, 164, 274
μ_r	Relative permeability of a material	49, 50
ν	Ratio of magnet length squared to face area	84
ξ	Inclined spring normalised displacement in the load bearing direction	182–185, 187, 188, 190
ρ	Resistivity	166, 167
σ	Conductivity	33
$\mathbf{\Phi}$	Magnetic flux vector	52
$\boldsymbol{\phi}_{z,y}$	'Force' between two magnetic nodes for orthogonally-magnetised cuboid magnets, used to calculate $\mathbf{F}_{z,y}$	63

Symbol	Description	Page
$\boldsymbol{\phi}_{z,z}$	'Force' between two magnetic nodes for cuboid magnets magnetised in the $\hat{z}$ direction, used to calculate $\mathbf{F}_{z,z}$	61, 62, 73
ϕ	Angular component of distance vector in cylindrical coordinates	xv, 158, 163, 164
φ	Inclined magnets planar rotation	116–123
$\boldsymbol{\psi}$	'Torque' between two magnetic nodes for cuboid magnets magnetised in the $\hat{z}$ direction, used to calculate $\mathbf{T}_{z,z}$	73
ω	Frequency	xix–xxi, 7, 8, 10, 39, 40, 202, 204, 205, 226, 228, 230–232
ω_d	Resonance frequency, $\omega_n\sqrt{1-\zeta^2}$	200, 206, 207
ω_n	Natural frequency, $\sqrt{k/m}$	xxiv, 107, 111, 179, 208, 224–226