Theoretical Aspects of Stochastic Signal Quantisation and Suprathreshold Stochastic Resonance

by

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For Juliet

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Abstract

Quantisation of a signal or data source refers to the division or classification of that source into a discrete number of categories or states. It occurs, for example, when analog electronic signals are converted into digital signals, or when a large amount of data is binned into histograms. By definition, quantisation is a lossy process, which compresses data into a more compact representation, so that the number of states in a quantiser's output are usually far fewer than the number of possible input values. Most existing theory on the performance and design of quantisation schemes specify only deterministic rules governing how data is quantised.

By contrast, *stochastic quantisation* is a term intended to pertain to quantisation where the rules governing the assignment of input values to output states are *stochastic*, rather than *deterministic*. One form of stochastic quantisation that has already been widely studied is *dithering*. However, the stochastic aspect of dithering is usually restricted so that it is equivalent to adding random noise to a signal, prior to quantisation. The term *stochastic quantisation* is intended to be far more general, and apply to the situation where the *rules* of the quantisation process are stochastic.

The inspiration for this study comes from a phenomenon known as *stochastic resonance*, which is said to occur when the presence of noise in a system provides a better performance than the absence of noise. Specifically, this thesis discusses a particular form of stochastic resonance known as suprathreshold stochastic resonance, which occurs in an array of identical, but independently noisy threshold devices, and demonstrates how this effect is essentially a form of stochastic quantisation.

The motivation for this study is two fold. Firstly, stochastic resonance has been observed in many forms of neurons and neural systems, both in models and in real physiological experiments. The model in which suprathreshold stochastic resonance occurs was designed to model a *population of neurons*, rather than a single neuron. Unlike single neurons, the suprathreshold stochastic resonance model supports stochastic resonance for input signals that are not entirely or predominantly subthreshold. Hence, it has been conjectured that the suprathreshold stochastic resonance effect is utilised by populations of neurons to encode noisy sensory information, for example, in the cochlear nerve.

Secondly, although stochastic resonance has been observed in many different systems, in a wide variety of scientific fields, to date very few applications inspired by stochastic resonance have been proposed. One of the reasons for this is that in many circumstances, utilising stochastic resonance to improve a system is sub-optimal when compared to systems optimised to operate without requiring stochastic resonance. However, given that stochastic resonance is so widespread in nature, and that many new technologies have been inspired by natural systems—particularly biological systems—applications incorporating aspects of stochastic resonance may yet prove revolutionary in fields such as distributed sensor networks, nano-electronics and biomedical prosthetics.

Hence, as a necessary step towards confirming the above two hypotheses, this thesis addresses in detail for the first time various theoretical aspects of stochastic quantisation, in the context of the suprathreshold stochastic resonance effect. The original work on suprathreshold stochastic resonance considers the effect from the point of view of an information channel. This thesis comprehensively reviews all such previous work. It then extends such work in several ways; firstly, it considers the suprathreshold stochastic resonance effect as a form of stochastic quantisation; secondly it considers stochastic quantisation in a model where all threshold devices are not necessarily identical, but are still independently noisy; and thirdly, it considers various constraints and tradeoffs in the performance of stochastic quantisers.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

20 February 2006

Signed

Date

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– Mark D. McDonnell

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Derek Abbott's 'Stochastic group' at The University of Adelaide. Photo taken in Santa Fe, USA, 19 June 2003. From left: Adrian Flitney, Mark McDonnell, Andrew Allison, Derek Abbott and Matthew Berryman.

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Nigel Stocks' Warwick University 'stochastic resonance' group. Photo taken at Warwick University, UK, August 2005. From left: Rob Morse, Nigel Stocks, Mark McDonnell, David Allingham and Sasha Nikitin.

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Dr Rob Morse, and his lab at The Mackay Institute of Communication and Neuroscience, at Keele University, UK, 17 August 2005.

Thesis Conventions

Typesetting This thesis is typeset using the LaTeX2e software. Processed plots and images were generated using Matlab 6.1 (Mathworks Inc.). WinEdt build 5.3 was used as an effective interface to the Miktex version of LaTeX.

Spelling Australian English spelling is adopted, as defined by the Macquarie English Dictionary (Delbridge *et al.* 1997).

Referencing The Harvard style is used for referencing and citation in this thesis.

Publications

Book Chapters

McDonnell-M. D., Stocks-N. G., Pearce-C. E. M., Abbott-D. (2006). Information transfer through parallel neurons with stochastic resonance, *Emerging Brain-Inspired Nano-Architectures*, Eds. V. Beiu & U. Rueckert, Imperial College Press (accepted 14 Jun. 2005).

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