AN INTEGRATED RISK EVALUATION MODEL FOR MINERAL DEPOSITS

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ABSTRACT

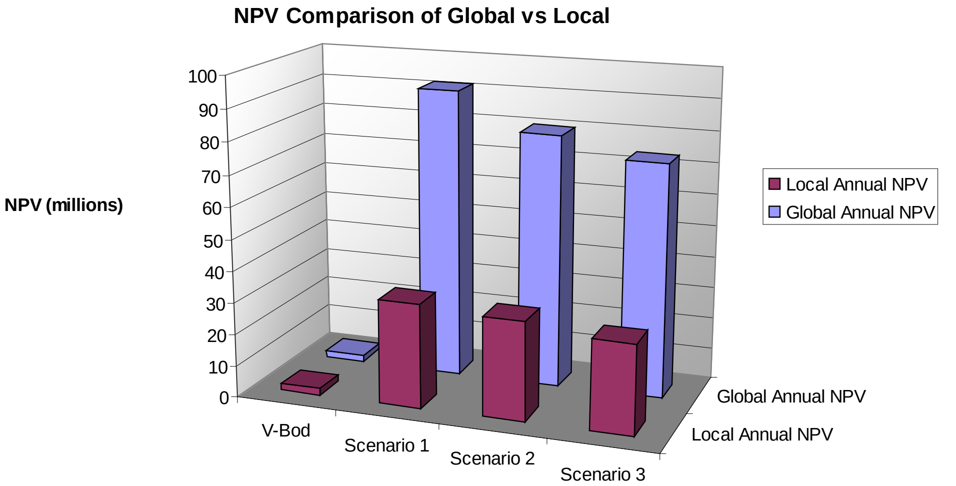
The core asset of most mining companies is its mineral resources and reserves. The company produces ore from its reserves, which is a subset of its mineral resources associated with varying levels of geoscientific confidence and uncertainty. One of the key evaluation challenges is to distil technical complexity into a financial model that is usually designed to focus only on one or two key valuation indicators, such as net present value (NPV) or internal rate of return (IRR).

The driver behind this research was whether conventional evaluation techniques for mineral projects can evaluate accurately both the spatial and temporal characteristics of project risks in financial terms, due to their inherent nature to understate the true variance, and under-value or over-value the actual NPV. How can conventional evaluation methods be compared to an innovative, integrated evaluation technique that quantifies the non-linear impacts of spatial resource variables on production constraints in financial terms, measured at the appropriate temporal scale?

To answer these questions, this research focused on developing an innovative risk evaluation methodology for two different diamond deposits and one gold deposit to incorporate spatial, non-spatial and financial data across the evaluation pipeline. The author developed an integrated evaluation modelling (IEM) framework based on a unique bottom-up methodology that follows every estimation block through the mining and processing value chain, i.e., it accurately captures the spatial variability of all relevant value chain variables in the ground and their correlated impacts on production constraints such as grade, density and processing characteristics. This variability is propagated through the processing value chain at a mining block (or selective mining unit, "SMU") scale.

The IEM approach revealed differences in NPV between a 'bottom-up' (or Local) evaluation method and a 'top-down' (or Global) evaluation method – see Figure 1. While the actual NPV for the virtual ore body (VBod) was CAD 2.1 million, the figure shows that the local evaluation method (bottom-up) more closely approximated the actual NPV of the project than the global (top-down) evaluation method, which materially over-estimated the NPV.

Global ‘top-down’ evaluation method compared to a Local ‘bottom-up’ method



	VBod	Scenario 1 (75m)	Scenario 2 (50m)	Scenario 3 (25m)
Global NPV (CAD million)	-	91.6	80.1	73.9
Local NPV (CAD million)	2.1	32.9	31.4	28.3

Figure 1 compares the Local (bottom-up) and Global (top-down) evaluation methods over three different sampling campaigns (75m, 50m and 25m spaced drill holes).

The author demonstrated that cash flow constituents derived from annual estimates in a top-down approach will not correctly reflect the asymmetries due to operational variability on a local, daily basis. The ‘bottom-up’ evaluation method represented a more accurate way of deriving annual cash flow estimates needed to make decisions on projects by accumulating the appropriate values from a bottom-up approach, i.e. daily, monthly, quarterly then derive annual estimates for NPV forecasts.

The two main advantages of the IEM methodology are that firstly, it accurately reproduces the spatial resource characteristics of blocks at the appropriate temporal scale; and secondly, direct linkages are created between the resource–reserve–financial models within a single software environment. This allows multiple scenarios to be rapidly assessed for a mineral project and the cost/benefits of implementing risk mitigation strategies to be easily evaluated.

This research also quantifies the financial impact of managerial flexibilities by evaluating selected hedging strategies that simultaneously consider production and economic uncertainties within an IEM framework. All modelled outputs are calculated in NPV terms using a modified DCF approach. The importance of linkages within an IEM framework is validated between unsystematic risks, with respect to key resource-to-reserve stochastic variables, and systematic risks considering the impact of foreign exchange rates.

The author concludes that the greater the variability of key systematic and unsystematic variables, the more the mine has to consider flexibility in its mining and processing schedules and management hedging strategies; but the real costs of attaining that flexibility needs to be evaluated using an IEM framework. The confidence in a NPV estimate for complex mineral projects cannot easily be quantified using any closed-form analytical or mathematical solution. Complex, non-linear relationships between resources, reserves, financial and economic parameters requires a simulation model based on an IEM framework to provide a robust solution.

STATEMENT OF ORIGINALITY

Submitted by Grant Nicholas to the University of Adelaide as a thesis for the degree of Doctor of Philosophy to the faculty of Engineering, Computer and Mathematical Sciences, March, 2014.

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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GRANT NICHOLAS DATE

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I am especially thankful for the earlier years of my studies working with Dr. Kleingeld, my expert panel and my colleagues as part of the De Beers, Mineral Resource Management R&D Group based in Wells, UK. Those years in Wells provided me with a unique opportunity to share ideas in an environment fostered towards challenging the conventional and seeking innovative, time efficient and more cost effective ways to evaluate diamond projects and turn them into operating mines. Special reference must be made to my colleague, Stephen Coward, who defied me to think differently and together we shared many ideas, wrote several papers, and solved numerous challenging problems in our work environments.

My expert panel, during the years in Wells and even thereafter, formed an important part of my learning process and strongly influenced my conviction to complete this thesis. Dr. Harry Parker’s overall understanding of the importance of the linkages between geostatistics, mine planning and financial evaluation was an inspiration for me to continue my thesis and I am deeply honoured that he always made time to offer me advice and words of encouragement.

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Lastly, I am most grateful to the De Beers Group, Quantitative Group (QG) and the University of Adelaide who all, at different times, assisted me with funding and support for my studies.

Thank you all so much.

GLOSSARY

AR Autoregressive time series model.

ARIMA Integrated Autoregressive-Moving Average time series model.

ARMA Autoregressive-Moving Average time series model.

AUD Australian Dollar.

CAD Canadian Dollar. Note that for market convention purposes, the foreign exchange rate (USD:CAD) is referred to in the 'Direct' sense, i.e. specifying the number of CAD required to buy or sell one United States dollar (USD).

Call option Provides the holder of the option with the right but not the obligation to buy the underlying asset by paying the exercise price agreed upfront in the contract. A call option is referred to be 'in the money' when the price of the underlying asset is greater than the exercise price and a profit could be made by exercising the option. Conversely, the call option is 'out of the money' if the price of the underlying asset is less than the exercise price.

CAPM Capital Asset Pricing Model.

CPHT Carats Per Hundred Tonne (a measure of diamond grade).

Correlation A measure of the dependency between two variables; or may be calculated as a measure of spatial dependency of a single variable at a distance interval.

Covariance A measure of the dependency between two variables; or may be calculated as a measure of spatial dependency of a single variable at a distance interval.

Conditional simulation A geostatistical tool which can be used to generate punctual or block 'realisations' of mineral grades. Each realisation is intended to honour the histogram and semivariogram of the true grade distribution, as well as honouring known data points.

Conditional distribution The probability distribution for a variable, given the known value of that variable at other locations in space.

DCF Discounted Cash Flow.

DTA	Decision Tree Analysis estimates the maximum and minimum project value by evaluating the probabilities associated with different options discounted back at the traditional discount rate.
European options	Those options that can be exercised only on their maturity date while options that can be exercised at any time are referred to as American or Real Options.
Exercise price	The amount of money invested to exercise the option if you are buying the asset (call option); or the amount of money received if you are selling the option (put option). As the exercise price of an option increases, the value of a call option decreases and the value of a put option increases. This determines the intrinsic value of the option.
FX	Foreign exchange rate.
Geometallurgy	It is a cross-discipline approach between geostatistics, geology, mining and metallurgy with the objective of addressing some of the complexities associated with determining the value of a resource and whether it is economic to exploit.
Geostatistics	Mathematical techniques used to estimate properties which are spatially dependent.
Heteroskedasticity	Non-constant variance.
Homogeneity	The property of a spatial series when its characteristics are independent of location. Homogeneity is equivalent to stationarity.
Homoskedasticity	Constant variance.
IEM	Integrated Evaluation Modelling approach, which models the linkages and dependencies between resources, mine planning and the financial model.
IRR	Internal Rate of Return.
Kriging	A collection of generalised linear regression techniques for minimising an estimation variance defined from a prior model. In contrast to classical linear regression, kriging takes into account stochastic dependence among the data.
Kriging variance	The minimised value of the estimation variance. It is calculated as a function of the semivariogram model and locations of the samples relative to each other and the point of block being estimated.

Kurtosis The kurtosis is a measure of the 'peakedness' of a data distribution around the mode. A kurtosis: equal to 3 suggests a normal, or Gaussian, distribution; < 3 implies a lower concentration near the mean than a normal distribution; and > 3 suggests that the distribution has an excess of values near the mean.

MA Moving Average time series model.

Markov process A stochastic process in which a prediction is determined solely by the closest n observations, and is stochastically independent from all remaining, more distant observations.

MCS Monte Carlo Simulations - any number of procedures that use simulated random samples to methods make inferences about actual populations.

Multivariate conditional simulation An extension of conditional simulation which also aims to ensure that the correct dependencies between simulated variables are honoured in each realisation.

NPV Net Present Value.

Nugget effect When the semivariogram does not pass through the origin and arises from the regionalised variable being so erratic over a short distance that the semivariogram goes from zero to the level of the nugget in a distance less than the sampling interval.

OLS The regression analysis method of Ordinary Least Squares.

Ordinary Kriging The general geostatistical estimation process often simply known as kriging. Unlike simple kriging, the mean is unknown.

PDE Partial Differential Equation.

Put option The converse of a call option – provides the holder of the option with the right but not the obligation to sell the underlying asset to receive the exercise price. A put option is referred to be 'in the money' when the price of the underlying asset is less than the exercise price and a profit could be made by exercising the option. Conversely, the put option is 'out of the money' if the price of the underlying asset is greater than the exercise price.

Random field The application of time series analysis to the spatial variability of theory geotechnical properties, and unlike time series analysis, random field theory is not confined to one dimension.

Range The distance over which the semivariogram becomes a constant.

Real Option The application of financial options, decision sciences, corporate finance and statistics to evaluating real or physical assets as opposed to financial assets (*cf*. definition of European options). A real option is the right but not the obligation to take an action (such as deferring, contracting, expanding or abandoning) at a predetermined cost, called the exercise price or strike price over the life of the option.

Regionalised A variable which has properties that are partly random and partly variable spatial, and has continuity from point to point, but the changes are so complex that it cannot be described by a tractable deterministic function.

Resource A 'Mineral Resource' is a concentration or occurrence of material of intrinsic economic interest in or on the Earth's crust in such form, quality and quantity that there are reasonable prospects for eventual economic extraction. The location, quantity, grade, geological characteristics and continuity of a Mineral Resource are known, estimated or interpreted from specific geological evidence and knowledge.

Reserve An 'Ore Reserve' is the economically mineable part of a Measured and/or Indicated Mineral Resource. It includes diluting materials and allowances for losses, which may occur when the material is mined. Appropriate assessments and studies have been carried out, and include consideration of and modification by realistically assumed mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors. These assessments demonstrate at the time of reporting that extraction could reasonably be justified.

ROV Real Options Valuation.

Simple Kriging The same as ordinary kriging, except that the mean is assumed known and thus, there is no need to impose the unbiasedness condition, which eliminates the final row from all matrices, as is the final column of the square matrix.

Skewness The Skewness is a measure of the symmetry of a data distribution. A skewness of zero suggests a symmetrical distribution, a positive value indicates a right-hand skew, and a negative value indicates a left-hand skew.

Spatial series A sequence of discrete or continuous data measured at specific locations - the spatial equivalent of a time series.

Stationarity A term used to denote different degrees of invariance in the characteristics of random fields. If the mean and autocovariance of the series change with the lag, and not location, the series is said to be weakly stationary. If all higher moments depend on the lag, and not position, the series is said to be stationary in the strict sense.

Systematic risks Risks related to economics, such as price and foreign exchange rates etc. that can be diversified.

Time series analysis A mathematical technique used to estimate properties which are temporally or spatially dependent. When applied to geotechnical engineering, time series analysis is usually referred to as random field theory.

Trend An abstract expression of the low frequency, large-scale systematic variation of a regionalised variable. The trend may also include bias in the test method.

Turning bands A simulation algorithm that can produce both non-conditional and conditional results. The method works by simulating one-dimensional processes on lines regularly spaced in 3D. The one-dimensional simulations are then projected onto the spatial coordinates and averaged to give the required 3D simulated value.

Unsystematic risks Project specific risks related to resource/reserve parameters such as grade, geology, density etc.

USD United States Dollar (see FX rate convention in definition of CAD).

USD/Carat United States dollar per Carat is an expression of revenue for diamonds.

Semivariogram A quantification spatial correlation of a variable, usually calculated from sample information.

VBod Virtual ore Body, which is an analogue of reality created through conditional simulations based on actual drilling results.

WACC Weighted Average Cost of Capital.

“As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.”

— Albert Einstein

Chapter 1 : Introduction

1.1 INTRODUCTION

The fundamental building blocks of most mining companies' business models are resources, associated with varying levels of geoscientific confidence and uncertainty. An understanding of this uncertainty is necessary to guide decision makers in the acquisition of or 'walking away' from new projects and assist in the optimal exploitation of reserves (Kleingeld and Nicholas, 2004). Complex mineral projects often have many uncertainties caused by the nature of estimating reserves based on limited data, problems in forecasting commodity prices and production costs, long evaluation periods during which economic and technologic conditions can change, uncertain regulatory and environmental costs and, in many cases, long project lives (Torries, 1998).

Given this context, mine management is often expected to make evaluation decisions at different stages of projects based on limited and uncertain data. The basic procedure of any evaluation is to compare the consequences or relative values of all possible alternative actions and then make informed decisions based upon the observed results. The challenge is exacerbated by having to distil technical complexity into a financial model that is usually designed to focus only one or two key valuation indicators, such as net present value (NPV) or internal rate of return (IRR).

While IRR was a popular metric in the 1970s and a1980s, NPV of discounted cash flows is a more widely adopted metric today. The payback period (years to get a positive cumulative cash flow, or positive cumulative discounted cash flow) and return on investment (ROI) metrics are also important considerations for investors when assessing the risk versus return of natural resource projects. Investment decisions are typically based on the estimated economic viability of a project and the appetite of the corporate investment committee for the associated project risk.

It is often expected that when decision-makers review a project that its NPV estimate appropriately incorporates the risk of a project, either in terms of embedding the risk within the cash flow revenue and cost streams or by adjusting the discount rate to incorporate a technical risk premium reflecting the project's risk. Industry is divided on this issue, either embedding technical risk by means of conservative assumptions in components of the cash flow, e.g. in the ramp-up period of a new mine; or making central estimates of cash flow components and then evaluating risk by sensitivity analysis. However, these are superficial ways to assess risk and there can be distinct limitations in adopting these methods in some cases. Furthermore, it will be shown that an incorrect valuation technique or decision criterion may be applied and unknowingly derive an answer that is mathematically consistent and may appear reasonable but is still inaccurate, and can mislead decision-makers. As this thesis shows, a simulation methodology is needed to determine the production and financial impacts of interdependencies of the uncertain components of cash flows.

This thesis is presented from the perspective of a project economist or project manager 'within' a company that has to evaluate whether a specific project is economically viable or not. The project is usually evaluated firstly as a 'stand-alone' project then assessed whether it contributes positively or negatively to the overall project portfolio to ensure that it offers fair return to the company's shareholders. This frame of reference is expanded to include a company's corporate strategy from a risk versus return perspective to understand the most appropriate time to invest into, or acquire, a project that on a stand-alone basis may be marginally attractive, but will position the company in a market the company feels will become more lucrative over time.

Given the vast amounts of capital spent on any one project, each project must be assessed in terms of its unsystematic risks (relating to resources and reserves) and systematic risks (market related such as prices and foreign exchange rates). While some aspects of market related risks are discussed in this thesis, only the USD:CAD (United States versus Canadian dollar) foreign exchange rate is explicitly modelled. Commodity prices could have been included in the context of systematic risks for this study but given that there are over 16,000 different price book categories all with their own supply-demand characteristics in De Beers, this would have been too complicated for diamonds. The focus of this thesis is on unsystematic, technical risks related to resources and reserves in order to quantify their financial impact on a project's NPV.

Although the focus of this research is mainly on diamond deposits, the theories and deductions derived from this study are not considered to be exclusive to diamonds. There are many estimation and evaluation similarities to non-diamond deposits, such as gold, base metals and iron ore. To demonstrate how these methodologies can be applied outside of the diamond industry, two of the three case studies examined in this thesis are diamond deposits while the third is a gold-copper deposit. A brief background to the main challenges associated with the estimation of diamond deposits and their financial valuation is provided below.

1.2 HYPOTHESES

The author hypothesized that there would be a material impact (in financial terms) if other non-temporal and non-spatial evaluation techniques were used to evaluate mineral projects instead of the proposed IEM methodology. This is especially true for mineral projects where there is significant resource variability caused by mining and/or processing constraints. The following hypotheses were postulated by the author:

1. Sensitivity analyses and conventional risk analyses using Monte Carlo simulations for the evaluation of production inputs into financial models are likely to produce material prediction errors.
2. The use of traditional evaluation methods based solely on the annual/global assessment of production parameters (i.e. not using a 'bottom-up' IEM approach) will understate the total variability of mineral projects and provide inaccurate NPV estimates.
3. A risk assessment that combines real option valuation (ROV) with spatial geostatistical modelling of resource parameters within an integrated, 'bottom-up' evaluation framework will provide a more accurate estimate of the 'upside' financial potential of marginal mining projects than traditional NPV methods based on geostatistical modelling alone.

1.3 BACKGROUND TO DIAMOND ESTIMATION AND EVALUATION

One of the key challenges faced by the project economist is to evaluate a project on the basis of sparse sampling data and provide a confidence limit on the evaluation. In addition, the appropriate discount rate must be selected that captures the overall risk of the project. The project economist is also faced with the dilemma of whether a global evaluation technique should be used, which could be used to produce a fairly rapid NPV estimate, or whether more sampling data should be acquired to conduct a detailed evaluation with more confidence in the NPV estimate. Does the improvement in accuracy validate the cost and time delay of acquiring the additional sampling data? If a particular risk has a considerable material impact on the NPV of a project, should this risk be mitigated and would the benefits of implementing this mitigation outweigh the costs?

One of the main aims of this research is to quantify the degree of accuracy associated with the NPV of an individual project and how best to reflect the associated risk. Mining is acknowledged as a complex environment with many sources of uncertainty ranging from sampling to economics. In order to optimize investment decision-making, an appropriately structured evaluation model must be developed. An evaluation model must be designed to encapsulate and integrate complexities across the evaluation cycle, with respect to sampling, resource estimation, mine planning and treatment, and financial and economic modelling. These complexities are diverse and range from sampling support and scale effects to understanding the impact of variability, uncertainty and flexibility on operational efficiency and economic viability (Kleingeld and Nicholas, 2004).

These complexities, combined with time and capital constraints, usually do not allow all components of evaluation to be integrated into a model. Hence, the model must strike a balance between simplified estimation techniques and incorporation of those components of the project that will make the most significant impact on the investment decision. These aspects may include technical parameters related to resources and reserves, legal, environmental, political and economic issues, and taking due cognizance of limited human resources and human errors in judgment.

The limited availability of representative sampling data to provide accurate production forecasts is an inherent problem for the evaluation of most mineral commodities. What makes diamond deposits even more difficult to estimate is that there are stochastic variables associated with both its estimation of resources and its revenue (USD/carat) that can lead to greater uncertainty in the final NPV estimate.

Retrospective studies that compare resource and reserve estimates with actual production are not always practical for diamond projects. Open-pit and underground diamond mining rarely depletes ore from a single block for a period lasting a month or more. Ore is extracted and treated from a combination of mining blocks that fulfill both the blending requirements of ore being sent to the treatment plant and the financial model requirements to maximize NPV. As a result thereof, it is impossible accurately to reconcile resource estimates with production to a block model resolution scale. Furthermore, varying economic parameters such as diamond prices, exchange rates, commodity prices and labour prices complicate accurate reconciliation.

While diamonds are not sampled in detail (at SMU scale) before production due to it being too time consuming and expensive, the author recognises that there are classes of deposits that are sampled in more detail from blast holes and/or reverse circulation holes, such as porphyry copper-gold-molybdenum deposits and volcanogenic massive sulphide (VMS) deposits containing copper, zinc, lead, gold and silver. For these deposits, the reconciliation is made between 'delivered to mill' based on measured or calculated head assays (Parker, 2012).

For most minerals, the following are required to prepare an estimate of the in situ mineral value:

- Delineation of the lithological units and their continuity;
- The type of mineralisation within each lithology; and
- Quantification of the mineral concentration within each lithology.

The steps highlighted above yield a resource estimate of the in situ mineral content or grade. The price of the commodity is usually determined by the market value, which can be affected by a complex relationship of investor confidence, political, socio-economic and other factors. There is usually no requirement to estimate the in situ value of the commodity as a function of the commodity's attributes.

The particulate nature of diamonds, their size, shape, quality, colour and value are vital factors in the accurate estimation and evaluation of diamond deposits. Diamond occurrences in nature are rare and are usually measured in parts per billion, whereas most other mineral commodities are measured in parts per hundred (in percentages). Diamonds are brought to the earth's surface in volcanic host rocks, principally kimberlite and to a lesser extent, lamproites. Most of these primary source rocks (or kimberlite pipes) do not contain diamonds, and those that do are very rarely economic, typically less than 1% of all known kimberlites (see Gurney et al., 2005 for more information on the economic concentrations of diamonds in kimberlites and lamproites).

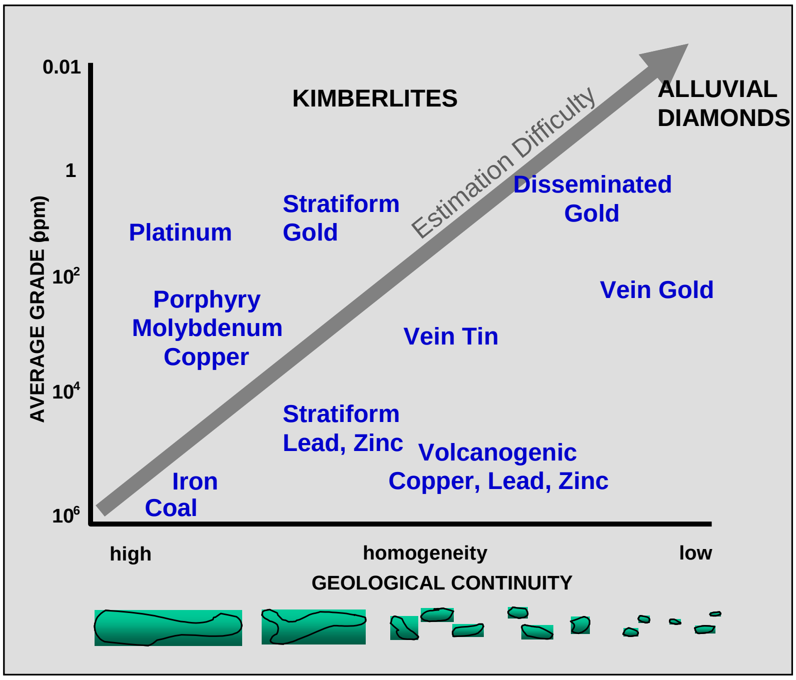


Figure 2. Sampling and estimation difficulty in relation to grade and geological continuity (after King et al, 1982)

Figure 2 illustrates the comparison between the degree of complexity associated with estimating diamond deposits versus other mineral commodities, relative to both geological homogeneity and average grade (ppm) plotted on the x- and y-axes, respectively.

Perhaps unique to diamonds is the necessity to take a few large bulk samples to infer the size frequency distribution of the diamonds and the assortment in terms of shape, clarity, colour etc. Based on the size of these bulk samples, the revenue suggested by the stones recovered may sometimes be increased to account for extremely rare, high-value stones that were missed in collecting the sample. Other variables such as the geological model, densities and grade (if estimated solely from macro diamonds recovered from large diameter drilling) are usually no more difficult to estimate that similar variables for most other mineral deposits. Where grade is estimated using micro-diamonds, there is considerable dependency on the interpretation of the size frequency distribution determined by the resource estimator.

Based on the author’s experience, four key phases of estimating diamond deposits can be identified that contribute to the overall estimation complexity (also refer to Kleingeld and Nicholas, 2004 for further information on complexities associated with diamond estimation and evaluation). While these are similar to the estimation phases for other minerals, the level of complexity for each individual phase combined with the correlations between phases are unique to diamond deposits. The four key phases are:

1. Delineation of the geological model; with respect to the outer pipe geometries and inner lithological boundaries to ascertain continuity both laterally and vertically;
2. Estimation of the total content curve used in grade estimation (usually lognormal, highly positively skewed distributions);
3. Estimation of densities and other geometallurgical characteristics that affect mineral processing within each lithological unit; and
4. Revenue estimation, which depends on a further four key variables: viz.
    a. Diamond size distribution
    b. Diamond colour distribution
    c. Diamond quality distribution
    d. Diamond crystal shape.

These additional sources of variability increase the complexity of evaluating diamond deposits which typically implies lower confidence levels overall. The sampling scale resolution typically has to be very fine (i.e. a tightly spaced grid with many drill holes) to detect the short-scale variabilities of each variable. This is usually practically impossible due to the high costs of sampling, length of time that it takes and physical environmental and geological restrictions, in some cases. As a result, there will always be a degree of uncertainty associated with the estimation of the mean and variances of these resource variables. An alternative method is sought to quantify the potential error in the project's NPV based on the existing sampling data.

1.4 SPECIFIC AIMS

1.4.1. In Scope

The purpose of this research was to develop an innovative risk evaluation methodology for mineral deposits to incorporate spatial, non-spatial and financial data across the evaluation pipeline in an integrated software environment. The drivers for this research focused on whether conventional evaluation techniques for mineral projects have the capacity to evaluate accurately both spatial and temporal characteristics of project risks in financial terms, due to their inherent nature to understate the true variance, and hence under-value or over-value a project's actual NPV. The author strived to understand how conventional evaluation methods could be quantitatively compared to an innovative evaluation technique that more appropriately captures the non-linear effects of spatial resource variables on production constraints taking into consideration the short (block-by-block) spatial and temporal scales.

This research includes the modelling of both unsystematic (project specific) risks for resources and reserves, and systematic (market) risks such as foreign exchange rate. The financial value of the mineral project is computed in conventional discounted cash flow (DCF) NPV terms and then compared to a real options valuation (ROV) method.

The author undertook the following research and development:

- ☑ Generated resource models (through geostatistical kriged estimates and conditional simulations) using ISATIS geostatistical software (Geovariances, 2008);
- ☑ Designed and developed the software environment to link resources to mining and processing constraints using MS EXCEL and VBA coding language;
- ☑ Created, and modified existing, financial models (in MS EXCEL, versions 2007 and 2010) to provide an integrated evaluation framework using VBA coding;
- ☑ Generated forward models for FX rates using a real options valuation (ROV) framework in VBA code (with kind assistance from CERNA, Ecole des Mines);
- ☑ Ran various risk analysis tests (sensitivity analyses, Monte Carlo Simulation analysis etc.) to compare conventional evaluation methods to the author's proposed methodology using Palisade's @Risk software (Palisade, 2008); and
- ☑ Developed, evaluated and linked selected FX rate hedging strategies to resource model uncertainties (using geostatistical conditional simulations) to assess the combined impact of both systematic and unsystematic uncertainties on a project's NPV.

The author developed this methodology using an integrated evaluation modelling (IEM) framework, (Nicholas et al., 2006) and compared the advantages and limitations of production and financial outputs with conventional risk analysis techniques based on linear kriged estimates, Monte Carlo Simulations (MCS) and sensitivity analyses.

The objectives of the research were to:

1. Assess the financial impact of the information effect on a diamond project valuation by systematically imposing a sequence of 'virtual' drilling grids onto an analogue or

virtual ore body (VBod) that represents a version of reality. The resource estimate and subsequent financial NPV estimate of the project were re-calculated each time as the number of drill holes increased. The evaluation focuses on the degree of variance and shift in the median NPV estimate as a function of increasing the information effect and attaining greater knowledge about technical risks of the project;

2. Estimate whether the non-linear effect of spatial resource variables on production constraints for a diamond project is material or not in production and financial terms. Furthermore, compare the temporal scale assessments of evaluating multiple spatial resource variables on a short-scale (block-by-block, daily basis) to that of a conventional longer-term, annual scale;

    a. The resource impact is assessed using geostatistical conditional simulations on a block-by-block basis to compare production and financial results for different temporal scales (viz. daily, monthly, quarterly and annually); and
    b. The correlated impacts of resource variables (linear and non-linear); and their interaction with reserve (mining and processing) constraints and their system dependencies are evaluated.

3. Calculate the combined impact of technical (unsystematic) risks with economic (systematic) risks on project value to assess whether it provides more value than the conventional valuation approach, which only considers systematic risks, to evaluate various hedging strategies. This approach focuses on capturing and modelling spatial resource uncertainties with reserve constraints and economic uncertainties to quantify the combined impact on decision-making and how the cost/benefits of risk mitigations strategies could best be evaluated. An economic forward model using a real options valuation (ROV) approach is used to represent foreign exchange (FX) rate uncertainty. The five scenarios considered are:

    • A flat nominal foreign exchange rate of 1.21 (reflecting management's simplified assumption of an average FX rate over a three year period);
    • Actual foreign exchange rates;
    • No hedging but stochastic spot foreign exchange rates following a Garman-Kohlhagen;
    • Hedging with zero-cost foreign exchange rate collars; and

• Hedging with calls evaluated using the Garman-Kohlhagen call option models (with an additional consideration for volatility uncertainty in the input parameters using a range of FX strike rates).

4. Evaluate the levels of predictive accuracy (and associated confidence limits) when using Monte Carlo Simulations (MCS) versus spatial geostatistical techniques to value mineral projects. There are several increasingly sophisticated MCS techniques to capture the uncertainty of resource, reserve and financial parameters. While MCS techniques may offer a degree of simplicity and significantly shorter processing time compared to geostatistical techniques, little has been documented in terms of their possible prediction errors;
   a. The use of sensitivity and MCS risk analyses techniques are compared with detailed spatial modelling techniques (examining differences at each stage of the evaluation pipeline, viz. resources, production and financial);
   b. The impact of correlation among parameters; number of simulations (iterations in MCS); probability density (shape) assumptions; and influence of the information effect (number of drill holes) are evaluated between MCS, sensitivity analyses and the use of conditional simulation techniques; and
   c. Different approaches to run MCS within the resource estimation stage, after the production (mining and processing) stage and within the financial model are examined and compared to capturing technical risks through conditional simulations.

5. Provide quantifiable confidence limits for cash flows per year and NPV estimates over the life of a project based on correctly accounting for spatial resource, reserve and economic uncertainties. In addition, identify high-risk periods (years) in the life of mine schedule where the combined production and economic uncertainties may be below an expected economic threshold or value-at-risk.

   a. Cumulative probability density curves are produced for the NPV of the project based on multiple resource scenarios generated by geostatistical simulations to quantify confidence limits; and
   b. Similarly, confidence limits are quantified for each cash flow period.

It is worthwhile to mention the differences between accuracy and precision in the context of this study. Accuracy is generally defined as the ability of a measurement to match the actual value of the quantity being measured. Precision, on the other hand, refers to the ability of a measurement to be consistently reproduced, and to the number of significant digits to which a value has been reliably measured (Taylor, 1999). The scope of this study focuses on measuring the accuracy of the estimated NPV in relation to the true NPV, while striving to reproduce results in a repeatable, objective and scientific manner, i.e. accuracy and precision are objectives of this study.

1.4.2. Out of Scope

For the purposes of this study it will be assumed that the main risks affecting project valuation are associated with resources and reserves, in terms of understanding resource variability and its consequential impact on production/engineering design and operational cash flows. This is supported by results from a CIM survey conducted by Smith (2000).

The author recognises that since the widespread adoption of geostatistics in the industry and disciplined approach to resource and reserve estimation imposed by the JORC code (first published in 1989 and the latest being the 2012 update), and other codes following the CRIRSCO template (CRIRSCO, 2013), the proportion of project failures related to inaccurate resource and reserve estimates has declined. In the last decade, the main problem has been the estimation of capital and operating costs, linked inextricably to rising labour costs, falling/fluctuating commodity prices, under-estimation of the length of time required for various environmental, financial and legal approvals to be in effect, and sometimes a case of poor engineering design for the mine/processing plant in relation to the assumed variability of the estimated reserves.

While the focus of this research is on evaluating the economic impact of resource and reserve risks, it is not intended to provide a comprehensive overview of all resource and reserve risks. It will be assumed that modifying factors such as political, legal, social and environmental enabling the successful conversion from resources to reserves have already been considered and are in place.

Holton (2004a) states that risk comprises two essential components, viz. exposure and uncertainty. While this research focuses predominantly on uncertainty, the concept of exposure is equally important. In general, projects are exposed to those scenarios (propositions) that have material consequences for a company. The issue “would we care?” questions the materiality of the risk proposition on a project.

Analysts tend to measure and characterize those risks that they perceive will have the biggest impact on NPV. How is the exposure to risks characterized if they have not been identified and measured? While this research focuses on the impact of resource variability on reserves and the financial model, it tends to concentrate on those resource variables that can and have been measured, and have the biggest material impact on business decision-making. In all three of the case studies discussed in this study, the key resource variables affecting the business have already been identified. However, the impact of their variability on the financial model is unknown, which is the focus of this research. It has been assumed that historical production and/or drilling information has been used to identify these key variables in each of the case studies.

Lastly, this research does not focus on the optimization of block selections within a resource block model based on a set of mining, processing or economic criteria. Typically, these optimization studies focus on optimization of production schedules (mining and processing) aimed at being incorporated into commercial software packages but while this work is admirable, it is often conducted under the hypothesis that the block values are accurate. The problem is that mining companies often do not drill in detail their high-grade areas and other areas scheduled for early production, which can materially affect the estimation of NPV. The author’s focus is on assessing the impact of resource variability on a set of given mining and processing constraints (rather than optimization) and expressing the outputs in clear, financial terms. It also assesses the combined impact of resource plus economic uncertainties on reserve constraints and risk mitigation decision-making.

1.5 ORGANIZATION OF THESIS

The first part of this thesis evaluates a diamond project with limited sampling data. Global evaluation methods are used to estimate a NPV. More sampling data are systematically acquired and the NPV is re-estimated each time to assess improvements in accuracies. Global evaluation methods are compared to more detailed, local evaluation methods contrasting the improvement in prediction accuracies with the cost of acquiring additional sampling data. Two different kimberlite deposits are examined; an underground mining operation and the other, an open-pit mine. In each case, the full sampling data sets are used to develop a virtual ore body (VBod) representing the 'actual' deposit.

Sampling campaigns are virtually drilled into the VBod and resources and reserves estimated, which form inputs into the financial model. The advantage of the VBod is that it provides a perfect example of a deposit where all values are known, thus allowing all subsequent resource and reserve estimates to be compared against it. This allows the economist the opportunity to quantify his/her improvement in accuracy as more data are acquired.

The first part of this thesis (up to chapter five) quantifies the risk in NPV terms and identifies the main time period during which this risk occurs. The second part of this thesis (chapter six) focuses on developing an evaluation method to mitigate the main risks in a project using a real options valuation (ROV) method. It evaluates a risk mitigation strategy by synchronizing managerial and operational flexibility with the uncertainty during these risky periods to mine the deposit in an optimal manner. The author considers that results from this particular exercise will be unique to this problem, however, the broader application of ROV to a project considering technical uncertainties will be common to other mineral projects.

This thesis comprises seven chapters, organized as follows:

Chapter 1. The specific aims of this thesis are discussed and the overall context (background) in which diamond estimation and evaluation problems are defined.

Chapter 2. A literature review discusses seminal papers in the areas of risk analysis, project evaluation and finance. A gap statement is established, which articulates that portion of the problem that the author is undertaking. The thesis objectives are then summarised at the end of this chapter.

In *Chapter 3* the experimental designs and techniques are described that are used to develop the IEM framework, which includes the generation of a virtual ore body (VBod) and spatio-temporal techniques to build resource, reserve and financial models.

Chapter 4 discusses the resource and reserve variance analysis results from experiments conducted in Chapter 3, comparing conventional risk evaluation methods with the author's proposed IEM modelling approach. Sensitivity analyses, Monte Carlo simulation analyses and geostatistical outputs are compared with each other.

Chapter 5. The most appropriate methods of capturing and modelling technical risks in the financial valuation of a diamond project are investigated. Relationships between the technical component of the discount rate, capital expenditure and techno-economic factors are quantified through heuristic experiments. These outcomes together with the VBod approach are used to provide a quantitative breakdown of the technical component of the discount rate.

Chapter 6 evaluates the combined impact of foreign exchange rate uncertainty with resource stochasticity on the NPV for a diamond project. Various hedged and unhedged FX rate scenarios are evaluated to identify the preferred management strategy. A real options valuation (ROV) approach is used to generate the FX rates to evaluate the mining flexibility option and quantify the cost/benefit relationship of implementing this real option.

The thesis concludes with *Chapter 7* which includes a final discussion, conclusions and recommendations for evaluating future technical risks in mineral projects.

Chapter 2 : Literature Review

2.1 INTRODUCTION

The core assets of most mining companies are their mineral resources and reserves. The discovery, estimation, evaluation, development and optimal management of mineral reserves are critical to ensure a profitable supply to meet market demand. The basis of this consistent supply is reserves. The essential building blocks of reserves are resources, associated with varying levels of geoscientific confidence (Kleingeld and Nicholas, 2004). Resource estimation generally includes several variables such as volume, densities of host and waste rocks, tonnage and grade. Although the accuracy of these estimates typically increases as the amount of data increases, the true values are never known. At any given stage of a project the accuracy of resource and reserve estimates is influenced by the uncertainties of quantitative (e.g. grade) and qualitative (e.g. geological complexity) data, which are affected by the amount, quality and spatial characteristics of the available sampling information.

Holloway (1979) identified four main characteristics of problem-solving that would require some form of analytical analysis, viz. when there is a large number of factors; more than one decision-maker; multiple attributes and uncertainty. He noted that where there is one or more of these factors present it is very difficult to integrate all aspects of the problem and ensure that all have been adequately addressed. Typical mineral project and mine evaluation problems are characterized by multiple attributes, which are often associated with both complexity and uncertainty.

Lawrence (1994) identified four main areas of mineral property valuation, viz. green field exploration areas; advanced exploration properties; pre-development projects and developing mines; and existing/operating mines. This thesis focuses predominantly on advanced exploration projects and operating mines. This is because there are usually more data available to support detailed evaluation approaches compared to early-stage green field exploration projects. The evaluation process can be divided into two related phases: the evaluation phase and the decision phase (Torries, 1998). Since decision-making and operations research are large fields in their own right, this study focuses primarily on the evaluation process and makes reference to the decision-making processes as needed. “Start with the innovative question in mind that drives the risk evaluation”, (Hatchuel et al., 2001).

Torries (1998) identified three general types of users of project evaluation results: private investors (including privately owned corporations), lenders and governments. Private investors are usually the project sponsors and operators. Lenders include commercial banks and institutional organizations, such as the World Bank, or downstream processors who lend money in exchange for exclusive rights to purchase the mine's products. Governments at both the national and local levels may be participants as lenders, contributors of equity, or taxing or regulatory agencies.

Each of these types of users has different investment and evaluation decision criteria. Therefore, each will interpret the results of a project evaluation differently, and each may use different evaluation methods (Torries, 1998). " . . . It is in the treatment of imprecise data and risk that valuers differ in their opinions of what method is appropriate for particular valuations" (Sorentino, 2000). Many mineral projects, and almost all large ones, involve all three of these types of users of project evaluation results. Not only must each party understand the project from its own perspective, each must understand the position of the others if an optimal, mutually acceptable position is to be reached.

Governments are usually most interested in the evaluation model to ensure that they receive their portion of royalties, corporate taxes and any other income derived from the exploitation of mineral projects. Federal and State governments also have a role to perform to ensure that local and regional communities benefit from the social infrastructure of mining development while making certain the environment is successfully rehabilitated at the end of the mine's life. Banks and lenders on the other hand are most concerned with the 'downside' risk, i.e. repayment of the capital borrowed plus all interest paid within the designated payback period. Private investors are attracted to dividend payments from a mining company, which represent higher returns than the stock markets, plus any 'upside' opportunities to increase profitability and/or increase longevity of stable dividend payments over an extended life of mine.

Uncertainties in each component of the evaluation model, such as revenue, costs and capital can materially affect the economic viability of a project, which influence the project portfolio of a company. Uncertainties for each resource model can result in larger variations between predicted and actual production outputs and cash flows. Correlations between variables plus the 'system linkages' between variables and their response to process constraints in the evaluation model must also be appropriately captured and modelled. This implies that it is vital to understand resource risks with respect to their stochastic inputs (such as statistical means and covariances) and their associated uncertainties. Mine designs, sequencing and scheduling requirements are based on resource models, which in turn, provide the run of mine (ROM) outputs into the treatment process.

In addition to technical risks relating to resources, the legal, political, social, environmental and economic risks must also be evaluated in order to convert resources to reserves. Project risks, such as plant ramp-up and time series modelling of project delays, are also important considerations. Other fundamental aspects such as technological limitations or improvements over time, people skills availability or the potential impact of 'bad management' on decision-making also need to be considered. Risks should be ranked according to their perceived probability of occurrence and potential impact on the project valuation. Numerous probability-impact techniques are available to assist in this regard (Vose, 2002) and (Aspinall and Brown, 2004).

Holton (2004a) states that risk comprises two essential components, viz. exposure and uncertainty. While it is acknowledged that some projects and/or companies have higher exposure to risk than others, the focus of this study is on uncertainty in the evaluation components, and its consequential impacts.

2.2 RISK ANALYSIS OVERVIEW

The use of numerous risk analysis techniques available in the industry today depends mainly on the applicability to the problem, time constraints, availability of accurate data and the level of competency of the analyst. Risk analysis techniques range from eliciting expert opinion using weighted combinations of expert judgments as adopted by Aspinall et al (2002) and subjective risk assessments, (Vose, 2002); to more quantitative modelling using Monte Carlo simulations and Geostatistical simulations (Ravenscroft, 1992); (Berckmans and Armstrong, 1997); (Dowd, 2000); (Dimitrakopoulos et al., 2002) and (Dowd and Dare-Bryan, 2004). Boundaries between subjective, semi-subjective and quantitative risk analyses are not always definitive.

A subjective or qualitative project risk assessment normally commences with a risk management plan that assists the risk analyst in identifying project objectives, principal stakeholders and provides a time scale for follow-up risk assessments. These approaches conventionally use techniques such as probability-impact matrices, risk registers and risk matrices. Some risk matrices go one step further and apply weighting factors to the impacts to identify those resource risks that are critical to the project. Potential Problem Analysis (PPA) or Failure Modes and Effects Analysis (FMEA) are used on stages of projects to facilitate proactive risk management of those projects.

A PPA seeks to characterize risk in a systematic way but is not intended to identify every conceivable risk or failure mode. PPA performs several risk analyses steps in a logical sequence. A risk register is a document or database that lists each risk pertaining to the project, along with a variety of information that is useful for the management of those risks. The aim of the risk register and PPA approaches is to produce a likelihood of risk occurrence table and a Magnitude of Risk Impact table which can be used to plot risks in terms of Probability and Impact.

Risk registers and PPA are extremely useful when the time available is limited, for example, when due diligence studies are being conducted. They also provide an excellent team-building opportunity and promote discussion of interconnected risks in various disciplines, which forms a basis of understanding for further quantitative risk analyses. While these subjective approaches encourage teamwork and provide a good framework for identifying problems and their potential impacts, they are primarily based on subjective opinions and the results cannot easily be reproduced and/or audited. As a result, these subjective risk registers (as stand-alone methods) are inadequate for capturing and modelling technical risks for the purposes of a quantitative risk assessment.

The need for an integrated approach to assess the uncertainty on oil and gas investment decision-making was recognized by Begg and Bratvold (2001). They introduced the concept of a Stochastic Integrated Asset Model (SIAM) which involved trading off some of the conventional technical rigour in favour of a more complete and accurate assessment of the impacts of uncertainty on the investment decision-making process. This approach incorporated non-spatial Monte Carlo simulations, which are better suited to identify and quantify those uncertainty parameters that affect the decision the most in early-stage mineral exploration projects rather than pre-feasibility or feasibility studies. The emphasis was on an efficient, holistic risk appraisal, rather than attempting to capture or model the spatial relationships found in mineral and energy resources or their correlated impacts on reserves, and ultimately on decision-making.

In a later paper Bratvold and Begg (2002) pointed out that over the preceding decade many of the publications on decision-making analysis tended to focus on quantitative methods at portfolio or asset project level in an attempt to provide more ‘quantifiable’ information to decision-makers. Most of these publications tended to present elegant mathematical solutions using Monte Carlo simulations, Real Options analysis or Markowitz efficient frontier optimization algorithms (Markowitz, 1952) to help quantify uncertainty and risk and to illustrate how decisions should be made. Unfortunately, many authors found the real world so complex that these models were of limited use (see Bratvold and Begg, 2002, for further reading on limitations on the use of Markowitz portfolio theory in the oil and gas industry).

Decision-makers preferred making decisions based on intuition, past experiences and repetition, ‘what’s worked in the past’. “Unfortunately, intuition and repetition are unreliable teachers at best. Research shows that the less competent people are, the less likely they are to know it. Overconfidence is a deeply rooted human characteristic” (Bratvold and Begg, 2002).

No mineral industry project is completely free of risk nor is its success completely certain; there is always a risk of failure. Risk cannot be eliminated (unless the orebody has been totally depleted) so the aim of risk analysis and assessments is to highlight the best risk mitigation strategy. A general strategy for dealing with risk comprises the following: identify the risk; assess and quantify the risk; reduce the risk; determine the minimum acceptable level of risk; reduce the risk to a minimum (where necessary); and manage the residual risk (Dowd, 1997 and Vose, 2002). In general, risk is analysed by project analysts (or engineers) and communicated to decision makers who, based on their perception of the risks, have to make informed investment decisions on the economic viability of a project, relative to the company’s project portfolio.

2.3 PROEJCT EVALUATION OVERVIEW

An evaluation framework should be designed to encapsulate and integrate the complexity across the evaluation cycle, i.e. sampling, resource estimation, mine planning and processing, and financial and economic modelling. This complexity is diverse and ranges from sampling support and scale effects to understanding the impact of variability, uncertainty and flexibility on operational efficiency and economic viability. These complexities, combined with time and capital constraints, usually do not allow all aspects of evaluation to be integrated into a single model. The model must strike a balance between simplified estimation techniques and sufficient incorporation of aspects of the project that will make a material difference to the investment decision (Nicholas et al., 2006).

Many of the well-established resource and reserve classification codes refer to a mineral resource as having some "reasonable and realistic prospects for eventual economic extraction" (JORC, 1999, 2004 and 2012; SAMREC, 2000, 2007 and 2009; and NI43-101, 2001 and 2011). The more recent versions of these codes make specific reference to the definition of resources that '… allow the application of modifying factors in sufficient detail to support mine planning and evaluation of the economic viability of the deposit' (for the definition of an Indicated Resource); and for the definition of a Measured Resource '… allow the application of modifying factors to support detailed mine planning and evaluation of the economic viability of the deposit'.

These codes offer guidelines for assessing the criteria required to define mineral reserves but do not stipulate any quantitative confidence limits associated with tonnages, grade and revenue estimates. The selection of measurement scales and their consequential impact on reserves and the financial model is likely to have a material impact on the economic viability of a mineral project. It is usually the subjective judgment of a competent person as to the appropriate resource classification based on the information provided.

Mineral project valuation has progressed substantially since the 1980s to 1990s with the emergence of CIMVAL (2003) and VALMIN (1998 and 2005) valuation codes and guidelines that govern the technical assessment and valuation of mineral assets and securities. They are not prescriptive and do not require quantitative confidence limits to be assigned to tonnages, grade and revenue estimates. Even after classification, the uncertainty of resources and reserves cannot easily be translated into quantitative risks that can be incorporated within a valuation model.

Mineral project evaluations can be placed in one of the following two broad classes: (1) determining the fair market value of a property for taxation or similar purposes or (2) determining value for investment purposes. Harrington (1987) defines value as "… the fair price that an investor would be willing to pay for a firm, or a portion of a firm, or any other asset." He claimed that value was determined by the size of the anticipated return; the date that these returns will be received; and the risk that the investor will take to obtain these returns.

Fair market value (FMV) can be defined as the value a willing buyer and willing seller may place on the property in the absence of any circumstances that would force the owner to sell or the buyer to purchase. However, FMV is usually very difficult to determine for mineral projects because of their uniqueness and lack of timely deals on which to base fair market value determinations. The investment value can be defined as the value at which a transaction would actually take place, in which case the actual investment amount may or may not equal the fair market value of a property. It is important to note that there may be a material difference in investment value between different investors, for example one investor may be interested to acquire the assets of another company because their own reserves are close to depletion and hence, place a higher investment value on the transaction. The difference between the two values depends on the specific investment requirements and constraints of the investor, (Torries, 1998).

2.4 RISK ANALYSIS APPLICATIONS IN MINERAL PROJECTS

Early sampling biases and estimation problems were identified by Krige (1951) on gold deposits in the Witwatersrand of South Africa. Later, Krige (1959) focused on the relationships between development values and recovery grades on the South African goldfields. This pioneering work by Krige in the 1950s used statistical frameworks to define the relationships between various factors such as vein width, the combined efficiency of sampling, measuring, assaying and mining, and recovery factors that affect recovery grades, and variations between predicted and actual production (referred to as resource to reserve reconciliation).

Krige (1972) defined uncertainties into two main categories, viz. ‘decision factors’ for which specific alternatives (or options) have to be taken either at the outset of a project or as the project develops; and ‘risk factors’ which were subject to errors of estimation, such as ore grades, tonnages, working and capital costs, inflation and escalation rates, plant recovery factors and waste dilution factors etc. Krige stated that with the exception of ore grades and tonnages, and possibly the calculation of plant recovery factors, the other financial and economic factors could not be defined in a completely objective way and were subject to human judgment errors.

Lowrance (1976) identified the main empirical components of decisions together with the characteristic value judgment corresponding to each component. Essentially, risk is perceived as the likelihood of an adverse event occurring. Estimation of risks is a scientific question utilising typical scientific research methods while the acceptability of risk is deemed more of a political question based on the perceptions of risk.

Previous literature, Knight (1921), Dasgupta and Pearce (1972), David et al., (1974), Vose (2002) and Kleingeld and Nicholas (2004) distinguished between risk (or variability) and uncertainty, where variability was defined as the ‘inherent stochastic nature of a mineral deposit’ while uncertainty refers to a lack of information, usually related to sampling data. David et al. (1974) work was particularly insightful as they showed that the return from open pits designed using conditionally simulated ‘real’ values was substantially above returns from open-pits designed using kriged estimates only. Authors, such as Levy and Sarnat (1984) often use the terms risk (or variability) and uncertainty interchangeably. They believe that in the area of financial investment, ‘probability beliefs are invariably subjective’, i.e. the market will impose a value for uncertainty.

In the areas of resources and reserves, it makes sense to distinguish between these two terms because they can be modelled separately. Conventionally, the focus has usually been on modelling or estimating variability. Geostatistical techniques are routinely used to estimate grade, geology and density resource models for most mineral commodities, Matheron (1973) and Krige (1951). Since geostatistical simulations were developed (Matheron, 1973 and

Journel, 1974), they have been used to model the inherent variability and compare the impact of different mining methods or support sizes on resources and reserves. Early work (Dowd, 1976); (Dumay, 1981); (Chica-Olmo, 1983); and (De Fouquet, 1985) focused on understanding the influence of technical aspects related to complex mining constraints and on quality control during production. As computer power increased, more simulations could be run and different types of simulation methods were developed that allowed more complex types of geology to be modelled.

The adoption of a holistic approach to ore estimation and evaluation was clearly elaborated upon by King et al (1982). One the main aims of the study was to reduce the possibility of gross errors in 'ore' reserve estimation through recognition that an ore reserve statement was an estimate, not a precise calculation. King (1950) emphasized the importance of geological structures in ore estimation and Miskelly (1981) focused on ore reserve reporting practices of major Australian mining companies. The Australian Mineral Industries Research Association (AMIRA) commissioned the Australian Mineral Development Laboratories (AMDEL) to make a study of ore reserve estimation in 1970 but King's report suggests that industry continued to perceive the problems of ore reserve estimation as "mainly computational".

King analysed some fifty new Australian mining ventures (coal excluded) which reached the production phase and revealed that reserves ultimately proved to be significantly less than predicted with differences ranging from serious to project abandonment. Those mines that had significant variations between predicted and actual production had estimation:realization ratios (in grade) of about 100:75 in a large gold mine, 100:70 in a major uranium mine, 100:55 in a sizeable copper mine and 100:80 in a small nickel mine. The study proposed that there is an overlap in the meaning of resources and reserves and identifies some of the key resource to reserve factors that should be included in reserve estimation, which could result in large resource to reserve reconciliation variations. The study focused on the estimation process, metallurgical, marketing and governmental factors that must be considered in order to achieve an estimate of what the reserve will yield. The study revealed that an ore reserve estimate should not only focus on the *in situ* estimate (resources) but also what will be fed to the mill or recovered (reserves). Ore estimation is the bridge between successful exploration projects (resources) and mine planning (reserves).

Although the areas of risk can be delineated as financial, technical and environmental (Dowd, 1997), it is difficult and can be misleading to separate them because they are highly interrelated when communicating risks to decision makers. The key to valid risk analysis is complete integration of all risks both within specific categories and between categories. It is essential to understand the nature of events and their associated uncertainties into risk models as drivers of the simulation procedures rather than generate probabilities of a risk occurring based purely on summaries of historical data.

Since the 90s, the impact of uncertainty on project economics became increasingly important as more marginal projects were discovered. (Ravenscroft, 1992); (Berckmans and Armstrong, 1997); (Dowd, 2000); (Dimitrakopoulos et al., 2002); (Dowd and Dare-Bryan, 2004); (Godoy and Dimitrakopoulos, 2011) and (Dimitrakopoulos and Asad, 2013) have all used a combination of objective functions and geostatistical techniques to evaluate the impact of resource risks on the mine plan and determine their probabilistic impacts on NPV. These techniques incorporate resource uncertainty in the scheduling optimization algorithm compared to traditional mine planning methods which could result in sub-optimal reserves.

Dimitrakopoulos and Ramazan (2004) presented a paper on uncertainty based production scheduling in open pit mining which emphasized the main limitations of traditional scheduling algorithms considering uncertain resource inputs. These limitations are a direct result from the input of uncertain resource estimates. Discrepancies between actual production and planning expectations arise through uncertainty about the orebody, in terms of ore grade, tonnages and quality. Traditional methods fail to consider the risk of not meeting production targets caused by uncertainty in estimated grades. Vallee (2000) reported that 60% of surveyed mines had an average rate of production less than 70% of designed capacity in the early years. Rossi and Parker (1994) reported shortfalls against predictions of mine production in later stages of production that were due mostly to orebody uncertainty.

Traditional production scheduling optimization methods do not consider risk in not meeting production targets which occur as a result of grade uncertainty and variability, leading to sub-optimal results. Dimitrakopoulos et al (2002) show the limits of traditional optimization in the presence of grade uncertainty, and the considerable conceptual and economic differences of risk based frameworks compared to methods ignoring geological risk. Ravenscroft (1992) discusses risk analysis in mine production scheduling, recommending the use of stochastically simulated orebodies to show the impact of grade uncertainty on production scheduling. He concluded that conventional mathematical based programming models cannot accommodate quantified risk, thus there is a need for a new generation of scheduling formulations that account for production risk.

A mathematical programming model was developed by Dimitrakopoulos et al (2002) based on linear programming (LP) that took into account geological uncertainty, equipment mobility and access required for scheduling and excavating mine blocks. In this scheduling approach, a probability is assigned to each block to represent the 'desirability' of that block being mined in a given period. The probability, calculated from simulated orebody models, represents the chances that a block will contain the desired grade, ore quality and quantity, including ore grades above given cutoffs, and recovery and processing characteristics.

Dimitrakopoulos et al (2002) compared simulation based model (SM) scheduling with traditional modelling (TM) and identified that the risk-based LP schedule performed substantially better than the traditional schedule when comparing the overall deviations in ore productions during the first two periods. Incidentally, the first few periods are generally the most important with respect to generating sufficient equity to pay back debts/loans and therefore require the highest confidence. Higher risk blocks are scheduled later in the life of mine plan where the time value of money has less effect than on early production periods.

Many of these papers emphasize resource uncertainty from a grade perspective. However, the author recognised that risk evaluation for diamond deposits should not only consider grade uncertainty but also geological, density and revenue per carat (in the case of diamonds) uncertainties together with mining, processing, financial and economic uncertainties (Kleingeld and Nicholas, 2004).

2.5 DISCOUNTED CASH FLOW (DCF) APPROACH

Over the past 15-20 years the techniques used in financial valuation of mineral projects have evolved. The addition of risk premiums for discount rates was popular in the mid-1980s. By the mid-1990s to 2000s most companies had adopted the Weighted Average Cost of Capital (WACC) method, sometimes with a country-risk premium. Conventional discounted cash flow (DCF) is used as the baseline for decision-making, but most mining companies now understand its limitations, Davis (1995) and Smith (2000). Firstly, the technical and financial parameters used as inputs in NPV calculations are subject to uncertainty; secondly, mine management can and do react to changing circumstances (such as rising or falling commodity prices) by adapting the mine plan. In some cases Monte Carlo simulations coupled with geostatistical orebody simulations overcome the first limitation; real options were developed to try to overcome the second one.

The DCF technique is a standard method of valuing financial and real assets where the net cash flows are discounted at some constant, risk-adjusted rate to the present value of the asset. The seminal paper by Smith (1982) described how significant sources of risk are addressed in project evaluations. Smith considered the discount rate to be a fundamental way of reflecting risk in discounted cash flow evaluations. The main constituents of the discount rate were identified as the real interest rate, mineral project risks and country risks. He noted that differences in opinion regarding the discount rate could result in variations of more than 50% in the NPV of a project.

Previous economic and finance theory proposed the use of corporate cost of capital as a discount rate. This value is the weighted average cost of the funds available to a company, including stock, debt and preferred shares referred to as the WACC from Sani (1997). The WACC is expressed as an interest rate and is calculated as follows:

$$r_{WACC} = w_d c_d + w_p c_p + w_r c_r + w_e c_e$$

where C_d is the cost of new debt

C_p is the cost of preferred stock

C_r is the cost of retained earnings

C_e is the cost of new (external) equity

$W_d + W_p + W_r + W_e$ are the respective weights which sum to one

Equation 1. Weighted Average Cost of Capital (WACC).

The derivation of the weights is not defined by a single method and may be calculated in a number of ways, such as setting the weights equal to the relative proportions of each type of finance in the company's total capital structure or according to the relative importance of each capital component to the company. If the weights are changed, it will change the derivation of the WACC and in turn, affect the discount rate.

For evaluations using an all equity basis only, the Capital Asset Pricing Model (CAPM) from Sharpe (1964), Linter (1965) and Treynor (unpublished), is most commonly used. The CAPM attempts to deal with risk and portfolio impacts at the same time. It suggests that risk can be dealt with through appropriately high discounting, i.e. it is a risk premium model that assumes that investors need increasingly high returns to compensate for increasingly high risk. The basis of this method is that the return on an individual corporate stock can be related to the stock market by expressing the relationship of the cost of equity as the sum of the risk free rate, and the products of the risk premium of market returns above the risk free rate and the beta factor (β) for the common stock:

$$r_{\cos tofequity} = r_{riskfreerate} + R_{marketreturn}\beta_{betafactor}$$

Equation 2. Cost of Equity as determined by the CAPM.

The beta (β) between a company and the market portfolio is defined as the covariance between the rate of return on the company and the market, divided by the variance of the market return, from Brealey and Myers (2003).

$$\beta = \frac{\mathrm{cov}(R_j, R_m)}{\mathrm{var}(R_m)} \text{ or } \beta = \frac{\rho_{jm}(\sigma_j \sigma_m)}{\sigma_m{}^2} \text{ or } \beta = \frac{\rho_{jm}(\sigma_j)}{\sigma_m}$$

Equation 3. Beta coefficient in the CAPM.

where R_j and R_m are the returns of the company project and market, respectively;

σ_j and σ_m are the standard deviations of the returns between the company project and market, respectively;

$\sigma^2{}_m$ is the variance of the market return; and

ρ is Pearson's correlation coefficient between the returns of the company project and market rate of return

For determining the beta of a new company project, no empirical rate of return is available and a 'suitable' proxy must be found. This is a subjective decision where the probability is very low for finding an exact match of a mining project that suitably produces the same expected rates of return over a specified time period, given the technical and economic risks.

The beta factor is a measure of a stock's volatility in relation to the overall market. It measures the part of the asset's statistical variance that cannot be removed by the diversification provided by the portfolio of many risky assets, because of the correlation of its returns with the returns of the other assets that are in the portfolio. A beta can be estimated for individual companies using regression analysis against a stock market index. It is important to consider the assumptions that underpin the beta calculation, viz.

- ☑ investors make choices on the basis of risk and return;
- ☑ all investors have the same expectations of risk and return;
- ☑ returns are measured by the mean (not the variance); and
- ☑ risks are measured by the variance.

Furthermore, when deriving the rate of return on the market, market capitalization weighted indices are preferred to equally weighted indices, Bradford (2003). This should consider the differences in support sizes (by evaluating a sufficient quantum of data) when determining a representative rate of return on the market. Biasness caused by 'thin-trading' on the stock exchange must also be taken into account in calculating the market return. If a stock is thinly traded, then it is likely that the month-end price may not arise from a trade on that day but may instead be the last-recorded price. Several researchers have devised techniques, such as the 'trade-to-trade' and Cohen estimators, for obtaining unbiased beta estimates in infrequently traded environments.

Blume (1971) and (1975) was the first to document that individual stock betas had a regression tendency towards the 'grand' mean of all stocks on the exchange. This regression bias arises when an estimation beta coefficient which is considerably higher than the average beta is more likely to be an over-estimate of the true beta than an under-estimate. Similarly, a very low beta is more likely to be an under-estimate than an over-estimate. A Bayesian type adjustment is necessary to correct for this regression bias.

As highlighted by Smith (1982), the beta factors measure the performance of company stocks relative to the stock market, but do not address the risks and characteristics of individual projects. He found that many mining companies used a discount rate of about 10% for feasibility studies of projects in low risk countries. However, there did not appear to be a theoretical basis for a discount rate in the 10% range, other than the fact that a 10% rate of return after taxes was deemed to be a reasonable rate of return on government bonds (3% - 5%, no inflation before taxes).

Three principal components for a mineral project were identified as the risk free interest rate, mineral project risks and country risk. The long term, risk free, real interest rate is based on the bond rate. Mineral project risks included risks associated with reserves (tonnage, mine life, grade); mining (methods, recovery, dilution, mine layout); process (labour, plant availability, metallurgy, recoveries, material balances); construction (costs, schedules, delays); environmental compliance; new technology; cost estimation (capital and operating); and price and market. Country risk refers to risks that are related to country specific social, economic and political factors. Smith (1982) proposed that the discount rate can be related to these three components by the equation:

$$d_{discountrate} = I_{riskfreerate} + R_{portionrate} + R_{countryrisk}$$

where $d_{discount\ rate}$ = project specific, constant dollar, 100% equity, discount rate

$I_{risk\ free\ rate}$ = real, risk free, long term interest rate (approx. 2.5%)

$R_{portion\ rate}$ = risk portion of the project discount rate

$R_{country\ risk}$ = risk increment for country risk

Equation 4. Derivation of the discount rate according to Smith (1982).

Using Equation 4, the risk portion of a project could be calculated. For example, if a project used a 10% discount rate as a base and country risk was ignored, the risk portion of a feasibility study level is 7.5% (10% - 2.5%). He derived the composition of these risks within the 7.5% risk portion; and showed the relative differences between these risk compositions for a given project phase; and lastly, calculated different risk portions for different levels of project stages, viz early exploration projects, pre-feasibility, feasibility and an operating mine. However, he recognized that his risk product values used a pre-determined value to calculate the prorate risk factors, viz. 7.5%, which would change if the 10% discount rate fluctuated. Sorentino (2000) stated that risk and uncertainty are often treated with conservatism in estimation and/or adjustment of the discount rate. Malone (1994) quoted that "… it is sound practice to be conservative ... final decision should represent a true value qualified slightly on the conservative side".

Some academics and practitioners have a view that project-specific risks (also known as unsystematic or ‘unpriced’ risks) such as geological and technical uncertainties are not correlated with the overall economy and can be completely diversified through the use of an investment portfolio (Samis et al., 2005). They believe that technical risks should not be accounted for in a project valuation model. This is usually the perspective of an investor who can selectively invest his/her money in various companies in order to acquire a diversified portfolio that meets with his/her personal requirements.

Decision-makers ‘within’ a company that only have a small number of project investments may not have the luxury of having a ‘well-diversified’ portfolio of projects. Often, projects may all be related to the same commodity, e.g. several iron ore or gold projects within a portfolio that are correlated by price, exchange rates etc. Given the relatively high investment costs for constructing and starting a mine (typically in the order of USD100s million to several billion dollars), capital and operating expenditures are inextricably linked to each site’s ability to operate effectively.

If a diamond or iron ore company is susceptible to a material decrease in commodity prices resulting in a drop in revenue, the company will likely focus its efforts to produce from its more cost effective operations in order to remain economically viable overall. This affects their ability to blend ore from different sites to attain a required threshold according to market expectations and they may be penalized accordingly, resulting in a drop in profitability for the company. It is also likely that sites with higher resource variability will result in more unpredictable production output, which may result in one or more sites (in the portfolio) not meeting planned production either in terms of quantity or quality of product (or both). This may affect the company’s entire portfolio and its ability to meet market expectation.

2.6 REAL OPTIONS VALUATION (ROV) APPROACH

Guzman (1991) noted that non-stochastic DCF methods do not take into account the flexibility of management response, based on better judgement which alters the original business plan and ultimately, changes the valuation result of a mineral project. Davis (1995) concurred with this and noted that a possible explanation of at least some of the shortfall in DCF versus ROV techniques is that the DCF approach fails to uncover the value that is attributed to active asset management, i.e. through managerial flexibility. Many mineral projects have projected reserves over the life of mine (LOM) that exceed 15 to 20 years. Conventional DCF calculations typically fail to reflect the potential value of these cash flows for longer LOM projects due to the time value of money effect resulting from the discounting rate. As a result, it is often not possible to accurately value mineral projects with long LOMs.

Miller (2002) observed that it is 'not widely appreciated' that conventional methods such as the internal rate of return (IRR) and DCF (NPV) require the assumption of perfect certainty of cash flows, even though this is rarely the case in reality. The NPV analysis of mineral assets, with respect to the minerals industry, fails to allow properly for the stochastic nature of mineral prices and cash flows. Alternative valuation methods, viz. real options that consider the stochasticity of mineral prices and cash flows, will enhance the NPV by including the 'active' or strategic management of mineral assets in response to these uncertainties, Guzman (1991), Lehman (1989), Mann et al (1992), Palm et al (1986) and Sick (1990). The two most important types of managerial flexibility that are over-looked in DCF analysis are 'operating flexibility' and 'investment flexibility'.

Operational flexibility includes any variation of operating parameters related to increasing or decreasing production supply (expansion and contraction); shutting-down; re-opening of treatment plants; re-optimizing of cut-off grades etc. Investment flexibility provides the ability to delay the start of projects should prices or technical risks be deemed too uncertain. The lack of valuing the managerial flexibility available to a mineral project will result in the mineral assets (and therefore the company) being undervalued using the traditional DCF approach, Davis (1995). The expanded value of the mining project is equal to the DCF value plus the value provided by the option premium. It is important to assess the cost of attaining this option premium (flexibility) against the benefit that it provides.

$$\boxed{\textit{Expanded Value} = \textit{DCF Value} + \textit{Option Premium}}$$

Copeland and Antikarov (2001) stated that it is unrealistic to believe that NPV captures the flexibility that decision makers have when they undertake projects and proposed that NPV systematically undervalues every project.

Lastly, the risk-averse investor may not recognize that most traditional DCF valuations assume 'flat' metal prices in the models whereas they could fluctuate considerably during the life of a project, Davis (1995). Real options analysis attempts to model the possible stochasticity of metal price over time. A newcomer to real options may be forgiven for asking why the DCF technique cannot work for options and flexibility included in the model by means of decision tree analysis (DTA). The standard process for valuing an asset is firstly to calculate expected cash flows; then secondly, discount them at the opportunity cost of capital to calculate their present value. While the first step is mostly feasible, finding the opportunity cost of capital is impossible because the risk of an option changes every time the stock price moves, Brealey and Myers (1991). The stock price is assumed to follow a random walk through the option's life.

For the above-mentioned reasons, an alternative valuation approach is necessary in which real options in terms of flexibility are considered while correctly discounting the values to attain the present value of an option.

According to Brealey and Myers (1991) the first person to have recognised the value of flexibility was Kester (1984) in an article in the Harvard Business Review. The following year, Mason and Merton (1985) reviewed a range of applications to corporate finance and in their seminal paper, Brennan and Schwartz (1985) applied option pricing techniques, first developed in finance, to the evaluation of irreversible natural resource investments using Chilean copper mines to illustrate the procedure. To simplify the mathematics, they assumed that the reserves were perfectly homogeneous and that the grades were perfectly known. From a mining point of view, these assumptions are unrealistic. Armstrong and Galli (1997); Carvalho et al (2000); and Goria (2004) have overcome this by combining geostatistics with option pricing.

Bratvold and Begg (2002) distinguished between real options thinking and real options valuation. Real options thinking focuses on assessing the value of the option to acquire information to reduce uncertainty and the value of flexibility (or options) to exploit, or at least mitigate, the impacts of uncertainties. They reserved the term real options valuation to mean calculating the value of risky cash flows from the perspective of an external investor, where the risk is priced using a portfolio of openly traded market instruments that carry a similar level of risk. Classical methods of calculating NPV not only ignore the value of real options thinking and valuation, they penalize any delay in making the investment. Experienced decision makers who use NPV are well aware of these limitations and often use “gut feel”, or “strategic considerations” to compensate.

In their paper, Brennan and Schwartz (1985) used a geometric Brownian motion based on Black and Scholes (1973) method with a convenience yield proportional to price in order to model the copper price. This was necessary to try to reproduce the natural variability of commodity prices over time. In contrast to many other commodities, diamond prices are not as volatile. Factors such as oil prices and the exchange rate are more volatile and can have a material impact on a diamond project’s value; the oil price affects costs and the exchange rate can directly influence the company’s revenue and operational costs. The author has chosen to focus on exchange rate for this study.

Many models have been developed for interest rate and foreign exchange rates, ranging from simple extensions of Black and Scholes (1973) through to Vasicek (1997) and on to the latest models with stochastic volatility. The book edited by Hughston (1996) provides a good overview of the subject. The author chose to use the Garman and Kohlhagen (1983) which is a simple extension of the Black and Scholes model – see Equation 5.

$$dS_t = \left(r_d - r_f\right) S_t dt + \sigma_S S_t dW_t$$

Equation 5. interest and foreign exchange rate equation.

In this model the drift term is replaced by the difference between the domestic and foreign interest rates. If S_t denotes the spot exchange rate at time t and r_d and r_f are the domestic and foreign interest rates, then σ_S is the volatility of the exchange rate and dW_t is a Brownian element. The Garman-Kohlhagen (GK) model is used to price European style foreign currency options and assumes that:

- Foreign and domestic interest rates and the exchange rate are constant;
- European-style options with a pre-determined expiration date;
- The market is efficient (i.e. there is no arbitrage);
- There are no transaction costs; and
- The exchange rate has a log-normal price distribution.

Most fungible commodities such as metals traded on stock exchanges, suffer from future price uncertainty. These metal markets are typically cyclical, also referred to as mean-reverting price models in real options. Forward modelling of these prices entails a complex blend of (macro and micro) economic theory, ROV principles and industry, competitor and market analyses combined with sound experienced judgement (Gentry and O'Neil, 2007).

Diamond prices are also subject to uncertainty. Future medium and long-term trends of diamond prices are highly uncertain and may be influenced by many factors such as competition, potential devaluation of stones due to conflict diamonds, the risk of synthetic diamonds on the overall trade and fluctuating market demand. The economic viability of projects that are revenue sensitive may be particularly susceptible to forward predictions of diamond prices. The assumption of flat real or flat nominal may be useful for baseline project evaluation but is unrealistic for assessing project risks.

Operational flexibility includes any variation of operating parameters related to increasing or decreasing production supply (expansion and contraction); shutting-down; re-opening of treatment plants; re-optimizing of cut-off grades etc. Investment flexibility provides the ability to delay the start of projects should price or technical risks be deemed too uncertain. For these reasons, ROV was developed as a better means of capturing and reflecting ‘real’ project flexibilities in the estimation of a NPV.

“Similar to options on financial securities, real options involve discretionary decisions or rights, with no obligation, to acquire or exchange an asset for a specified alternate price. The ability to value real options (e.g. to defer, expand, contract, abandon, switch use or otherwise, alter a capital investment) has brought a revolution to modern corporate resource allocation”, Trigeorgis (2002).

A practical definition for real options analysis is defined by Mun (2002), viz. “the application of financial options, decision sciences, corporate finance and statistics to evaluating real or physical assets as opposed to financial assets”.

A real option is the right but not the obligation to take an action (such as deferring, contracting, expanding or abandoning) at a predetermined cost, called the exercise price or strike price, for a predetermined period of time, i.e. the life of the option, Copeland and Antikarov (2001). They identified six main variables that influence the value of real options:

i. *The value of the underlying risky asset* – in the case of real options, this may be a project, investment or acquisition.
ii. *The exercise or strike price* – the amount of money invested to exercise the option if you are buying the asset (call option); or the amount of money received if you are selling the option (put option). As the exercise price of an option increases, the value of the call option decreases and the value of the put option increases.
iii. *The time to expiration of the option* – the value of the option increases as the time to expiration increases.
iv. *The standard deviation of the value of the underlying risky asset*. The value of an option increases as the volatility of the underlying asset increases – i.e. there is more upside potential.
v. *The risk-free rate of interest over the life of an option*. As the risk-free rate goes up, the value of the option also increases.
vi. *The dividends that may be paid out by the underlying asset*. Dividend payouts will decrease the option value.

A call option is defined as the right to buy the underlying asset by paying the exercise price, which was agreed upfront in the contract. At the time of exercise, the profit on the option is the difference between the value of the underlying asset and the exercise price. A put option is the converse of a call option – it is the right to sell the underlying asset to receive the exercise price. A call option is said to be 'in the money' when the price of the underlying asset is greater than the exercise price and a profit could be made by immediately exercising the option. If the price of the underlying asset is below the exercise price, the option is 'out of the money', Copeland and Antikarov (2001).

European options are those options which can be exercised only on their maturity date while options that can be exercised at any time are referred to as American or Real Options.

The main types of options can be summarized from Copeland and Antikarov (2001):

i. *Simple options* such as deferring or abandoning a project, contracting (scaling back) or expanding a project.

ii. *Switching options* are portfolios of American call and put options that allow their owner to switch between two modes of operation at a fixed cost.

iii. *Compound options* are options on options such as phased investments of a project where each phase is an option that is dependent on the exercise of the previous option.

iv. *Rainbow options* are options that are affected by multiple sources of uncertainty, not only price stochasticity. A mining project that has a combination of technical, economic and market uncertainties is a typical example.

v. *Compound rainbow options* are often necessary to model real-world applications.

The holder of an option on real assets is analogous to the scenario where the owner of an American call option on a financial asset has the right, but not the obligation, to acquire the asset at the strike price on or before the exercise date, and will exercise the option if and when it is in his/her best interest to do so, Trigeorgis (2002). The decision makers of a company (such as the board of directors) have the right but not the obligation, to make a capital investment in a project on or before the anticipated date when the opportunity will cease to exist, in order to acquire the present value of expected cash flows generated from the project. The real investment opportunities (or real options) of a mineral project corresponds with the call options on stocks.

Call option on a stock	**Real option on a project**
Current value of stock	Gross present value of expected cash flows
Exercise or strike price	Investment cost
Time to expiration	Time until opportunity disappears
Stock value uncertainty	Project value uncertainty
Riskless interest rate	Riskless interest rate

Table 1. Comparison between a call option on a stock and a real option on a project.

The option of including flexibility within a project provides management with an opportunity to adapt its future actions depending on the future environment. Trigeorgis (2002) described management flexibility as introducing an asymmetry in the normal probability distribution of NPV, making it more lognormally, or positively, skewed. This expands the investment opportunity's true value by improving its upside potential while limiting downside losses relative to initial management expectations. Most of the important payoffs of managerial flexibility can be captured in a simplified way by combining simple options as building blocks. Where many real options need to be considered in a project valuation, then the total investment opportunity can be seen as a collection of real options or compound real options.

In the absence of managerial flexibility, 'static' or traditional NPV would best value the asset. Management would make an initial capital outlay, *I*, to generate a higher present value of cash flows, *V*.

$$\text{NPV} = V - I$$

The difference between *V* and *I* is the current value of the investment. Management may delay capital investment in the project but this does not necessarily imply flexibility has been included in the project valuation.

Although the value of the immediate investment (i.e. the NPV) may be perceived to be important, the actual value of the investment opportunity is of greater importance. Therefore, an investment opportunity may still be economically desirable even if the investment is unprofitable (i.e. NPV < 0). The opportunity to invest is formally equivalent to a call option on the value of a project, *V*, with the initial investment outlay, *I*, as the exercise price.

Trigeorgis (2002) calculated the value of the investment opportunity for non-traded assets based on the Black-Scholes option-pricing formula, adjusting for a cash dividend payout (or return shortfall, δ).

$$C(V,\tau,I) = Ve^{-\delta\tau}N(d_1) - Ie^{-r\tau}N(d_2)$$

Equation 6. The value of an investment opportunity according to Trigeorgis, 2002.

where *V* is the value of a completed project
I is the one-time investment outlay
τ is the time to expiration before the investment opportunity disappears
δ is the resulting rate of return shortfall between the expected equilibrium rate of return, required in the market by investors, and its actual growth rate
r is the risk free interest rate

$N(d_1)$ and $N(d_2)$ are cumulative normal probability density functions (as defined in the Black-Scholes formula)

A conceptual real options framework was introduced by Trigeorgis (2002). It illustrates that projects may fall into different categories of evaluation complexity depending firstly, on whether the owners have a proprietary or a shared project, whereby the latter implies that management must take account of competition in their decision-making. Projects are then divided into simple options that can be evaluated as stand-alone investment opportunities and compound options, which comprise a sequence of interrelated simple options. The last strategic question faced by management refers to the discretionary nature of the decision, with respect to the timing of the investment decision. Management must distinguish between investment opportunities that allow them to defer their investment decisions ('deferrable' real options), after receiving additional information, and projects that involve an immediate investment commitment ('expiring' investment opportunities).

2.7 PROBLEM DEFINITION

The application of DCF and NPV techniques to evaluate projects is standard practice for most financial practitioners today. However, there are a number of fundamental assumptions and limitations of the input parameters of the DCF technique, which have been highlighted earlier in this literature review that can materially affect the project valuation outcome. According to Miller (2002), there are three main limitations of the DCF valuation technique when applied to projects of uncertainty:

1. The selection of an appropriate discount rate poses a problem;
2. DCF techniques tend to ignore the value of management flexibility; and
3. Investment decisions are typically viewed as "now or never" type decisions rather than as options that may be delayed.

There appears to be confusion among authors in terms of the best use and applicability of the DCF technique to value mineral projects. Ballard (1994) seems content that risk can be accommodated through a discount rate estimated based on the capital asset pricing model (CAPM) from Sharpe (1964), Linter (1965) and Treynor (unpublished). Runge (1994) on the other hand believes that uncertainties in mineral valuation are too project specific to be assessed using the CAPM. Butler (1994) regards Monte Carlo simulation methods as being impractical and is satisfied with discrete sensitivities instead. Other authors such as Lonergan (1994) and Winsen (1994) insist that management flexibility be incorporated into the valuation analysis.

Decision tree analysis (DTA) and Monte Carlo simulations (MCS) can be used to model uncertainties associated with the input parameters of the DCF equation, and while it is possible to allow for correlations between variables in some methods, this often adds to the complexity of the model and can result in erroneous results in some cases (see Nicholas, 2007). MCS can simply be defined as a mathematical method used to model uncertainty in one or more parameters of a model that calculates the expected, probability outcome. The user specifies the input probability distributions for each parameter, defines the correlations (if any) between parameters, and then runs the MCS to produce multiple realisations (draws) from each defined probability density/mass function (pdf) to calculate the expected output.

MC simulations are generally used to provide confidence intervals around an expected output, where in the interest of reducing modelling time or for very large evaluation models, or the lack of appropriate data, expert opinions can be used to define probabilistic ranges for specified parameters. Probabilistic outputs are typically generated which are used to assist in understanding the key risks for mineral projects. On the other hand, an IEM approach requires more development and modelling time than MCS and DTA techniques but provides a unique platform to incorporate technical linkages between variables at the appropriate scale, specifically focusing on relationships between resources, mining and treatment processes and the cash flow model.

The author recognises the popularity of MCS as a risk analysis tool to model the components of cash flows for mineral deposits (see chapter five of this thesis), including the modelled correlations between variables. MCS can effectively be combined with DTA to provide a more quantitative risk analysis of staged decisions for project evaluation, for example a project manager evaluating whether to design and build a processing plant at various throughput capacities with the outcome dependent on multiple assessment stages considering capital costs, operating costs, ore variability and various mine production rates etc. To this extent, DTA and MCS risk analysis techniques could be combined with an IEM approach to assess risk in a more quantifiable manner that appropriately captures the spatial correlations and their net impact on production in a staged approach.

Time series models could also be used to include correlation among parameters, such as the commodity price. However, they typically assume a fixed project life and discount the future cash flows at the traditional discount rate to derive the present value of the investment. Combinations of these techniques all introduce different aspects of uncertainty, subjectivity and complexity into the NPV result by making certain assumptions and/or assuming rigidity in the model.

The selection of an adequate risk-adjusted discount rate is subjective and is usually a function of the beta (β), derived from a twin project with similar expected cash flows but not necessarily similar extraction and/or treatment risks. The selection of an 'adequate' discount rate appears to be inextricably linked to the perceived risk of the project. Thus, care must be taken to select the appropriate valuation method to ensure that risk is not 'double accounted for' in both the components of the cash flow model and the discount rate applied. While real options also make certain assumptions that are similar to the conventional DCF approach, it ultimately produces an evaluation result that is more tractable based on the inclusion of stochastic variables and managerial flexibility while considering the owner's prerogative to exercise his/her option(s) at any feasible time.

Galli and Armstrong (1999) compared option pricing, decision trees and Monte Carlo simulations for evaluating projects. They evaluated how each method handled uncertainties such as reserves, oil price and costs; how they incorporated the time value of money; and whether the methods allowed for managerial flexibility. They concluded that despite certain obvious differences, the methods are different aspects of a more general project evaluation framework with NPV as the static base case scenario.

An alternative approach to reflect project risk adds capital expenditure (Capex) over the LOM (or specifically during the risky periods) in the cash flow model. Techno-economic factors may also be applied to the revenue component of the Inferred Resources in an attempt to compensate for project risk. Conservatism may 'creep in' where one or more of these risk factors are unwittingly included or 'double accounted' for in the valuation model. At the same time, critical risks could be overlooked and incorrectly accounted for in the evaluation model. The covariance relationships among resource variables and their interaction with reserve constraints are complex and cannot easily be modelled intuitively through expert opinion or subjective modelling methods. The problem is exacerbated when uncertainty is considered with respect to both the mean and variance of each resource and reserve parameter. For these reasons, it is necessary to reflect correctly the resource and reserve complexities through an integrated evaluation model (IEM) that more realistically captures the operational risks to which a project may be exposed to.

Resource variables are spatially distributed and geostatistical techniques are used to provide estimates at an appropriate estimation unit size. The adverse consequences of estimating values of small blocks from wide-spaced drilling and their impact on pit design and scheduling (Armstrong and Champigny, 1989), (Allard et al., 1994) and (Dowd, 1994) must be considered. These unit sizes should correspond to the selective mining unit (SMU) at which the reserve will be depleted to ensure that an IEM can be constructed at an appropriate scale.

Although previous publications, such as David (1974) referred to mining units, the SMU was defined by Parker (1980) to refer to the selective mining unit or the smallest volume that could be practically segregated to ore or waste. These were typically rectangular prisms. Polygonal boundaries were first used to evaluate blast holes but these were found to be over-optimistic with regard to dilution and ore loss along boundaries. Kriging or inverse distance models of small blocks were then tried with success. The next advance was to use hermitian polynomial correction for change of support applied to the Nearest Neighbour (point support) models to produce grade-tonnage curves for various SMUs (using rectangular prisms). The grade-tonnage curves were matched against production (measured by grade control polygons), which led to definitions of SMUs related to production rate, not bucket width (see Parker, 2012).

An IEM methodology is essential to ensure that short-scale variabilities associated with resource variables reflect the impact of mining and processing constraints at an operational time scale, which is usually on an hourly or daily basis. If this temporal scale is ignored, material errors could be incurred in the estimation of the NPV.

Mining depletion (at short-scale) of reserves is influenced by the modelling of resources at a specified estimation unit size, i.e. the estimation unit size must be small enough to capture some of the inherent variability of the resources estimates at short-scales while taking cognizance of the adverse consequences of estimating values of small blocks from wide-spaced drilling. Ideally, estimating or simulating within small SMUs will provide a more accurate representation of the short-scale spatial and temporal risks that could be encountered when the mine plan is depleted.

The larger the block size or panel is, the 'smoother' the estimate will be resulting in short-scale variabilities not easily being modelled or detected. This can create a false sense of security that all risk 'averages' out over a large panel scale or annual time period but this is not necessarily true and short-scale variabilities can materially impact upon the business model as demonstrated in Nicholas et al. (2006, 2008). The use of large blocks can result in an over-estimation of tonnage and under-estimation of grade, particularly where there is a final stage of detailed sampling to select ore from waste. NPV can be materially in error if pit limits and production schedule are based on the grades and tonnages of large blocks with wide sample spacing lines (see Zhang, 1998 and Parker, 2012). The error is exacerbated where processing costs are high. Waste that will be segregated by further sampling information is incorporated in the large-block estimates, and the associated processing costs attributed to it, which can negatively impact the cash flows. Consequently, the scale at which resource risks are defined has a direct bearing on the mining and treatment (reserve) risks and ultimately, the overall project risk.

The key components of a conventional DCF style calculation for a diamond project are:

1. *Revenue*, derived from the calculation of extracted ore tonnages, recovered carats, grade and revenue per carat and subject to a foreign:local exchange rate.
2. *Costs*, which comprise variable and fixed costs; the former are usually estimated from the unit cost per item multiplied by the quantity of items. Both the unit cost price and the estimated quantities are subject to uncertainty.
3. *Capital* estimates may broadly be categorized into construction capital costs and working and on-going replacement capital, which are related to resource and reserve uncertainties. An inversely proportional relationship is generally established between the capital expenditure (capex) and the operational expenditure (opex) depending on management strategy and their appetite for risk.

4. *Discount rate* approach for diamond projects can vary radically between companies. One method entails the use of the weighted average cost of capital (WACC) plus any additional premium for technical and/or country risks. While the WACC and country risk component can often be derived through market measures or ratings from financial houses, the technical risk component is unique to each project and the derivation thereof is mostly subjectively derived.

Time affects the estimations of each of the above-mentioned cash flow components. The inverse relationship between capex and opex is one of the most important considerations that management has to comprehend in developing their project into fully-fledged operating mine. Projects that have strong operating margins, i.e. where their opex profile is relatively low in comparison to the associated revenue income, can afford to have more of a contractor-style model with operating leases leveraged off their operating margins (e.g. mining equipment leases with banks or vendors such as Caterpillar etc.). Management of these projects may elect this operating model to preserve their balance sheets, i.e. by not having to allocate costs to capex, they do not require debt from banks that may result in high gearing (debt to equity) ratios penalizing the company by analysts or rating agencies; or increased capex costs may result in the project being un-fundable. Conversely, projects that have low operating margins may need to capitalise expenses on their balance sheet to preserve their 'skinny' operating margins.

These capex versus opex considerations have a profound impact on management's ability to successfully get the project into production, and furthermore, to optimize the NPV of the project. The mineral evaluation process must consider that operating decisions made early in the life of a mine will likely affect the remaining LOM and influence financial returns. Average mining and treatment grades, capex and opex relationships and operational flexibilities are key considerations that will materially impact the NPV. For example, management's decision to install only one (instead of two or more) crushers as part of their overall processing circuit may reduce the initial capex requirement but severely limit the processing plant's throughput ability and potentially affect the grind size going into the mills, which could reduce overall recoveries with the net effect of reducing the operating margin.

The combination of the magnitude and timing of operating and capital costs will ultimately influence the project evaluation analysis. The timing and magnitude of mining revenues depends upon factors such as ore reserves, production rates, metallurgical recoveries, commodity prices and markets. These variables are often extremely difficult to estimate with high confidence. The overall operating environment is another area of concern. In recent years, the national and international operating environments of mining properties have been severely impacted upon by environmental and other regulatory requirements (Gentry and O'Neil, 2007). These constraints have invariably increased operating and capital cost requirements and, in some cases, delayed production activities resulting in lower than expected NPVs.

The challenge is to try to capture correctly the main risks in an evaluation model and quantify their impact on NPV with an associated confidence limit. Finding the most appropriate method and position in the evaluation model to reflect these risks is part of that challenge. The method of increasing the technical risk premium (project risk portion) of the discount rate that is applied to cash flows based on work by Smith (1982) is not recommended as a viable approach to capture technical risk in project evaluation.

The standard NPV formula is well known where *CF* refers to the cash flow in each period *i* and *r* is the discount rate. This equation can be rewritten as a weighted sum to illustrate the impact of the discount rate on the variance of the DCF (see Equation 7).

$$NPV = \frac{\sum CF_i}{(1+r)^i} - I_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$DCF = \sum CF_i * \left(\frac{1}{(1+r)^i}\right) \quad \text{or} \quad \text{DCF} = \sum CF_i * w_i \qquad \ldots\ldots\ldots\ldots(2)$$

Equation 7. Discounted Cash Flow (DCF) equation for deriving the Net Present Value (NPV)

When risk analyses are conducted to ascertain the impact of the uncertain cash flows on a project’s NPV, the mean net cash flow in each period, *i*, will be reduced by the weighting factor, *w*. This penalizes cash flows in later years. The weighting factor, *w*, increases exponentially as a function of time, *i*. If technical risk is included in the discount rate, it implies that technical risks increase exponentially over time, which in most mining scenarios, is untrue. Usually, as more information is gained over time about the ore body complexity, the mining and processing methods are adapted to become even more efficient than in earlier years where less information was known. Thus, in reality (and using ‘real options thinking’) technical risks actually decrease over time contrary to the DCF (NPV) formula in the equation above.

ROV is acknowledged by the author as an improvement upon conventional DCF (NPV) techniques with respect to catering for flexibility options. However, it includes its own set of subjective assumptions and is often perceived as a ‘black box’ approach by conservative mining managers, thus limiting its practical application to mining projects. Real options should be marketed as a complementary tool or enhancement to DCF and NPV analyses rather than a replacement technique.

ROV application for mining projects seems to have more benefit in modelling economic parameters such as commodity prices and interest rates than addressing technical spatial and temporal scale uncertainties, raised in this literature review. For these reasons, an alternative evaluation method was sought that could correctly capture spatial and temporal scale technical attributes from resources and reserves in the financial model; and did not deviate too far from the conventional DCF (NPV) approach; and allowed flexibility options to be easily evaluated.

In order to quantify the impact of the selected scale on valuation, it is recommended that the process incorporate a quantitative impact assessment. This assessment should include the modelling of unsystematic (specific) risks for resources and reserves, and systematic (market) risks, such as foreign exchange variability and costs of commodities such as oil, steel, concrete. This would allow confidence limits around project valuation to be quantified as objectively as possible in a transparent and scientific integrated, evaluation framework.

2.8 GAP ANALYSIS

The problem of estimating the NPV of a mineral project, especially for diamond deposits, based on a limited quantity of sampling data is complex and not yet clearly understood (as discussed in the ‘Problem Definition’ – see Section 2.7). For the purposes of this study, it will be assumed that the main risks affecting project valuation are associated with the technical and financial aspects of resources and reserves (Smith, 2000). The main reasons for this are:

- Uncertainty exists around the means and variances of spatial variables and their correlated impacts on production estimates. Many conventional financial models are based on some form of kriged resource estimate, which is recognised as the best linear unbiased estimate. The combined responses of multiple non-linear resource realisations on mining and processing constraints and their cumulative impact on the cash flow model have not yet been meticulously documented.

- Variances within, and covariances between, spatial resource variables and their combined impacts with non-spatial mining and treatment parameters are often characterised by complex relationships. They are influenced by several parameters, such as limited data, geological complexity, and mining and processing requirements. The evaluation challenge is further complicated when accounting for the temporal scales at which production outputs are measured and collated, which can be critical to projects that have high resource variabilities combined with limiting mining and treatment constraints.

- Literature exists for risk quantification and modelling of spatial resource variables using geostatistical techniques (Dowd, 1976, Parker, 1977, Thurston, 1998, Journel and Huijbregts, 1978, Dimitrakopoulos et al., 2002b, Krige, 1951). Authors such as Dowd and Dare-Bryan (2004) highlighted spatial linkages between resource modelling and mining (blasting) to predict dilution/spreading of grade in muckpiles. Often production figures are calculated from annually derived estimates that do not necessarily capture the impact of short-scale variabilities on planned mining and processing constraints. This may under or over-estimate the true financial value of a mineral project. The combined effects of spatio-temporal scales are evaluated at a block by block (SMU) scale in this thesis.

- A robust methodology has not yet been documented to evaluate the financial costs/benefits of operational and management flexibilities in time as a risk mitigation strategy, given the combined effects of spatial resource uncertainties, mining and treatment constraints and economic uncertainties, e.g. foreign exchange rates.

- Quantitative methods for estimating the accuracy and confidence limits for NPV of mineral projects have been attempted before, but no consistent quantifiable method exists. The limitations of Monte Carlo Simulations (MCS) and sensitivity analyses as risk analysis tools for NPV estimation have been documented by Nicholas et al. (2007). Other approaches adjust the discount rate applied to cash flows in an attempt to represent project and/or technical risk (Smith, 1982). These techniques may unwittingly introduce material biases in the project value as they do not appropriately consider the spatial covariances between mine blocks or the short-scale temporal scale effect on operational constraints. They also tend to focus mainly on 'downside' risks and ignore the 'upside' opportunities.

As a result of the above-mentioned complexities, the confidence in a NPV estimate cannot easily be quantified using any closed-form analytical or mathematical solution. Complex, non-linear relationships between resources, reserves, financial and economic parameters requires a simulation model to be developed to provide a solution.

2.9 OBJECTIVES

The main objective of this research is to compare quantitatively conventional evaluation methods with an innovative, 'spatially-aware' IEM evaluation technique that captures the non-linear effects of the response variables (such as recovery) related to production constraints taking into consideration the short (block-by-block) spatial and temporal scales. Using this IEM approach as a platform, the author also aims to understand the combined financial impact in NPV terms of economic uncertainties (assessing foreign exchange rate variability) with the impact of resource variabilities (grade, geology, density etc.) on reserve constraints.

Research in this thesis transcends conventional discipline boundaries and covers the areas of geostatistics to generate resource estimates and conditional simulations; mine planning to constitute reserve estimates; financial theory and economics; and real options theory to evaluate risk mitigation methods using spatial resource simulations. This thesis focuses on quantifying the financial impact of the non-linear relationships between stochastic spatial resource variables, production constraints and uncertain economic parameters.

Empirical work is conducted to quantify inaccuracies in the NPV estimates by comparing global versus local evaluation methods for mineral projects. By using a virtual orebody (V-Bod) as a completely known reality, the NPV can be re-evaluated as more sampling data are acquired, and confidence limits calculated. The author also synthesizes the production and financial impacts of limited sampling data on resource estimation and the temporal scale at which reserves are evaluated.

Chapter 3 : Experimental Designs

3.1 INTRODUCTION

3.1.1 Experimental Overview

This chapter describes the experimental designs that need to be developed to provide solutions to the estimation and evaluation problems outlined in chapter two of this thesis. Begg and Bratvold (2001) identified the need for an integrated approach to assess the impacts of uncertainty on oil and gas investment decision-making. Later, Kleingeld and Nicholas (2004) recognized the need for a similar integrated evaluation approach to understand the impact of resource variability on the business model for diamond deposits.

For diamond evaluation, input resource parameters such as geology, grade, revenue per carat and density all have associated uncertainties. Each of these parameters may be composed of several contributing variables that are uncertain, for example the mineralization, structural and genesis models for geology. These parameters in turn may be correlated with each other. Standard practice usually involves using a single resource and reserve model whereupon sensitivity analyses are conducted but these do not adequately capture the range of variation associated with the compounding effect of resource uncertainties.

For these reasons and, as discussed in chapter two of this thesis, an integrated evaluation modelling (IEM) methodology was deemed necessary to capture correctly the correlations and system linkages of resource variables on the mining and processing constraints of a diamond project (and later, extrapolated to other non-diamond mineral projects). The IEM approach attempts appropriately to capture, replicate and model the key linkages between resources, reserves and the financial model. Complex resource estimation problems are often expressed through ‘simplified’ mathematical equations to solve a global or local geostatistical problem.

The production and financial impacts of non-linear resource-to-reserve relationships cannot be approximated using a closed-form mathematical solution as each project has its own set of resource and reserve variables, which interacts with mining and processing constraints in a sequential, non-linear and unique way. The effect of a non-linear valuation function was first recognized by Parker and Switzer (1975) and led to the use of conditional probability distributions for each block. Later, Dimitrakopoulos (1998) noted that mining transfer functions are usually non-linear. An average type block model may not provide an average map of the response of the uncertainty; and the simulation technique selected for modelling must be evaluated in terms of mapping the response uncertainty. In addition, there is a direct (and often iterative) relationship between reserve constraints (e.g. cut-off grade classes, mining and processing routes and costs) and assumptions in the financial evaluation of a project.

Each deposit may have several resource variables (e.g. grades of one or more primary or secondary variables, density and rock types) that all have inherent but different variabilities. These resource variabilities could materially affect a project's forecasted production estimates, i.e. the non-linear impacts of actual variabilities of the ore body on the designed mining and processing constraints. Thus, the NPV of a project is directly influenced by the actual variability of its resources in relation to the planned mining and processing constraints, and management's ability to mitigate any negative-impacting issues affecting its business plan. The greater the perceived variability of key variables, the more the mine has to plan to adapt its mining and processing schedule to accommodate variability in the ore feed.

The author considers that failure to account correctly for spatial and temporal risks, by estimating the 'average' annual production totals instead of accumulating the effects of the short-scale (e.g. daily) interactions of resource variables on the mining and processing constraints into annual production totals, may result in material errors in estimating a mineral project's value.

This evaluation consideration may be defined as:

$$\mathrm{E}\left[f\left(x\right)\right] \neq f\left[\mathrm{E}\left(x\right)\right]$$

Equation 8. A mathematic expression of the mining evaluation conundrum.

Equation 8 implies that the expected value 'E' of some function $f(x)$ is not equal to the function (f) of the expected value $\mathrm{E}(x)$. For the purposes of this thesis, the function $f(x)$ may be thought of as a transfer function, i.e. the modification of resources to reserves by considering the appropriate mining and processing constraints for a particular mineral project.

The expected value E(x) refers to the expected value derived from conditional simulations of the resource.

Equation **8** translates into two different problems, defined as follows:

1. If each conditional simulation representing a resource realisation for a specified variable (e.g. gold grade) is run one at a time through the mining and processing transfer function to derive financial outputs, and the expected value of these outputs calculated, then this does not equate to running only the expected value of all the resource realisations (statistically referred to as an 'E-type' estimate) through the same transfer function to produce an expected NPV; and

2. If each mining block (or selected mining unit, SMU) is run through a transfer function on a block-by-block basis adopting a 'bottom-up' evaluation approach that accounts correctly for temporal scale in calculating production tonnages and revenues, this may not be equivalent to estimating the average annual production tonnages and revenues derived conventionally from running annual averages through the same transfer function using a 'top down' evaluation approach.

The first problem defined above compares the impact of resource realisations (using conditional simulations) and an 'E-type' estimate with a conventional linear estimate, such as ordinary kriging. This demonstrates the production and financial impacts of considering resource variabilities, through non-linear resource modelling. The second problem does not necessarily require the use of conditional simulations to demonstrate its impact and makes use of kriged estimates as the best linear, unbiased estimator of resources. It aims to compare relative NPV estimation accuracies for a conventional 'top-down' evaluation approach with a 'bottom-up' evaluation approach using an IEM.

3.1.2 Outline of Case Studies

Three different case studies are discussed in chapter four of this thesis to demonstrate the production and financial impacts on non-linear, resource-reserve-financial relationships of an open-pit diamond operation, an underground diamond operation and an open-pit gold mine. While a unique IEM was developed for each mineral project, the author conducted variance analyses to attempt to define a generic evaluation approach for future evaluation methods. For each case study, outputs were expressed in production totals, cash flow and NPV terms.

The three case studies demonstrate different perspectives of risk evaluation complexities and support the hypothesis that the spatial characteristics of some mineral deposits need to be evaluated on a block-by-block basis to determine their non-linear impact on production constraints and the financial model. This is a key determinant of the IEM approach as, although there are common trends for most mineral projects, the unique relationships between the degree of resource variability, the combined impact on production constraints and correlations with financial and economic parameters, will likely require each mineral project to be uniquely assessed.

The objective of the first case study is to evaluate the impact of several compounding mining constraints on the resource model, based on an ordinary kriged estimate, using an IEM approach versus conventional evaluation approaches. The second case study quantifies the impact of several processing/plant constraints on resource variability (modelled using conditional simulations) for an open-pit diamond operation. The purpose of the third case study is to evaluate the combined non-linear resource-reserve effects of mining and processing constraints on resource variability (modelled using conditional simulations) for an open-pit gold operation. An additional aim of this last case study is to demonstrate that the IEM approach can readily be extended to other mineral commodities besides diamonds.

A synopsis of each case study is provided below.

1. A Canadian underground diamond operation:

The aim of this case study was to assess the impact of several compounding mining constraints on the resource model, considering the geometrical variability of the top surface of the dyke (or an ‘ore-bearing’ vein), the thickness of the deposit, and grade. The author developed a unique ‘block-by-block’ spatial evaluation technique to evaluate the impact of these resource variabilities using ordinary kriging of the deposit, which was then run through an IEM to generate financial outputs.

One of the challenges of this deposit was that the drilling data were spaced too far apart (grid spacing ranged from 25m to 75m) to accurately detect short-scale variability of the geological geometry. This was confirmed by a series of geological cross-sectional maps (or 'face maps') taken from the side-walls of the main mine drive. To simplify the evaluation problem a geological face-map from one of the main exposed mining areas was assumed to be representative of the entire deposit. A detailed short-term, spatial mine plan was not available at the time so a simplified short-term mining sequence and schedule was imposed on the resource; this schedule adheres to the relevant mining and processing constraints.

The author re-developed the financial model because the mine's existing financial model was not amenable to risk analysis; i.e. many of the cash flow input parameters (i.e. revenue and operational costs) were grouped together and multiple risk and sensitivity scenarios could not easily be carried out.

2. An open-pit diamond operation:

The objective of this case study was to assess the combined impact of three resource variables (grade, density and yield variability) on the NPV estimate using conditional simulations for each variable and to evaluate flexibility options for mitigating risks predominantly from a processing perspective. While ordinary kriged estimates were accepted as the best linear, unbiased estimator, independent conditional simulations of the three key variables were generated so as to more effectively quantify the resource variabilities.

An IEM approach was used to assess the impact of resource uncertainties at the scale of planned operational depletion, considering firstly the mining sequence and schedule, and then the impact on the treatment plant recovery model. The main objective was to express financial risk as a function of resource and reserve uncertainties at the appropriate temporal scale.

3. An open-pit gold operation:

The objective of this case study was firstly, to assess the short-term impact of grade variability (modelled using geostatistical conditional simulations) on both mining and processing constraints; and secondly to run each realisation through the financial model to produce confidence limits around production outputs, cash flow and NPV. Particular emphasis was placed on how best to evaluate the impact of mining selectivity and stockpile management for variable grades represented by geostatistical simulations. Secondly, this case study also demonstrated that the author's IEM methodology was applicable to other mineral commodities, besides diamond deposits.

The author also evaluated the impact of stochastic variables on cut-off grades in terms of their impact on production totals and financial indicators. Static global cut-off grades were compared to scenarios where the variable cut-off grade was determined by stochastic parameters to assess the impact on the planned cut-off grade policy.

3.2 BACKGROUND AND OVERVIEW

This section provides an overview of the geostatistical, risk and financial backgrounds that are pertinent to the development of experimental designs and case studies presented in this thesis. This is not intended to be a comprehensive assessment of each discipline but rather to highlight the appropriate issues that need to be understood to facilitate the coherent understanding of chapter four and the relevant case studies.

3.2.1 An Integrated Evaluation Model (IEM) Approach

The IEM methodology was developed to provide a more accurate evaluation method to assess risks for mineral projects across the entire resource-reserve-financial platform. The IEM aims to highlight previously unforeseen risks and opportunities in the conventional business models of mineral projects by directly linking resources to mining and processing constraints with the financial and economic model. Results from this approach can be demonstrated to be materially different from conventional evaluation approaches, which are based on linear production estimates that ‘average out’ in quarterly, annual increments. The IEM methodology was designed to add value to a single mineral project (e.g. evaluating Conceptual/Pre-Feasibility/Feasibility risk studies) or could be applied to existing operations (e.g. evaluating expansion/contraction/reconciliation studies). Although beyond the scope of this thesis, the approach was also extended to evaluate risks within a portfolio of projects to determine the combined effect of resource variabilities or production limitations on the overall business model of a large mining corporation.

The IEM methodology was based on a 'bottom-up' approach that is unique in that it follows every block through the mining and processing value chain. It captures correctly spatial variability of resource attributes in the ground (grades, deleterious elements, processing characteristics, etc.). This variability is then propagated through the processing value chain at a mining block (or SMU) scale. The 'bottom-up' (as opposed to 'top down') evaluation approach is necessary to capture correctly the non-linear resource characteristics and their correlated impacts on the reserve model, with specific regard to the key mining and processing constraints. Results are easily translated into clear, financial metrics with confidence limits that provide decision-makers with a clearer understanding of the material risks and opportunities in their business models.

The two main attributes of the IEM methodology are firstly, it correctly deals with the spatial resource characteristics of block models at the appropriate temporal scale; and secondly, direct linkages are created between the resource–reserve–financial models within a single software environment. This allows multiple scenarios or 'options' to be accurately and rapidly assessed for a project and the cost/benefits of implementing risk mitigation strategies easily evaluated. Some of the key benefits of the IEM methodology are:

- ☑ Confidence limits around all production (tonnes, grade, metal) and financial outputs (NPV, IRR, cash flows per month/quarter/year) can be statistically quantified;
- ☑ Costs/benefits of various risk mitigation strategies (e.g. stockpiling/blending) can be quantifiably assessed;
- ☑ The financial impact of reducing/increasing production (e.g. initial processing plant scaling and planned expansions) can be evaluated relatively straight forwardly;
- ☑ A more realistic short-and long-term assessment of the impact of mining and processing constraints on resource variables can be produced in clear, financial terms;
- ☑ Different financial scenarios (e.g. taxation structure, price or cost forecast, exchange rate) can be evaluated by correctly considering the resource-to-reserve spatial and temporal scale attributes of a particular project; and
- ☑ The bottom-up approach within the IEM framework can relatively easily be extended to evaluate and cost different processing pathways, energy requirements/carbon footprint etc.

The IEM toolkit is not an 'off the shelf' software solution but is rather a bespoke solution of MS Excel spreadsheets with Visual Basic Application (VBA) programming codes that link each component of the evaluation pipeline together. This was later developed into a web-enabled, dot.net programming environment linked to a SQL database for mining consultancy purposes but is beyond the scope of this thesis. In the IEM framework, resource models (such as the conventional kriged estimate and multiple resource realisations) are linked directly to the mine plan outputs derived from mine sequencing and scheduling software (e.g. NPV Scheduler, Whittle and Mine2-4D). This customized software captures both the mining constraints (e.g. mine call factor, mining rate, ore selection) and processing constraints (e.g. plant call factor, recovery factor/model, ore blending requirement) per period and applies these constraints in the appropriate sequence to each mine block (or SMU) for each resource realisation.

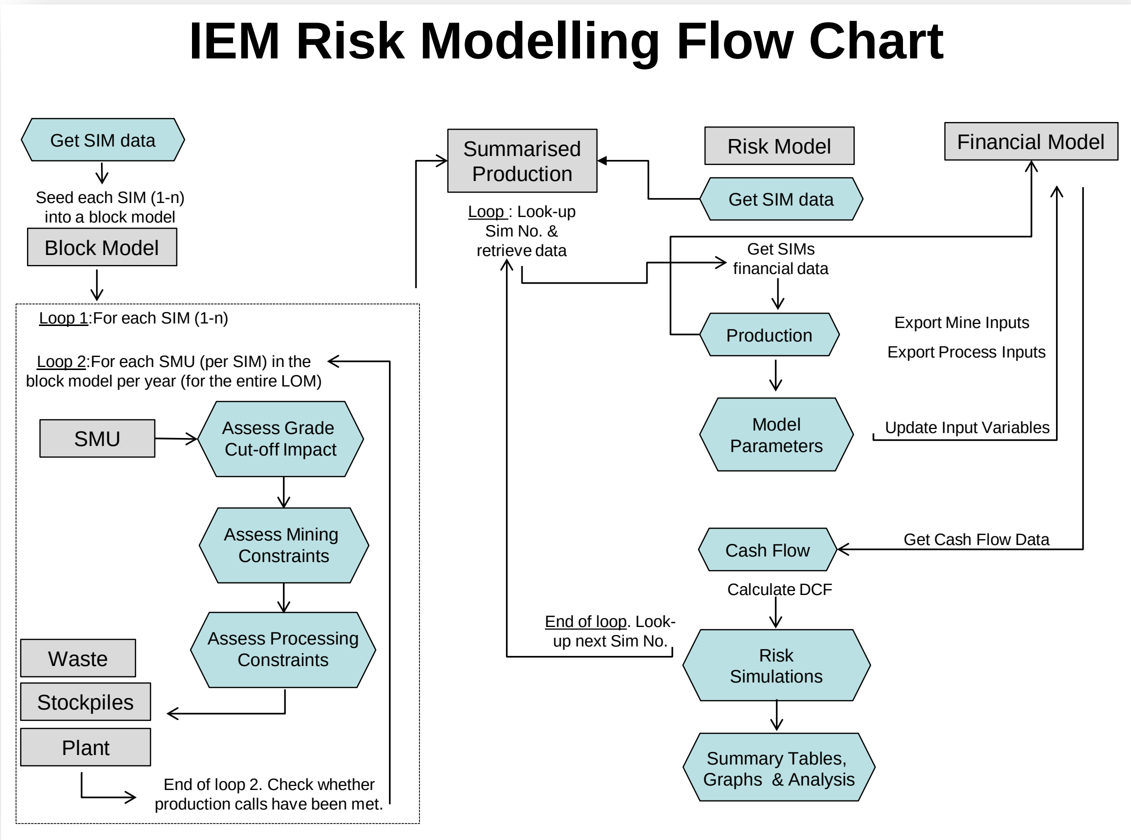


Figure 3. Programming flow chart for an Integrated Evaluation Model (IEM) Approach. The left side of this diagram collects *n*-number of conditional simulations, 'Get Sim data' and seeds this data into each SMU in the 'Block Model' then the algorithm is run to calculate the impact of production constraints 'Access Grade Cut-Off Impact' for each conditional simulation per SMU and results stored in the 'Summarised Production' tab. The

right side of the diagram pertains to running each of the ‘Summarised Production’ outputs per simulation through the ‘Cash Flow’ model to generate financial outputs for each ‘Risk Simulation’ and is captured by the ‘Summary Tables, Graphs and Analysis’ tab.

Figure 3 provides an overview of the programming logic needed to develop an IEM approach. While this is a generic approach, the details of resource modelling and reserve constraints are unique to each project and the approach customized accordingly. In the design and development of an IEM framework there are several considerations that are characteristically similar for each of the three case studies discussed in chapter four.

The geological, sampling data, resource, reserve and financial modelling characteristics common to most project evaluation studies are discussed below.

3.2.2 Geological Modelling

The particulate nature of diamonds, their size, shape, quality, colour and value are important factors in the accurate estimation and evaluation of diamond deposits. Diamond occurrences in nature are rare and are usually measured in parts per billion, whereas most other mineral commodities are measured in parts per million, parts per thousand or in percentages (Kleingeld and Nicholas, 2004).

Diamonds are brought to the earth’s surface in volcanic host rocks, principally kimberlite. Most of these primary source rocks or kimberlite pipes do not contain diamonds, and those that do are very rarely economic (see Gurney et al., 2005 for further information on the economic potential of kimberlites and lamproites). Depending on whether diamonds are contained in kimberlites or placer deposits, they are either free or locked up in the host rock. Though diamond is the hardest natural substance, it is brittle, which makes it susceptible to breakage during its release in either sampling, extraction or treatment.

Geological modelling is an essential first step in the estimation process, as typically the variability of grade and ore type between lithologies is much higher than the variability within individual lithologies. The importance of a good geological model forms the foundation for estimation and evaluation modelling, and the use of geostatistics to assist in developing the geological model has been recognised in the past by numerous practitioners such as Parker (1997).

In developing a geological model for kimberlites, there are two main considerations, viz. defining the overall pipe geometry and the quantity and dimensions of internal lithologies within the overall pipe geometry. The delineation of the pipe geometry requires the outer boundaries of the kimberlite pipe to be demarcated in order to distinguish between kimberlite and waste (or country) rock. In practice, delineation of the pipe geometry is very dependent on interpolation between relatively few pierce points from core drilling. The ore/waste contact is usually sharp, while the internal boundaries are often gradational and require interpretation. This high degree of interpolation and interpretation can result in uncertainty around the volume estimates.

Diamond-bearing material within a kimberlite pipe is variable and is the product of different depositional processes and the mixture of country rock fragments and kimberlite-derived constituents. As a result, different kimberlite lithologies can be recognised within the pipes. Lithological boundaries define zones of similar geological and diamond emplacement characteristics. Uncertainty is introduced into the lithological boundaries as it is based on interpolations between only a few intersections from core drilling.

An understanding of these lithological zones and the boundaries between them is essential for estimation purposes. This is necessary so that the geostatistician can model a semivariogram using only samples that fall within the boundaries of the delineated lithology. Kleingeld and Nicholas (2004) postulated that where lithological zones are appropriately delineated in diamond pipes, uncertainty in grade estimates and/or revenue estimates could be substantially reduced, which will improve the accuracy of the overall estimate. The definition of lithological zones and the boundaries between them are defined from multiple datasets, including geological, geochemical, geophysical and structural. Each of these has uncertainties. Geological zones must be defined at a scale appropriate to the sampling, evaluation and mining processes.

Previous unpublished work by Parker and Brisebois (1999) simulated the kimberlite pipe radius in an ‘unrolled’ space using polar coordinates by means of a simple kriging (SK) sequential gaussian algorithm. The conditioning data were pierce points of the pipe boundary in core holes. Their client would not provide permission for them to publish the work, pulling it out of a presentation in a conference held on conditional simulations in Perth in 1999.

Later, Deraisme and Farrow (2004) were among the first authors to publish a paper on defining lithologies for kimberlite estimation and used the concept of simulating the sampling of an ore body, combining geostatistical simulations with geological modelling to produce a risk analysis of the outer kimberlite pipe boundaries. Their studies focused mainly on the quantification of uncertainties in the geological modelling of kimberlite pipes. A simulation approach was used that provided maximum flexibility to reproduce realistic limitations of the sampling procedure. The idea was to simulate several 'possible' pipes with their internal geology and estimate them by kriging. While the use of geostatistical simulation and geological modelling is not new, this paper was the first published article to address the specific problem of analysing uncertainty related to kimberlite pipe boundaries.

Future development of many kimberlite mines depends on mining at deeper levels at which sampling and operating costs are likely to increase significantly. Investment decisions require accurate resource estimates associated with quantified confidence limits. Usually, one of the first sampling decisions requires the selection of the 'right' number of drill holes to sample the pipe representatively to yield data to calculate estimates within acceptable confidence limits.

Once a geological model has been developed, the required sampling strategy for grade and revenue determination must be defined. This involves establishing sample support size (volume), sample frequency (density) and sample spacing (spatial distribution). The sample size used is a function of the complexity of the orebody and the required level of confidence. During exploitation, selective mining is undertaken locally to 'footprint' (also referred to as fingerprinting in some industries) the detailed diamond characteristics per lithology to help in forecasting the diamond assortment for planning purposes.

Numerous tools are used to produce an estimate for the dollar per carat revenue and the diamond assortment profile such as size frequency distributions (SFD), cumulative pareto-type distributions and extreme value modelling. Further modelling may be necessary to account for diamond breakage in the recovery process, under-recoveries due to plant inefficiencies and differences between bottom cut-off sizes between sample and production plants.

3.2.3 Sampling Considerations

The primary objective of including more samples in a deposit is to reduce uncertainty associated with resource and reserve variables. The natural variability of these variables within the deposit cannot be reduced by additional sampling, only uncertainty can be reduced. Variability must be managed via the scheduling process in mine plans to allow selective mining of the orebody. The collection of data (via drill holes, conveyor belt sampling etc.) for estimation and modelling purposes increases the amount of information (i.e. reduces uncertainty) but there are several sampling challenges which can lead to potential estimation errors if not addressed correctly.

The sampling theory developed by Pierre Gy between 1949 and 1951 related mainly to the sampling of broken ore (see Figure 4). Gy later elaborated upon his earlier work (1977, 1982 and 2004) focusing on the sampling of particulate and discrete materials. Gy's sampling theory focused mainly on the quantity of material that should be taken from a mass of broken rock or mineral to be representative of the original mass. Later, Bongarcon and Gy (2001) delved into more detail on components of the total sampling error such as the liberation factor, which is of particular importance in the calculations. While these broken ore sampling techniques are relevant to metallurgical and processing evaluation stages, they are not directly applicable to the *in situ* sampling of kimberlite deposits using drilling methods.

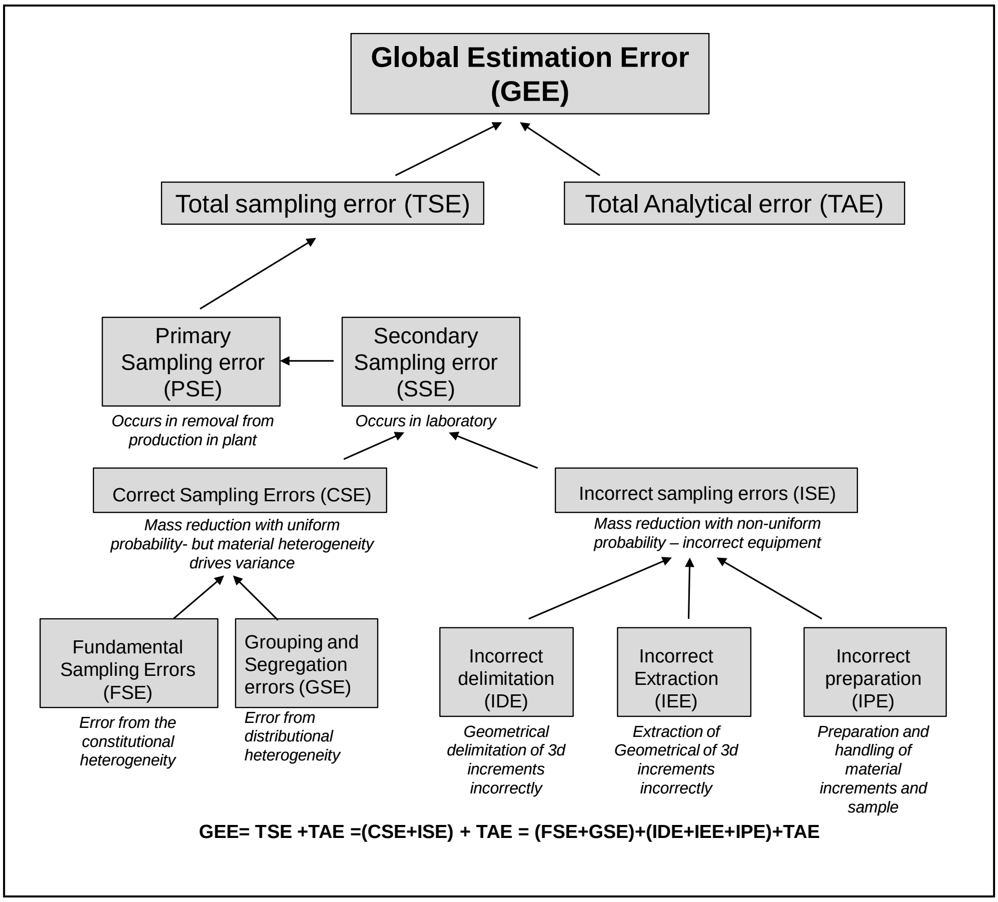


Figure 4. Relationship of sample related errors developed by Pierre Gy, 1949 to 2004.

Kleingeld and Lantuéjoul (1992) recognized that the sampling of a deposit where the mineralisation was irregularly distributed (or 'spotty') presented a number of unique sampling challenges. The first of which was a geometric problem where a sampling bias may occur if the 'spotty' mineralisation patches were not intersected often enough, whereas in mining all the 'spotty' patches would be extracted. The second challenge was that the mineralisation within each patch was highly variable. Kleingeld and Lantuéjoul identified that the impact of the second factor could be reduced if a sample of the same size as the patch was taken. The combination of these two factors was considered to account for the observed natural variability of the grade within a kimberlite deposit.

Thurston (1998) later studied techniques to develop a quantitative understanding of the variability of sampling protocols used to evaluate kimberlite deposits. He simulated a number of different sample protocols using a simulation method that took into account the statistical and spatial characteristics of the diamond distribution within the kimberlite deposit (Kleingeld et al., 1996). He found that the number of samples and sample size were important depending on whether global or local estimation was being carried out, and could be directly related to the appropriate confidence limit and accuracy of the estimation technique.

For the purposes of this thesis, it has been assumed that resource estimates do not contain any material sampling errors. This assumption is based on the premise that for diamond deposits the nugget effect and coefficient of variation for grade would be low due to sufficient number of drill holes that have intersected the deposit to mitigate against potential sampling bias due to ‘spotty’ mineralisation; and secondly, that the majority of drill holes are large diameter drill holes (LDD) recovering sufficient macro diamonds (often corroborated by micro-diamond estimation analysis assessing the total ‘diamond’ content curve distribution). It should be noted that assessing the uncertainty in revenue per tonne (from a sampling error perspective) is a topic in its own right that deserves its own thesis and has been eliminated from consideration in this research.

3.2.4 Resource Estimation

3.2.4.1 Diamond Estimation Complexities

Kleingeld (1987) first discussed the difficulty of diamond estimation and evaluation in relation to other mineral commodities in his doctoral thesis on discrete values, then later Thurston (1998) referred to it in his doctoral thesis. Figure 2 ranks the complex nature and difficulties of estimating diamond deposits compared to other mineral commodities as a function of their concentration and homogeneity. Kleingeld attributed this increased estimation difficulty due to their extreme value characteristics based on limited sampling data and the stochastic nature of the variables used to estimate both their grade and value (revenue per carat). The particulate nature of diamonds, their size, shape, quality, colour and value are important factors in the accurate estimation and evaluation of diamond deposits.

Kimberlite grades are usually expressed in carats per tonne (cpt) or carats per hundred tonnes (*cpht*) given their concentration in parts per billion. The grade variable ‘*cpht*’ is calculated as a function of carats per cubic metre (cpm^3) and density (specific gravity, SG) as shown in Equation 9. Note that cpm^3 depends on both spm^3, which is the concentration of diamonds (or ‘stones’) within a cubic metre of ore and the carats per stone, which is a function of stone size.

Sampling challenges that are applicable to other mineral commodities are applicable to both the stones per cubic metre and the carats per stone. It is often more complicated to attain high confidence in the estimation of the carats per stone relative to the stones per cubic metre, because in the case of the former, it is highly improbable that large stones will be recovered from drill holes given the relatively small sample cross-sectional area. To address this problem, large bulk samples comprising several thousand tonnes of ore may be needed to retrieve a sufficient quantity of large stones from the size frequency distribution (SFD) range to cater for these extreme values.

$$cpht = \frac{cpm^3}{density}$$

where $cpm^3 = spm^3 * \frac{carats}{stone}$, and $spm = stones/m^3$

Equation 9. The calculation of diamond grade in carats per hundred tonnes (cpht).

Inputs into estimating the diamond revenue model are uncertain, and hence the revenue model itself is uncertain and is influenced by the mineralisation, geological and emplacement models. Lithologies delineated on the basis of geology could represent a difference in revenue models. The uncertainty associated with diamond revenue modelling is different, and in many ways more complex, than price stochasticity affecting other mineral commodities, such as the gold price, although diamonds have a degree of price stochasticity as well. Diamond valuation has four main attributes to consider rather than only one in the case of the gold price, for example. The four main attributes are size, colour, model (or shape) and quality. Each of these attributes is associated with inherent variability, and hence, a larger sample of diamonds is required to estimate the average USD/carat value than to estimate the stone density distribution or the stone size distribution which together constitute the grade.

3.2.4.2 Random Functions and Variables

In order to understand the variabilities associated with diamond estimation and evaluation, it is first necessary to introduce the concepts of regionalised variables, random functions and random variables. Sichel (1947, 1952), Krige (1951) and De Wijs (1951) appear to be the first to use statistical methods to consider geometrical relationships between blocks and the samples used to estimate them. These three authors focused on the significance of the distribution of regionalised variables combining some of the key concepts which Matheron (1963) later formalised as geostatistics. Any variable distributed in space is referred to as a regionalised variable. Matheron (1963) discussed the concept of a regionalized variable as an actual function, taking a definite value at each point in space.

From a mathematical perspective, a regionalised variable is a function which takes a value at every point in the space of regionalization, however, the function varies irregularly from point to point in the space of regionalization. This local point to point irregular behaviour can be interpreted in terms of random variables, which is a variable taking on a specific value according to a certain probability function. The average behaviour of these random variables suggests a structure which in turn proposes a functional representation. One way of interpreting the characteristics of a regionalised variable is in the probabilistic terms of a random function (Dowd, 1978).

The concept of the random function was, as far is known, introduced by Matern (1960) for his work in experimental design and analysis in forestry, while Matheron introduced the name 'geostatistics' in 1962. The random function is the set of random variables at all possible locations. The unique outcome that exists at every location is a realisation of the random function. Whilst the actual *in situ* grade is a unique realisation of the random function, an infinite number of realisations share the same geostatistical properties. The random variables are spatially correlated and the sample values provide realisations of this correlation. Because the outcomes of the random variables are known at data locations, this information may be used to calculate measures of spatial continuity and infer a spatial continuity model for the random function. This model may then be used to minimise the variance of the probability distribution at unsampled locations given neighbouring data.

Matheron (1963) recognized that in order to estimate Z in a way that minimises the error and is unbiased, it is necessary to accept that grade distribution is the outcome of a process which is effectively 'random'. Grade may be considered as a random variable that assumes a series of outcome values at unsampled locations. The series of possible outcome values for the random variable at each location is determined by a probability distribution. The variance of this probability distribution is zero at datum locations, and is centered on the measured grade (assuming accurate measurement).

Values of spatial variables are measured at specific locations x. These values $z(x)$, at locations x are interpreted as particular realizations of random variables, $Z(x)$, at the locations. The set of auto-correlated random variables $\{Z(x), x \epsilon D\}$ defines a random function. Spatial variability is quantified by the correlations among the random variables (Dowd and Pardo-Iguzquiza, 2002). The definition of a random function expresses both the random and structured aspects of a regionalised variable. At any point x_i, $z(x_i)$ is a random variable. However, any pair of random variables $z(x_i)$, $z(x_j)$ are to some extent correlated in space, and this correlation expresses the spatial structure of the regionalised variable $Z(x)$.

For most practical applications of geostatistics, there are limited data (usually from drill holes) sampled from a number of points and hence only a finite number of points x_1, x_2 … x_n are available at any time. There are only n components of $z(x_1)$, $z(x_2)$ … $z(x_n)$ of the random function $z(x)$ available. At each point x_i, where i = 1 to n, only one realisation of the random variable $z(x_i)$ is available, implying that there is an infinite number of possible random functions that this single, limited realisation could represent. Any practical solution must therefore be limited to some family of random functions that requires the estimation of only a small number of parameters; where this family establishes a model of the random function.

Dowd (1978) considers three families (F_1, F_2 and F_3) of random functions with each family defined in terms of increasingly restrictive hypotheses. For F_1, the family of strictly stationary random functions defines a strictly random function as one in which has a distribution law which is invariant under translation, i.e. the probability distribution is assumed to be identical everywhere so that all random variables in the random function have the same distribution. For F_2, the family of second order stationary random functions defines a second order stationary random function as one in which the expectation $E[z(x)]$ exists and is independent of the support x; and a covariance function exists for any pair of random variables $z(x)$ and $z(x+h)$ which depends only on the separation distance h. This second order function implies that only the mean and variance are assumed to be the same everywhere, however, it also assumes that the variance is known, which is often not the case in practical terms.

To overcome this problem, a further restriction is made. For F_3, a family of intrinsic random functions is defined whereby the expectation $E[z(x)]$ exists and is independent of the support x; the increments $E[z(x) – z(x+h)]$ are stationary for all vectors h, and are independent of x. This implies that the increments have a finite variance, $D^2\ [z(x+h) - z(x)] = 2\gamma(h)$ which is known as the variogram. This hypothesis implies that the two experimental values $z(x_i)$ and $z(x_{i+h})$ at two different points x_i and x_{i+h} are two different realisations of the same random variable $z(x)$. The intrinsic stationarity implies that the mean and variance of pairwise differences is assumed to be the same everywhere, i.e. the mean is zero. This family of intrinsic random functions (F_3) is used in linear geostatistics as in practice there is usually sufficient homogeneity of the regionalised variable to allow these assumptions to be made.

There are several tools available to characterise spatial correlation of the random function, of which the semivariogram is most commonly used in practice. The ‘semi’ part of the term semivariogram refers to half (or ‘semi’) of the variance, $D^2\,[z(x+h) - z(x)]$ because this gives the variance per point when the points are considered in pairs. Thus, $\gamma(h)$ can be interpreted as the variance of the variable at the given separation vector, h, which means that only pairs that are spatially separated by the lag h, are considered. The semivariogram uses the pool of pairs to quantify the variance of the distribution for that separation distance:

$$\gamma(h) = \frac{1}{2n(h)} . \sum_{i=1}^{n(h)} \left[Z(x_i) - Z(x_{i+h}) \right]^2$$

where $n(h) =$ the number of couples separated by distance h

and $h =$ distance between sample pairs

and $Z(x_i) =$ value of sample at location x_i

and $Z(x_{i+h}) =$ value of sample at location x_{i+h}

Equation 10. The geostatistical semivariogram

The experimental semivariogram’s nugget variance indicates the variance of the combined distribution of pairs of points separated by an infinitely small distance, where an infinitely small distance is zero and the value of the variogram for $h = 0$ is zero. For an experimental variogram the nugget variance is the variance at distances less than the sampling interval, while for the model variogram it is the variance of the differences for distances $h > 0$. The latter is inferred from the former. At this separation distance the probability distribution of either random variable is unaffected by any knowledge of the other random variable, the sill of certain types of semivariograms tends towards the variance of the sample dataset. While semivariograms calculated from data may take a variety of forms, very often they have asymptotic behavior, reaching a relatively constant plateau at some positive value. This plateau is called the sill. The value of h at which this sill is reached is called the range.

In order to deduce the variance of the distribution of squared differences in grade between sampled data points; and the variance of the distribution of squared differences in grade between sampled data points and unsampled data points, it is necessary to know the semivariogram value at separation distances other than those used as lags in the experimental semivariogram. A (positive definite) random function model is fitted to the experimental semivariogram. The semivariogram models only the variance of squared differences between random variables as a function of separation of distance; the distribution (shape) is not specified by the semivariogram model. In this study semivariograms were modeled as single or multi-structured models using spherical, exponential or Gaussian functions.

3.2.4.3 Domaining Considerations

Consideration of appropriate estimation domains is critical in resource estimation, where domaining is usually a practical sub-division of the orebody into zones to which a specific variogram may be applied to the variable within the zone. Estimation domains are generally closely related to geological, structural and/or weathering units. Vann (2005) identified the following factors that need to be considered in defining estimation domains:

1. Distribution of lithology;
2. Distribution of weathering surfaces;
3. 'Structural architecture' of mineralisation;
4. Sampling and analytical precision; and
5. Spatial distribution of grade *within* mineralised structures.

The variography of grades by means of semivariogram modelling is crucial when making stationarity domaining decisions. Domaining decisions should be undertaken by a geologist and a geostatistical expert based on assumptions about the homogeneity of the zones over which the estimation is to take place.

In the case of kimberlites there may be distinct lithologies (also referred to as domain boundaries) that need to be considered within a kimberlite pipe for diamond estimation purposes. Often these domain boundaries are distinguished from adjacent lithologies in terms of mineralogy (associated with a change in geochemistry or structural event such as a dyke intrusion or breccia event) and/or chemical weathering at boundary interfaces. Given the extreme value nature of diamond grade estimation and stone size valuation based on limited drill hole data, the delineation of these domain boundaries are important properties of the phenomena under consideration.

3.2.4.4 Linear versus Non-linear Estimation Methods

There are a multitude of geostatistical estimation methods that can be applied to mineral projects. It is good practice when selecting the appropriate estimation method to consider the geology of the deposit (e.g. geological structure, mineralisation trends, domaining) in relation to the available sample data, such as drill hole spacing, sample compositing and quality of the data. Vann and Guibal (1998) note that in many mining projects, sample grades are highly positively skewed and that significant 'deskewing' of the histogram and reduction in variance can occur when going from sample to block support, where blocks are of considerably larger volumes than samples. This 'change of support concept' is well known in geostatistics where the SMU dimensions in relation to the drill hole sample spacing and panel support size should involve some consideration for linear versus non-linear estimation methods.

Examples of linear interpolators are Inverse Distance Weighting (IDW) and Ordinary Kriging (OK). OK can in principle be thought of as a linear regression expressed by Equation 11.

$$y = mx + c$$

where, m is the slope of the line

and, c is the value of y when x is zero

Equation 11. Linear regression equation.

Some of the key limitations of linear estimation techniques (such as kriging) are that they were developed to produce the 'best linear unbiased estimator' and not to estimate a distribution around the grade value (except in the case of a normal distribution), necessary for estimating reserves. The use of linear estimation for highly skewed grades may also be problematic when some indication of the distribution may be required to gain a better understanding of the mean in relation to the spread of the data. In some instances, such as geometallurgy, for non-additive variables there is no linear relationship between the value of the variable when measured at different scales (e,g. between the core and block). Implicitly, even if every possible core sample within a block was measured for a particular variable, the average of all these billions of values would not be the value of the block. In these scenarios the use of non-linear geostatistical estimation techniques would be more appropriate.

Vann and Guibal (1998) note that non-linear estimation was developed for the specific problem of estimating recoverable resources. Non-linear interpolation attempts to estimate the conditional expectation, and the conditional distribution of grade at a location, as opposed to simply predicting the grade itself. In the case of a Normal distribution ordinary kriging (OK) and simple kriging (SK) estimate the conditional expectation. Non-linear geostatistical estimators are those that use non-linear functions of the data to approximate the conditional expectation. The conditional expectation can be obtained through the following probability distribution where the probability of the grade at location X_o is a function of the known sampling information at locations $Z(X_i)$.

$$P\left[Z\left(x_0\right) \mid Z\left(x_{i=1 \text{ to } n}\right)\right]$$

Equation 12. Conditional probability distribution

Some of the more well-known non-linear estimation methods are Disjunctive Kriging (DK); Indicator Kriging (IK) and Multiple Indicator Kriging (MIK); Probability Kriging (PK) and Lognormal Kriging (LK); and Uniform Conditioning (UC). For the purposes of this study, the linear estimation called 'kriging' and non-linear simulation method known as 'conditional simulation' will be elaborated upon.

3.2.5 Kriging

Matheron honoured Danie Krige (1951) by coining the term 'kriging'. There are several types of kriging algorithms applied to geostatistical estimation problems such as Indicator Kriging (IK) where indicators lie in the range [0;1] and for estimations, the indicators define sample values above or below a threshold and once the estimates are obtained, may define probabilities or block proportions; and Multiple Indicator Kriging (MIK) which involves kriging of indicators at several cutoffs (Journel, 1982); Disjunctive Kriging (DK) is where a function is defined as a linear combination of indicators and these indicators are cokriged taking into account existing correlations between indicators at various cutoffs (Matheron, 1976); Lognormal Kriging (LK) which requires data to be strictly lognormally distributed in order to take the logs of the data and calculate the resulting conditional expectation (Dowd, 1982); and Probability Kriging (PK) which is an attempt to correct the order relationship problems identified with using MIK by cokriging the indicators and the rank transform of the data, (Verly and Sullivan, 1985).

Ordinary Kriging (OK) is by far the simplest and most frequently used linear estimation method and has been used by the author to develop estimates in each of the three case studies.

Kriging may be defined loosely as an optimal regression against observed *Z* values of surrounding (real) data, and weighted according to spatial covariance values (derived from the semivariogram of the data). If the true grade of a block is denoted by Z, then the standard kriging equation used to estimate, Z*, of volume, V, is estimated by the weighted sum of *n* sample grades.

$$Z^* = \sum_{\iota=1}^{n} \lambda_i z_i$$

where Z^* = estimated block value

λ_i = an unknown weight for the measured value at the i^{th} location,

where the sum of the weights provides an unbiased estimate of Z,

and yields the minimum estimation variance

z_i = measured value at the i^{th} location

Equation 13. Kriging equation.

The goal of kriging is to determine the weights, λ_α such that the estimation variance is minimized (Equation 14) under the unbiasedness constraint (Equation 15).

$$\sigma_E^2(u) = Var\left[Z^*(u) - Z(u)\right]$$

Equation 14. Estimation variance.

$$E\left[Z^*(u) - Z(u)\right] = 0$$

Equation 15. Unbiasedness constraint for the Kriging equation.

For simple kriging, it is assumed that the trend component is a constant and known mean, m(u) = *m* as shown in

$$Z_{SK}^*(u) = m + \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{SK}(u)\left[Z(u_\alpha) - m\right]$$

Equation 16. Simple Kriging equation.

For ordinary kriging, rather than assuming that the mean is constant over the entire domain, it is assumed that it is constant in the local neighbourhood of each estimation point, i.e. $m(u_\alpha) = m(u)$ for each nearby data value $Z(u_\alpha)$ that is used to estimate Z(u), see Equation 17.

$$\begin{aligned} Z^*(u) &= m(u) + \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u)\left[Z(u_\alpha) - m(u)\right] \\ &= \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) Z(u_\alpha) + \left[1 - \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u)\right] m(u) \end{aligned}$$

$$Z_{OK}^*(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) Z(u_\alpha)$$

$$\text{with} \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) = 1$$

Equation 17. Ordinary Kriging equation.

It is generally accepted practice for OK to be implemented as a means to minimize conditional bias, see Matheron (1963), Ravenscroft and Armstrong (1990), and Krige (1994) and (1996). However, minimizing conditional bias results in increased smoothing, yielding locally inaccurate predictions of the recoverable tonnes and grade above cut-off. The smoothing is partly a function of the drilling density, but also depends on block size, search volume and semivariogram. It is well understood that kriging results in smoothing. Because of the smoothing associated with kriging, the variance of kriged values *Z** is less than that of the true grades values *Z*; the histogram of *Z** has more values around the mean but less extreme values than the histogram of the true grades, *Z*.

It is well known that for a kriged estimate, estimates above the mean on average overestimate the actual values, and estimates below the mean on average under-estimate the actual values (for examples refer to Journel and Huijbregts, 1978; and Goovaerts, 1997). Smoothing is acceptable within the stated objectives of kriging because the globally unbiased condition only requires that estimation error is zero on expectation over the entire domain of interest. This is a weaker criterion than conditional unbiasedness, which requires that estimation error is zero on expectation at any grade intervals. The conditional unbiased criterion also entails global unbiasedness; global unbiasedness does not necessarily entail conditional unbiasedness.

3.2.6 Conditional Simulations

Geostatistical simulation methods aim to reproduce both the *in situ* variability and the spatial continuity of the input data set. Simulations replicate the spatial structure of a data set as a whole rather than providing optimum local estimates of an attribute. The simulated model is said to be conditionally simulated, if it reproduces the values at sampled points and reproduces the same dispersion characteristics of the original data set, i.e., the mean, variance and covariance or semivariogram (Costa et al., 2000).

“The illusion that a sound estimation algorithm suffices for ore reserves evaluation comes from the lack of understanding of the trade-offs involved when defining the goodness of criterion of an estimate” (Journel and Kyriakidis, 2004). This implies that no estimation method, unless it was based on completely exhaustive sampling data, can provide an estimate that is ‘good’ for ‘all’ purposes. Most traditional estimation algorithms that involve distance based weighting algorithms including kriging, are aimed at providing local (rather than global) accuracy.

To some extent this estimation conundrum is described by the Heisenberg uncertainty principle (Heisenberg, 1958) which maintains that although one cannot examine two complementary observables at the same time, it is important to know them both to understand the behaviour of the system. Although Heisenberg formulated the principle for microscopic quantum mechanics, there is a conceptual analogy between this inequality principle and the geostatistical estimation challenges that are encountered in today’s mining world.

Mathematical techniques are used to develop estimation algorithms in order to provide a level of local precision to geostatistical estimates. However, the mathematical models introduce inherent uncertainties into the algorithms due to estimates (rather than known values) of parameters. By selecting a larger model, the description of the system improves but the local precision may become worse.

Figure 5 compares conditional simulations with kriged estimates (ordinary kriged and co-kriged, with copper and gold) for a block cave development. Each conditional simulation represents a plausible version of reality and may look similar to each other on a global scale, however, they have distinct local variations in relation to each other and when compared to the ordinary kriged (OK) and cokriged (CoK) copper-gold estimates. While kriging represents the best, linear unbiased estimator it is not designed to represent the variability of the deposit. Simulations are used for the analysis and solution of problems in which variability is a critical factor (Dowd, 1996).

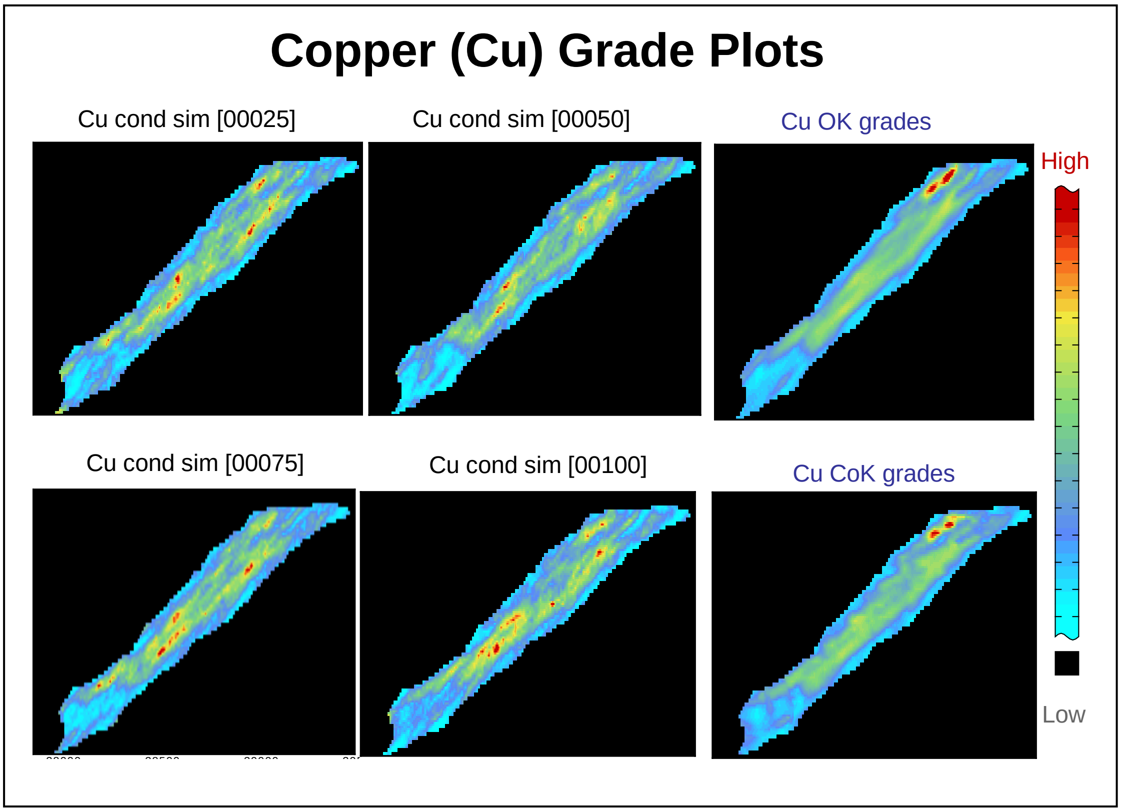


Figure 5. Grade plots for copper comparing four conditional cosimulations (number 25, 50, 75 and 100) with an ordinary kriged estimate (Cu OK grades) and co-kriged estimate (Cu CoK grades) – Nicholas, 2009. The cosimulations and cokriged estimates considered both copper and gold. High grades are indicated by warmer colours (red) while colder colours (blue) reflect lower grades.

Conditional simulations allow the generation of sets of realisations that each reproduce the histogram, spatial variability and known data values of the variable of interest. Each simulation is independent of the others and equally likely to be drawn from the set as referred to by Journel and Huijbregts, (1978); Goovaerts, (1997); Dimitrakopoulos, (1998); Dimitrakopoulos et al., (2002); and Journel and Kyriakidis, (2004). The distribution of simulated values at each node represents the conditional cumulative distribution (ccdf) of the variable at each node, taking into account all of the factors that influence that distribution.

Journel (1974) described the Turning Bands method in 1974 and later, Parker (1978) ran Sequential Indicator Simulations (SIS) to model the uncertainty of ore shoots for the Mt Razorback tin deposit in Tasmania; followed by Sequential Gaussian Simulation (SGS) of grade and thickness at a uranium mine in New Mexico. Parker (1978) recognised that a SGS method would produce an auto-correlated realisation of random variables in space. SGS uses simple kriging to estimate the local conditional probability distribution (*lcpd*). This process assumes that the *lcpd* is the classic normal distribution with the mean and variance of the *lcpd* being equal to the simple kriging estimate and estimation variance, respectively.

Similar to the Turning Bands simulation technique, described by Journel (1974) and Montoglou and Wilson (1982), the application of this method requires normalization of the data and back-transformation of the results. SIS is based on the estimation of the *lcpd* using Indicator Kriging (IK). SIS does not make any assumptions on the shape of the *lcpd*, which is explicitly estimated. Consequently, SIS is slower and more tedious than the SGS technique.

Conditional cosimulation is the multivariate extension of conditional simulation. Conditional cosimulations reproduce the above properties for each variable, but also reproduce the spatial correlations. The scenarios in which cosimulation is preferable to simulation are not necessarily analogous with scenarios in which cokriging is preferable to kriging. In particular, the benefits of cokriging are very limited where the variables of interest are isotopically sampled. There are, however, compelling reasons why cosimulation is preferable to independent simulation whether or not the orebody is isotopically sampled.

The *ccdf* of each variable at each node provides a measure of uncertainty for that variable at that location. Although the *ccdf*s of multiple variables can be simulated independently for the node, they in themselves cannot be used as a measure of combined uncertainty (for example the probability all of the variables exceeding given cutoff grades at the node). Because multivariate conditional simulations reproduce the above properties for each variable and the spatial correlations among variables, they generate the *ccdfs* for each variable and the combined probability distribution at each node. The latter can be used as a measure of combined uncertainty.

The application of conditional simulations in mine planning optimization studies has been recognized by several practitioners in the past, such as Ravenscroft (1992), Dowd (1994), Dimitrakopoulos (1998) and Smith (2001). Conditional simulation techniques have been used more regularly in the diamond placer industry than in kimberlites and in the former case, have led to techniques such as the Cox process being applied by Kleingeld *et al* (1996). In the case of kimberlites, simulations were focused mainly on grade, density and 'ore' thickness uncertainties. These are mostly segmented models and do not form part of an integrated model that quantifies the financial impact of correlated variables on the business model.

3.2.7 Ore Reserves

Statistical techniques for the evaluation of *in situ* ore reserves on a panel-by-panel basis were proposed around 60 years ago by Krige (1951) and Sichel (1952). Previous resource estimation sections within this thesis described the *in situ* estimate, which depended mainly on sampling, assaying, geological interpretation and estimation modelling techniques. 'Recoverable estimates' or 'ore reserves' involves additional considerations such as the choice of mining method, judgements or predictions of recovery, mining and recovery dilution, mining and recovery throughput rates, mining and recovery efficiencies with specific regard to the impact of contaminants/deleterious elements, and of the effects of non-technical aspects such as socio-political, legal and environmental factors (King et al., 1982).

Reserves relate to the proportion (tonnage) and average grade of those SMUs of size $|v|$ that will be selected as ore within any given panel (V). The author notes that the term 'ore reserve' has strict definitions in a number of public reporting codes, e.g. JORC, SAMREC. A more general use of the term 'reserve' is used here. The 'optimal' SMU and panel size is selected from the mine plan after considering optimal bench heights, equipment type (height, size, quantity), mining dilution, mining rate etc. which forms part of the mine plan design, sequencing and scheduling considerations. Free selection of constant SMU size $|v|$ is usually assumed within each panel. However, the panel V could be of any size, at the maximum limit so as to include the entire deposit (D) or as small as to only include one SMU (Journel and Kyriakidis, 2004).

The panel *V* should not be so large as to include vastly different mineralisation zones, thereby contravening implied geostatistical stationarity, which may result in increasing local estimation errors. Neither should the panel *V* be too small (contain only a few SMUs) as the grade histograms derived from the SMUs within each panel may not be reliable and could introduce large estimation errors. As noted by Journel and Kyriakidis (2004), the smaller the panel *V* size, the less the error averaging but the larger the uncertainty about its reserves will be. In many mining operations, the entire mining panel may not be depleted in a single year, which may introduce evaluation errors in the cash flow model due to local estimation errors within a panel.

Figure 6 contrasts the increased variabilities associated with SMUs with larger-scale panel grades.

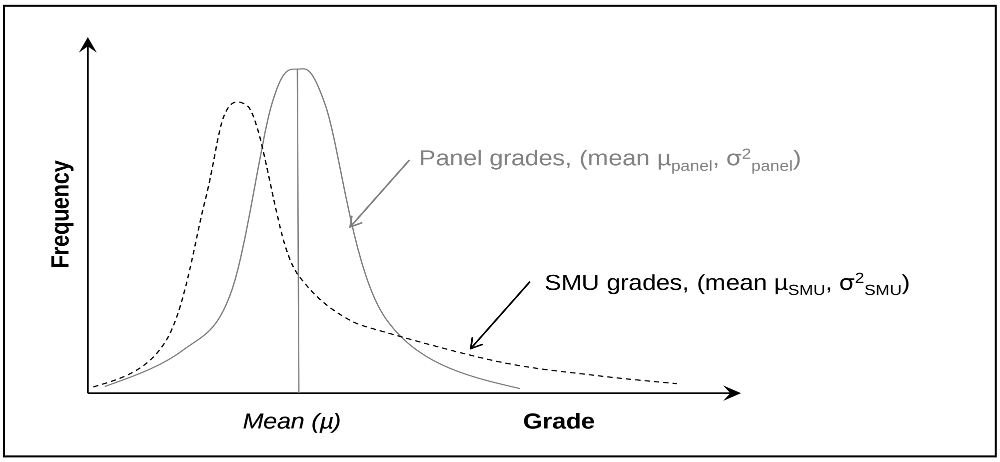


Figure 6. Smaller scale SMU grades will have higher variabilities than the larger scale panel grades, which are likely to have a degree of smoothing associated with it. Adapted from Journel and Kyriakidis (2004) based on the concept of the 'support effect'.

It is not uncommon for mine planning engineers to develop a long-term mine plan focused on an annual block sequence and schedule for each year in the LOM plan. This implies that the production tonnes, grades and total carats produced (or metal quantities) have been calculated from the mine plan for each year but the exact sequence in which each SMU will be mined has not been determined. For advanced projects and operating mines, usually a shorter-term plan (on a monthly basis) is developed for the first two years where production outputs have been calculated on a monthly, rather than an annual, basis. A medium-term (quarterly plan) may be developed for the next 1 – 2 years while the remaining LOM is not likely to have more detail than on an annual basis.

Mine planning optimisation techniques, such as the Lerchs-Grossmann (LG) algorithm, (Lerchs and Grossman, 1965) are based on mathematical models that assume inputs into a mine plan are known. This assumption is untrue for the evaluation of most mineral deposits but even more so for diamond deposits, as there are more variables to consider that are uncertain. Resource models are not deterministic but are in fact associated with varying degrees of uncertainty and resource variables may have varying degrees of correlation between them, e.g. kimberlite density, grade and recovery (liberation) may be positively correlated in some situations. The base or footwall of kimberlite dykes can be associated with higher density material, with more stones (higher grade concentrations) in the footwall yielding better processing recoveries (high grades often result in high recoveries).

Traditional production scheduling optimization methods do not consider the risk of not meeting production targets which occur as a result of grade uncertainty and variability, leading to sub-optimal results. For some more modern cases, the mine plan is reviewed from practical and probabilistic perspectives, which may include identifying mine blocks associated with greater technical risks and scheduling these blocks later in the project's life-of-mine schedule. Discrepancies between actual production and planning expectations arise through uncertainty about the orebody, in terms of ore grade, tonnes and quality. Ravenscroft (1992) discussed risk analysis in mine production scheduling, recommending the use of stochastically simulated orebodies to show the impact of grade uncertainty on production scheduling. He concluded that conventional mathematical based programming models could not accommodate quantified risk, and identified a need for a new generation of scheduling formulations that account for production risk.

Dowd (1994) identified the use of correlated Monte Carlo simulation to conduct sensitivity analysis of deposit uncertainty, price and costs to understand the impact on optimal open pit design. Dimitrakopoulos (1998) recognized the value of conditional simulation algorithms for modelling ore body uncertainty in open pit optimization. Later, Smith (2001) identified that the uncertainty associated with production variables could be quantified in each block using conditional simulations and that stochastic programming (SP) methods could be used to extend linear and integer programming-based production scheduling algorithms into a stochastic optimization paradigm. In this SP approach to scheduling, the distribution of the production variable in each block is used as input into a single optimization which will determine a sequence of block extraction that accounts for the deposit uncertainty.

Godoy and Dimitrakopolous (2004) developed a mining transfer optimisation algorithm with objective functions that consider orebody uncertainties in relation to financial, mining and treatment criteria such as the maximum NPV pit shell, discounted cash flow etc. A mathematical programming model was developed based on linear programming (LP) that took into account geological uncertainty, equipment mobility and access required for scheduling and excavating mine blocks. In this scheduling approach, a probability was assigned to each block to represent the 'desirability' of that block being mined in a given period. The probability, calculated from simulated orebody models, represents the chances that a block will contain the desired grade, ore quality and quantity, including ore grades above given cutoffs, and recovery and processing characteristics.

The first few years of a mining project are generally the most important with respect to generating sufficient equity to pay back debts/ loans and therefore require the highest confidence. Higher risk blocks are scheduled later in the life of mine plan where the time value of money has less effect than on early production periods. This type of risk based scheduling presents the decision maker with an option to either plan a higher optimal NPV with lesser regard for technical risks or select a lower NPV but 'safer' option and exploit the orebody based on the greatest technical confidence.

Other workers have considered objective functions and simulated annealing techniques to focus on quantifiably maximizing value (and/or reducing costs) by prioritizing the sequence of mine blocks (Dimitrakopoulos and Ramazan, 2004). Open-pit mine planning is generally more flexible than underground operations using reverse-stoping and block caving techniques etc. This is because to some extent, open-pit operations can adapt their mine designs to accommodate an uncertain resource model whereas underground mining defines an 'almost' irreversible plan that cannot easily adapt.

Where conditional simulations have been used to express the uncertainty of resource models, a number of variables such as grade, density and revenue per carat exist for each simulated realisation that may or may not be correlated with each other and depends on the geological model. Dimitrakopoulos et al. (2002) developed 'envelope optimisation' methods, focusing mainly on grade, and using geostatistical conditional simulations to produce an output envelope of NPV solutions. While these methods are useful in identifying an optimal envelope of possible solutions and highlighting the error in focusing only on one estimated

NPV, the mean of all the realisation outputs is not an optimal mine design. For each simulated realisation, an optimal pit may be designed resulting in the optimal block sequence and schedule based on the maximum contribution per block. But which realisation is representative of reality based on a range of simulated realisations and which one should the mine plan be based on?

For the above-mentioned reasons, the IEM methodology adopted in this thesis assumes that an 'optimal' mine plan has been derived from the base case resource estimate (e.g. the kriged linear model). This mine plan (notably the block sequence and schedule) is imposed on each conditionally simulated resource realisation. The aim of which is to determine the impact of resource variability on the selected mine plan, which represents the 'best case' business model for a particular operation. Note that where there is insufficient detail on the specific SMU sequence within an annual period, a practically constrained mining logic was applied whereby each adjoining SMU must be mined (including a top-down sequence based on z-elevation for open-pits) rather than adopting a completely random, selective SMU approach that may be totally impractical to implement in reality.

3.2.8 Estimation Bias and Selectivity

There are three main potential sources of error in estimation. These are global bias, local or conditional bias, and the problem of selectivity. Global bias can occur when the mean grade of the estimated blocks is not equal to the actual mean grade of the region (see Figure 7A). Global bias can be largely avoided by carrying out sufficient regularly spaced sampling, using sampling techniques that do not introduce bias. Domaining and estimation using a representative 'cloud' of samples around a block are good preventers of global bias Local or conditional bias is the overestimation in high, or underestimation in low, estimated blocks or panels (see Figure 7B). The problem of selection (in Figure 7 C) is usually defined as a function of the correlation cloud where some blocks or panels will be selected but are in fact waste (X_1) while other blocks will be left in the ground when they are in fact ore (X_2).

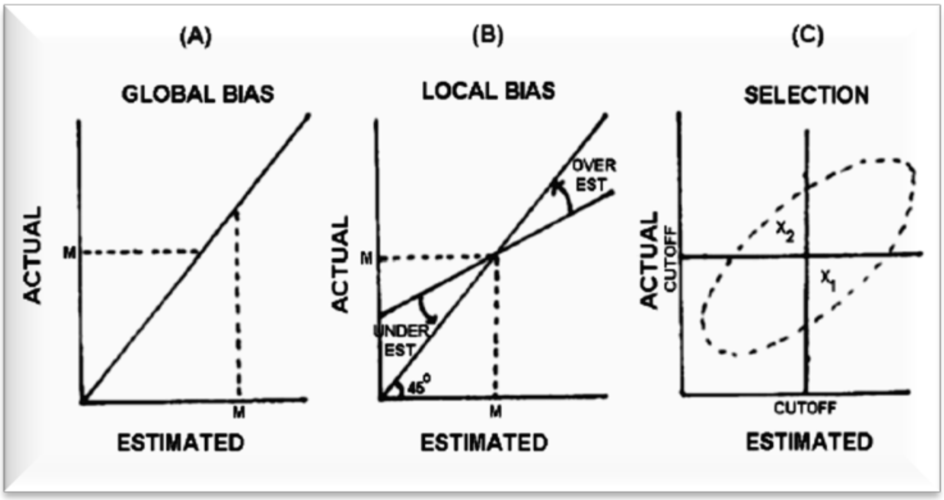


Figure 7. Global and local bias versus selection graphically depicted.

The ‘bottom-up’ evaluation approach proposed in this thesis has more to do with ‘selection’ (in Figure 7C) than global or local bias, although local bias will likely influence the selection of SMUs sent to the plant. The error introduced by not adhering to a ‘bottom-up’ approach is not solely an estimation error as previously noted but rather a function of how each SMU (within the block model) at a local estimation scale interacts with the planned reserve constraints, with the latter usually designed to consider the average variability per annum rather than at a smaller SMU scale.

The optimal selection of SMUs that contribute to a production target (e.g. annual ore tonnes) will be influenced by the estimated grade value allocated to each SMU, plus the grades of deleterious variables within that SMU that will affect the efficient processing of that SMU or panel. This implies that a SMU with the highest grade may not always be the most profitable as it may cost more to process it if the deleterious elements have high grades.

Based on the argument above, the author proposes two additional categories of error estimation to those depicted in Figure 7, which will be elaborated upon further in this thesis. The first is that of an ‘Evaluation Bias’ depicted in Figure 8 which demonstrates the impact of short-scale variability within each SMU on the planned production constraints in any given period (month/quarter/year) of evaluation, i.e. this figure represents the non-linear impact of evaluation considerations on production estimates, which is not to be confused with the impact of selectivity as a function of inaccurate grade estimation relative to cut-off grade shown in Figure 7 (c).

The second is the concept of 'Scheduling Errors' shown in Figure 9, which highlights potential errors that can take place when selecting blocks for processing based on the well-known 'time value of money' approach applied within a conventional discounted cash flow (DCF) financial framework. This figure should be considered as a logical extension of the 'evaluation bias' concept highlighted in Figure 8 with the key differentiator being that Figure 8 depicts the impact of scheduling bias in NPV terms (not only production estimates per period) taking cognisance of production risks in relation to the time value of money.

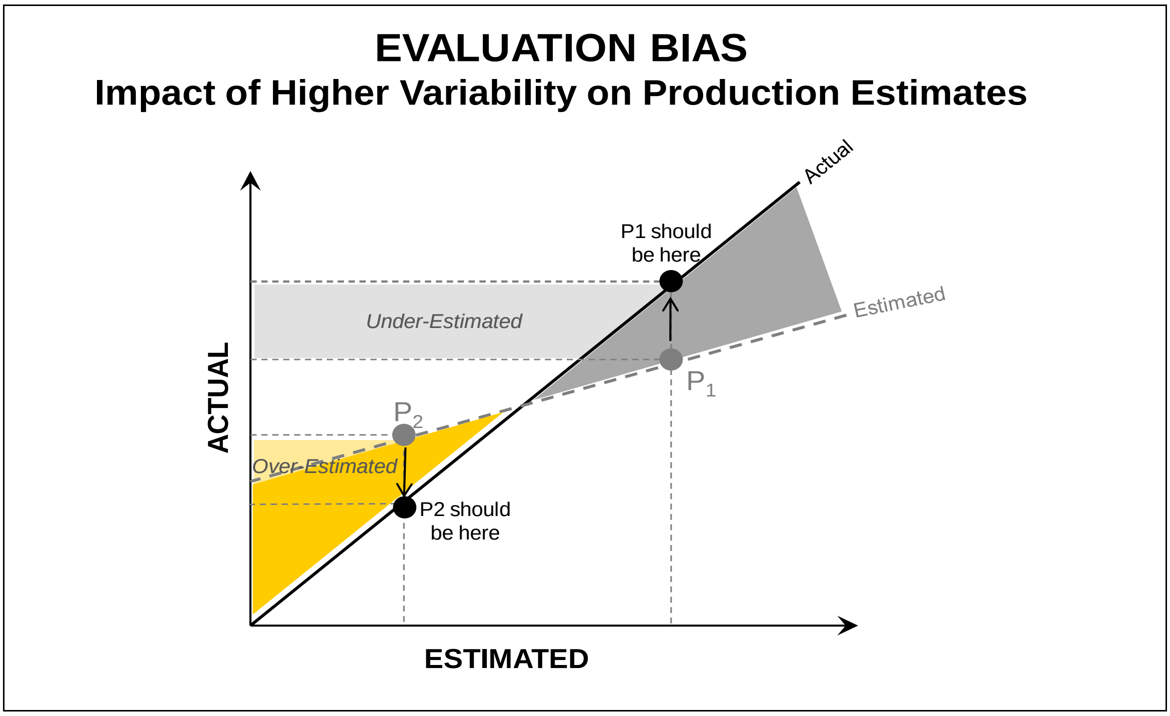


Figure 8. Depiction of evaluation bias on production estimates due to the non-linear impact of selectivity and short-scale variability on reserve constraints. For example, higher than expected 'actual' variability of grades results in there being greater opportunity to select those blocks that have higher grades than the average estimated grade to feed the plant (where the mine generates more tonnes than actual capacity at the processing plant). In this case the production estimate, P1 has been under-estimated (grey shaded area) and should actually result in higher production figures for the relevant period. Conversely, P2 (yellow shaded area) shows the impact of blocks where the production estimate, P2 has been over-estimated due to the increased variability of grades in blocks or processing constraints resulting in lower than expected production outputs for that period.

The evaluation bias described in Figure 8 depicts the potential bias that can occur when ‘higher than expected’ variability in the ore is encountered. This could have a negative impact (production totals are over-estimated) or a positive impact (production totals have been under-estimated).

Point P1 in Figure 8 depicts a scenario in which the metal/production totals (within any given period and/or over the LOM) have been under-estimated, i.e. there is a positive skewness of increased metal grades and/or lower grades for deleterious elements within the collective SMUs for any given period (estimated through non-linear models), which is greater than the expected business case scenario (linear model). The increased variability derived from the SMUs is still within the reserve constraints and allows ‘higher than expected’ grades (or metal content) to be realized, i.e. the mine produces more ore than what the plant can process, and only the ‘best’ SMU grades (or metal content) are selected from the ROM stockpiles to feed the plant. This actually produces higher-than-expected production totals (average grade, metal content etc.) than originally estimated. Thus, the initial production estimate was under-estimated.

Point P2 in Figure 8 shows a scenario whereby the metal/production totals (within any given period and/or over the LOM) have been over-estimated, i.e. the increased variability (through non-linear modelling) of metal grades and/or deleterious elements from the collective SMUs for any period is greater than the expected business case (based on some form of linear model). Typically, mining and processing constraints are designed on the average variability estimated from the business case, which in most cases will be smoother than the actual variability encountered within the SMUs at shorter time scales.

Higher grades in some SMUs may be associated with higher-than-expected deleterious elements (e.g. arsenic) that have to be blended carefully to avoid exceeding plant thresholds. Thus, the increased variability from these SMUs will exceed one or more reserve constraints, inhibiting recovery of higher grades (metal content or revenue) but still getting the lower grades, which results in an overall decrease in the average recovered production. Therefore, the initial production is over-estimated.

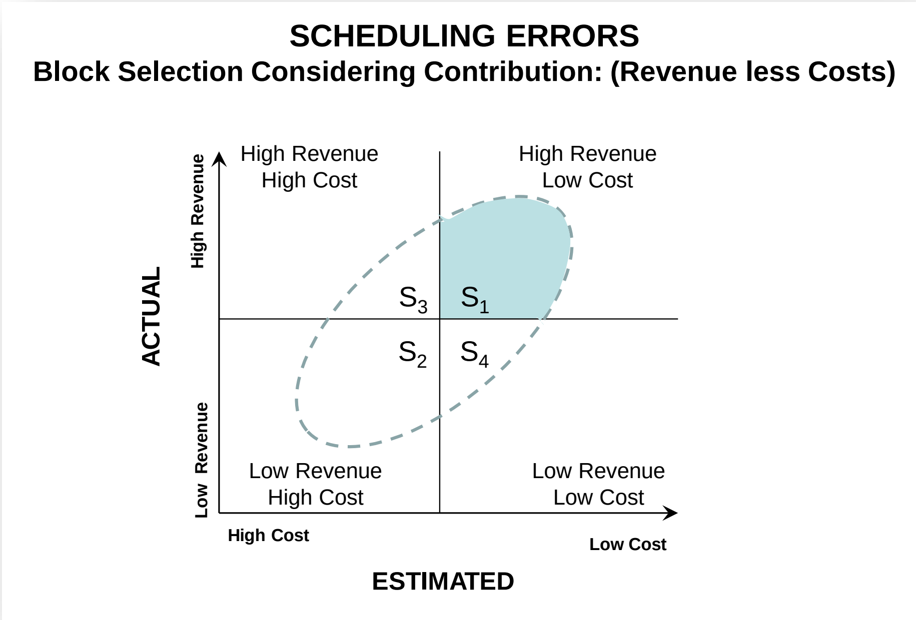


Figure 9. Scheduling errors that can occur when the short-scale temporal impact of block selection is considered in terms of contribution to evaluate the effects in the 'time value of money', for a conventional DCF analysis.

The scheduling errors shown in Figure 9 occur when selecting SMUs (as part of a panel or ROM mix) to be sent to the plant for processing. In this figure, sectors S1 to S4 assume that all the SMUs referred to are already above a designated grade cut-off and have been classified as ore, i.e. there has been no misclassification of ore versus waste at this stage. Note also that the figure refers to 'revenue' and not grade alone because revenue is considered to be a function of volume multiplied by grade, by density and by price (in its simplest form).

Sector S1 refers to those SMUs that have the highest financial value, i.e. high revenue and low cost. These SMUs should ideally be scheduled first in the LOM plan to achieve maximum financial value with respect to the time value of money. Sector S2 refers to those SMUs with the lowest potential value, i.e. low revenue and high cost. These SMUs should be scheduled last in the LOM plan. Sector S3 refers to those SMUs which were mistakenly estimated as S1 but are actually high revenue plus high cost. These SMUs should not be scheduled first in the LOM plan (nor should they be last in S2 either). Sector S4 refers to those SMUs which were mistakenly estimated as S2 but are actually low revenue plus low cost. These SMUs should not be scheduled last in the LOM as they are of slightly higher value (nor should they be first in S1).

3.2.9 Financial Modelling

The main cash flow components that have been considered in the financial modelling stage of this thesis are listed below. They are based on a conventional DCF NPV style calculation for a mineral project (i.e. diamond, gold, base metal projects etc.):

- ☑ *Revenue*, derived from the calculation of extracted ore tonnes, recovered carats (or metal), grade and revenue per carat (or USD metal commodity price) and the foreign:local exchange rate.
- ☑ *Costs*, which are a function of both variable and fixed costs; the former is usually derived from the unit cost per item multiplied by the quantity of items. Both the unit cost price and the estimated quantities are subject to uncertainty.
- ☑ *Capital* estimates that may broadly be categorized into construction capital costs and working and on-going replacement capital, related to resource and reserve uncertainties. An inversely proportional relationship is often established between the capital expenditure (capex) and the operational expenditure (opex) depending on management strategy and their appetite for risk.
- ☑ *Discount rate* approach for diamond projects, especially, can vary radically between companies. One method entails the use of the weighted average cost of capital (WACC) plus any additional premium for technical and/or country risks. While the WACC and country risk component may be derived through market measures or ratings from financial houses (e.g. NM Rothchilds and Sons, Standard and Poor), the technical risk component is unique to each project and the derivation thereof is mostly subjective.

Each of the above-mentioned cash flow components is associated with uncertainty. Time also affects the estimations of each of these cash flow components. Management may decide to invest more capex in the early stages of an operation to reduce opex if the mine has a long LOM forecast and an acceptable risk profile. Conversely, operations that are associated with high risks and marginal returns may only receive minimum capex with relatively higher opex to allow management to assess how the economic viability of the operation develops over time. The mineral evaluation process must also consider that operating decisions made early during the life of a mineral deposit will likely affect the remaining LOM and influence the financial returns. Average mining and treatment grades, capex and opex relationships and operational flexibilities are key considerations that will materially impact the NPV.

In some cases, a conventional DCF NPV financial model (typically in Microsoft Excel spreadsheet format) may be developed by a mining company. This financial model is often designed to evaluate only one set of production inputs and usually comprises a ‘stand alone’ set of MS Excel spreadsheets with no direct link to the production inputs, let alone dynamic resource model inputs or stochastic economic inputs linked directly to the financial model. The assumption of ‘flat real’ or ‘flat nominal’ prices, which implies that management assumes prices are constant and fixed for the duration of the evaluation period, may be simple but unrealistic for project evaluation.

Financial models are typically developed by the project engineer and/or accountant who have usually not been personally involved in the mine planning or resource estimation stages of the specified project. As a result, technical risks and opportunities within the project (specifically within the resource and reserves) may not be appropriately captured within the financial model. One method to compensate for technical risks involves increasing the discount rate of the project, which implies that the impact of technical risks increase at an exponential rate over the LOM of the project. Little regard for the upside opportunities from a resource and reserve perspective is encapsulated in the model.

Furthermore, this deterministic conventional DCF NPV model is not designed to quantify flexibilities (such as operational or investment flexibility options) to evaluate correctly the ‘downside’ risks and ‘upside’ opportunities. Operational flexibility includes any variation of operating parameters related to increasing or decreasing production supply (expansion and contraction); shutting-down; re-opening of treatment plants; re-optimizing of cut-off grades etc. Investment flexibility provides the ability to delay the start of projects should prices or technical risks be deemed too uncertain.

The author recognised the limitations of a deterministic conventional DCF NPV financial model and adapted these models to accept multiple production inputs with system linkages between various software platforms based on the IEM methodology. Visual Basic Application (VBA) code was used within a MS Excel environment to compile code that imported the appropriate inputs from sources (e.g. resource models) into another (e.g. reserve model) and run these through the financial model to generate a range of financial outputs.

The main advantage of using VBA code was that resource or reserve models could be produced in any software format and could easily be imported into the financial model. In some instances, the author had to recreate the financial model because it was not adaptable to running multiple risk analysis scenarios. Specific detail around the necessary adaptations to each financial model and the system linkages forming part of the IEM is discussed in Chapter four on a case by case basis.

3.3 SUMMARY

Standard practice for evaluating mineral deposits often involves using a single resource and reserve model whereupon sensitivity analyses are conducted but these do not adequately capture the range of variation associated with the compounding effect of resource uncertainties. Complex resource estimation problems are often expressed through 'simplified' mathematical equations to solve a global or local geostatistical problem. However, the production and financial impacts of non-linear resource-to-reserve relationships cannot be approximated using a closed-form mathematical solution as each project has its own set of resource and reserve variables, which interacts with mining and processing constraints in a sequential, non-linear and unique way.

Failure to account correctly for spatial and temporal risks, by estimating the 'average' annual production totals instead of accumulating the effects of the short-scale (e.g. daily) interactions of resource variables on the mining and processing constraints into annual production totals, may result in material errors in estimating a mineral project's value. The author recognised these constraints and developed an IEM framework, which encapsulates a simulation model that attempts appropriately to capture, replicate and model the key linkages between resources, reserves and the financial model.

It is demonstrated in the next chapter that the interactions between resources and reserves follow a complex, non-linear pattern, which is specific to each project, and has to be appropriately accounted for by simultaneously considering spatial and temporal scales of data in the evaluation model. The system architecture and design of this IEM approach is common to all three case studies in this thesis and, by inference, to many other mineral commodities such as gold and base metals. This is a key determinant of the IEM approach as although there is some commonality between mineral projects, the unique relationships between the degree of resource variability, its combined impact on the production constraints and its correlations with financial and economic parameters will likely require each mineral project to be uniquely evaluated.

Chapter 4 : Variance Analysis using an Integrated Evaluation Model

4.1 INTRODUCTION

Managers of mining projects are often challenged to make informed decisions based on the financial metrics of projects that are typically derived from several sources of data, comprising unsystematic and systematic risks. The challenge is exacerbated by having to distil various sources of technical uncertainties into a financial model that is usually designed to capture production outputs, summed annually in a cash flow model to produce a single net present value (NPV) or internal rate of return (IRR) figure.

While it is frequently assumed that the appropriate technical expertise is incorporated into the design process at each stage of a project, it remains a challenge to incorporate accurately the spatial and system correlations between technical and financial processes and aptly capture the risks and opportunities in the financial output. It is even more demanding to capture and express risks of the project in 'easily digestible' financial terms. This challenge is magnified when the evaluation assessment has to incorporate several risk scenarios into 'one version of the truth' that is easily understood by decision-makers.

Geostatistical techniques are routinely used to incorporate resource risks such as grade, geology and density for most mineral commodities, (Matheron, 1973) and (Krige, 1951). Since geostatistical simulations were developed (Matheron, 1973 and Journel, 1974), they have been used to model the inherent variability and compare the impact of different mining methods or support sizes on resources and reserves. Early work (Dowd, 1976); (Dumay, 1981); (Chica-Olmo, 1983); and (Fouquet De, 1985) focused on understanding the influence of technical aspects related to complex mining constraints and on quality control during production. As computer power increased, more simulations were run and different types of simulation methods developed that allowed more complex types of geology to be modelled.

Since the 90s, the impact of uncertainty on project economics has become increasingly important as more marginal projects were discovered. (Ravenscroft, 1992); (Berckmans and Armstrong, 1997); (Dowd, 2000); (Dimitrakopoulos et al., 2002) and (Dowd and Dare-Bryan, 2004) have used a combination of objective functions and geostatistical techniques to evaluate the impact of resource risks on the mine plan and determine their probabilistic impacts on NPV. These techniques incorporate resource uncertainty in the scheduling optimization algorithm whereas traditional mine planning methods do not and could result in sub-optimal reserves. It is worthwhile reiterating that simulations depend on the available data, thus, if conditioning data are not representative, the results can be misleading. This is particularly a problem when simulation data are being used at an early stage of project development and therefore, should be reviewed in advance to ascertain its reasonableness.

There is a need for an integrated evaluation model to aid management to make more informed decisions based on quantifiable information that appropriately incorporates systematic risks (price and FX rate uncertainties) and unsystematic risks (resource and reserve). Three case studies are described below that demonstrate the value of an integrated evaluation model (IEM) framework to obtain greater insight into project dynamics and to provide quantitative confidence limits around production and financial outputs.

4.2 CASE STUDY 1: ASSESSMENT OF RESOURCE VARIABILITY ON MINING CONSTRAINTS FOR AN UNDERGROUND OPERATION

4.2.1 Technical Overview

This case study demonstrates the impact of the scale of measurement in NPV terms on the evaluation of an underground diamond mine with several mining challenges based on resource and reserve uncertainties. Scale of measurement refers to dimensions in both space and time that are related to the key variables of the project, such as volume (vein thickness), grade, density, costs, revenue and foreign exchange rates. This is critical to a valuation assessment for a mineral project, as it will be demonstrated that for certain deposits with complex resource characteristics and limited operational flexibility, the valuation is materially affected by the use of large-scale, annual average estimates for key resource and reserve variables. An integrated evaluation modelling (IEM) methodology is recommended using short-term, operational scale numerics that are accumulated into annual estimates to derive more realistic NPVs.

It is unrealistic to create predictions of resource and reserve estimates on a small block scale when sample data are limited and spread over a large volume. In many cases production estimates of tonnages and grades are computed on an annual basis rather than a shorter-term scale (e.g. daily or weekly). The sum of the local reserve depletions in a year is not equal to the total expected production derived from the average global reserve depletions. This is most applicable for mineral projects that have a high degree of short-scale geological and mineralization variability but only limited sampling data. The effect is amplified when resource variability has a substantial impact on mining rate and treatment efficiencies. The problem is further exacerbated for marginal projects which usually cannot afford the cost and potential time delays of spending additional evaluation capital on attaining close-spaced sampling data.

As the scale of data acquisition changes (i.e. more or less data are acquired), the mean and variance of the data will typically change. The impact of scale on a single variable depends largely on the distribution of the underlying phenomenon, e.g. grade or density. If many sample data were acquired, the shape of the distribution (specifically, the means and variances) for each variable would be well-defined. In most cases of evaluation, however, only limited sampling data are acquired and as a result, changes in the means and variances of individual resource variables could have a material impact on the project value.

Two different evaluation approaches were selected in this case study to demonstrate the impact of measurement scale, viz. ‘top-down’ and ‘bottom-up’ techniques. The former refers to annual forecasts that are calculated from depleting resource estimates through a global mine plan. Average expected values per annum are used as inputs into the mine plan to produce a NPV estimate. An alternative approach uses a bottom-up evaluation technique whereby additional sampling data allow finer resolution resource models to be created. These finer scale models provide a way to carry out a quantitative assessment of the impact that resource variability has on daily mine output. Annual cash flow forecasts are derived from accumulations of daily depletions based on localised resource estimates.

While it may appear that these two methods would produce similar NPV results, there are cases where they do not. A case-study of a Canadian underground mine is presented where diamonds are contained in an irregular dyke that intrudes into a fractured granitic host rock. Two sources of uncertainty were modelled. Firstly, geology was evaluated as a form of unsystematic (specific) risks due to the uncertain thickness of a mineralized dyke and its undulating top surface. Secondly, economic uncertainty, in the form of foreign exchange rate volatility between the US dollar and the Canadian dollar, was integrated into the evaluation model as a systematic (market) risk.

4.2.2 Geology and Resource Modelling

The actual geometry of the dyke is deemed to be particularly variable. On the regional scale (hundreds of metres) the dyke appears to be a continuous, gently dipping sheet, although three areas of offset have been identified by surface seismic imaging (McBean et al., 2001). At a more local scale (10-100m), orientation changes and splits and large splays have been observed, which are thought to be structurally controlled. On a small scale (0-10m), the dyke is typically controlled by two different host rock features. Within the strongly foliated metavolcanics, the kimberlite appears to roll and undulate on a small scale matching the foliation, while in the granitic host rocks, local variations occur along a primary set of joints that are flat lying but affected by secondary jointing resulting in an angular step-like nature to the dyke (McBean et al., 2001).

To assess the impact of geological variability on project valuation, the author simplified the this problem by assuming that dyke thickness and shape variability derived from face-mapping in the development tunnels were representative of the entire deposit. A VBod was created using a non-conditional geostatistical simulation from a combination of drilling information, bulk-samples and face mapping from an exposed part of the dyke. It is assumed to be the ‘reality’ on which various sampling campaigns were conducted to generate sample data. Although sampling data were available, the author did not want to run a conditional simulation designed to honour sample data because the simulated variance would be restricted to that calculated from known drill hole data.

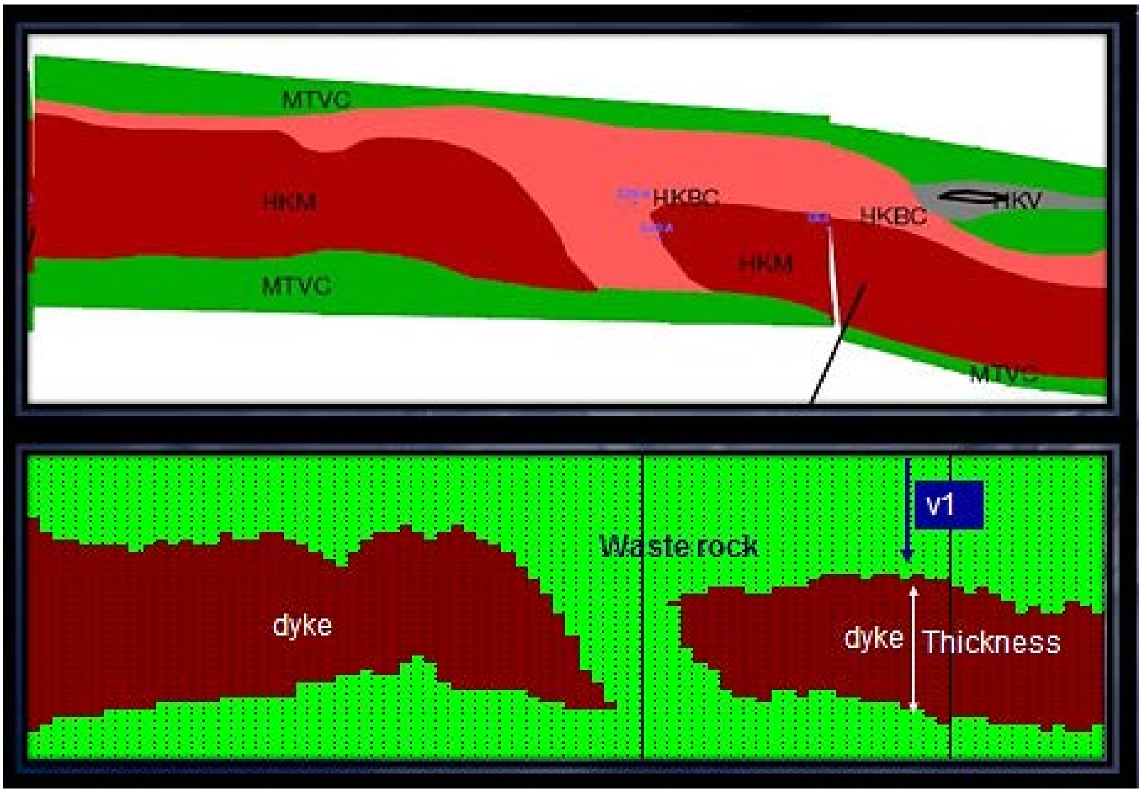


Figure 10. Geological representation of a Virtual Ore Body (VBod) derived from a combination of drill hole data and geological face maps. In this figure "v1" represents the height from the topographic surface to the top of the dyke (mineralised zone) and "Thickness" parameter represents the mineralised thickness.

Figure 10 shows the three variables considered in the evaluation model for this case study:

- The geometrical variability of the top surface of the dyke (v1), which is a vector measured from the surface topography to the top of the 'lumpy' orebody;
- Thickness representing the estimated volume of the dyke; and
- Grade (in carats per hundred tonnes).

Despite the limitation of this approach that only a single VBod was created due to time constraints, it is still deemed appropriate for the purposes of this study as it provides a system to quantify the estimation accuracies between the selected evaluation methodologies. Sample data were used as input to generate kriged estimates and conditional simulations for grade, dyke thickness and geometric surface undulations of the dyke.

If drilling data were limited, it may understate the true variance of the deposit. Hence, an unconditional simulation was used to try and model the full range of possible variances. For simplicity, one simulation is assumed to be reality (the VBod) rather than as a single realisation of a particular orebody. Comparisons were made between the two techniques and the VBod. Three sampling campaigns were conducted on the VBod and resource and reserves estimates were recalculated each time using the additional information to assess the impacts on differences between the top-down and bottom-up approaches.

Sampling data in any evaluation model are fundamental in producing estimates that reflect reality. Although including more spatially representative samples typically reduces uncertainty associated with both the mean and variance of resource estimates, it does not alter the natural variability within the deposit. In some cases drilling more sample holes may not necessarily reduce the variance of the estimated grades derived from sample-sized volumes of the deposit, it could reveal an increased in the estimated variance. The variance estimated from a finite number of samples will vary as the number of samples varies.

Koch and Link, (1970-71) discussed how variability of the sample mean depends both upon the variability of the original population and upon the sample size; variability decreases as the sample size increases. Thurston (1988) described considerations for sampling kimberlites and identified how the uncertainty in local and global estimates change with different sampling configurations, both in terms of the size of the sample and in terms of the number of samples taken. The limitations of designing a sampling campaign for multiple variables are discussed by Kleingeld and Nicholas, 2004.

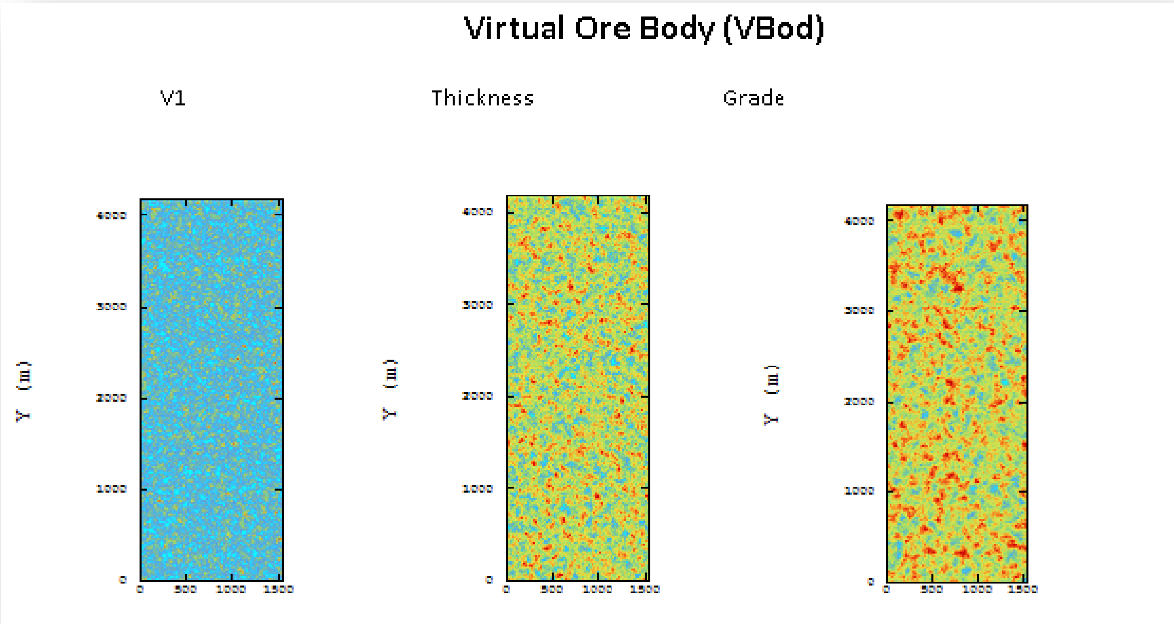
Core drilling was used to delineate geological variability on three different grid densities; 75m by 75m, 50m by 50m and 25m by 25m, creating scenarios one, two and three, respectively. A 50m by 50m drilling grid was used to sample for grade, using large diameter drilling (LDD). Grade did not have any material variability between scenarios and thus, a single sampling campaign sufficed. The same grade estimates were applied to each scenario.

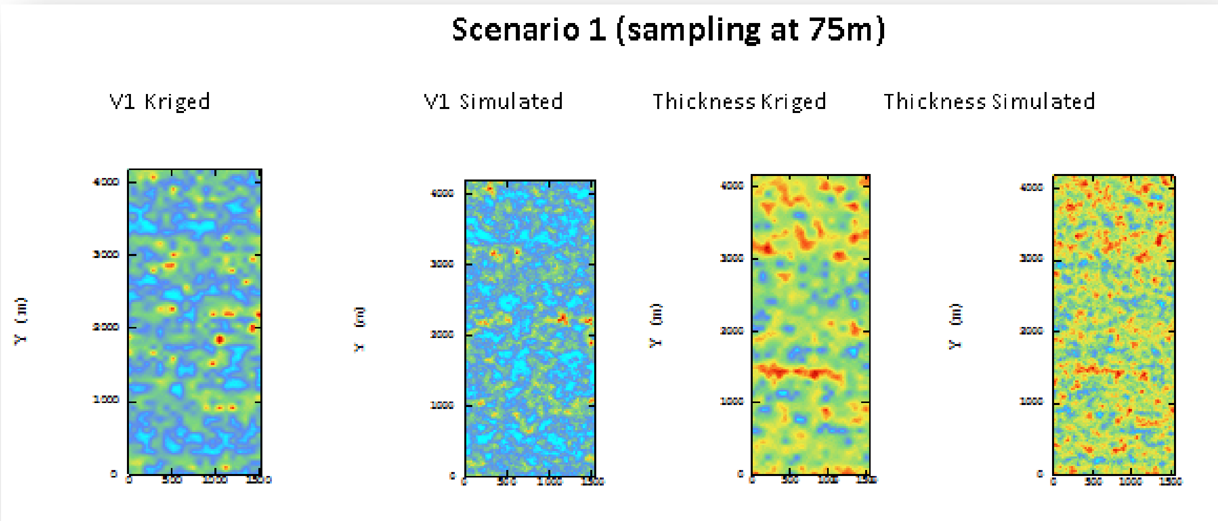
Table 2 describes the design of the simulated sampling campaigns on the VBod; sampling occurred at point support and simulation grid nodes were 4m by 4m in dimension.

	V-bod	**Scenario 1**	**Scenario 2**	**Scenario 3**
Description	reality	wide-spaced	moderate	detailed
Grid Dimensions	4m x 4m	75m x 75m	50m x 50m	25m x 25m
No. of samples/ nodes	399 360	1 136	2 556	10 224
Sample % of V-Bod	-	0.28%	0.64%	2.56%

Table 2 summarises the three sampling campaigns and the VBod.

A single mine plan was created based on combined kriged estimates (for grade, v1 and thickness) and overlain onto each estimate and simulation to determine the reserves. All output was fed into the financial model. Base maps of the VBod and each sampling campaign are shown in Figure 11 (colours towards the red end of the spectrum represent higher values while colours toward the blue end represent low values).





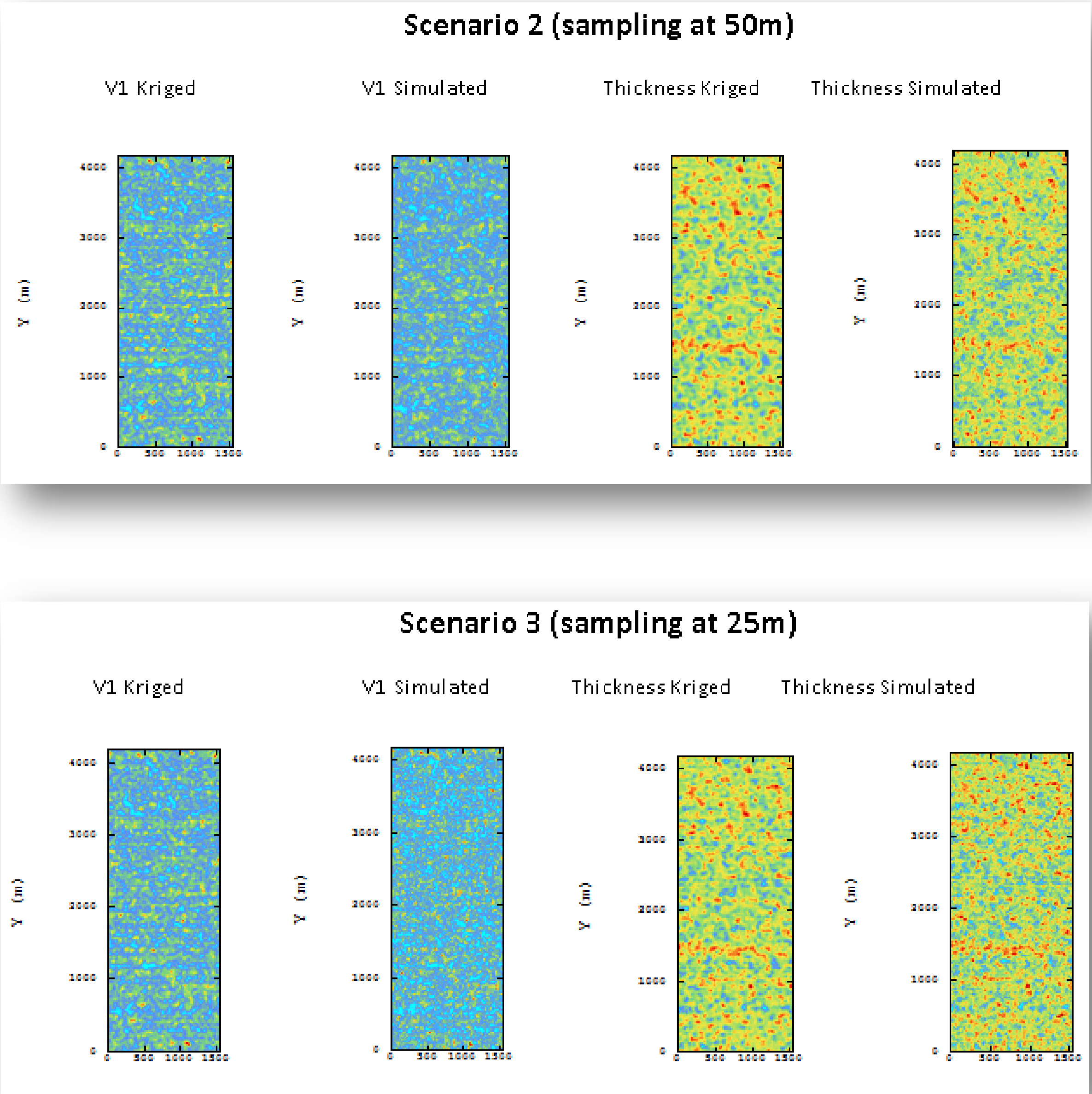


Figure 11. Compares the thickness and v1 base maps for the kriged and simulated outputs of each scenario (sampling at 25m, 50m and 75m) with that of the VBod. Grade was held constant between scenarios. Note that v1 represents the vector measured from the surface topography to the top of the dyke (or orebody vein) and represents the degree of undulation of the orebody

	V-bod	Scenario 1 (75m)		Scenario 2 (50m)		Scenario 3 (25m)	
		Kriged	Sim 1	Kriged	Sim 1	Kriged	Sim 1
Mean Thickness	1.70	1.70	1.66	1.70	1.71	1.70	1.69
Variance Thickness	0.23	0.11	0.18	0.11	0.15	0.13	0.20
Mean v1	1.88	1.90	1.90	1.90	1.91	1.90	1.91
Variance v1	0.18	0.09	0.17	0.09	0.10	0.09	0.17
Mean Grade	191	195	187	195	195	195	192
Variance Grade	1 985	1 062	2 860	1 062	1 523	1 062	2 004

Table 3 shows the statistical differences between the VBod and each scenario for grade, dyke thickness and the geometrical variability of the dyke surface (v1).

In Table 3 the key variables that were modelled were dyke thickness, v1 and grade with the modelling emphasis placed on thickness and v1. Interestingly there is little increase in variance for the v1 kriged estimates (or the simulated data in Sim1) but in some ways this is expected given the smoother undulating surface of the dyke, which has been modelled by the geologist based on available face maps and drilling data. The kriged thickness estimate on the other hand shows an increase in variance from scenarios one to three, i.e. as the drilling density increases, more variability of the localized thickness parameter has been detected.

4.2.3 Reserve Modelling

The degree of resource complexity may have less material impact on an operation's financial outcome for operations that are generally unconstrained in terms of mining and treatment thresholds (assuming that resource estimates have been estimated accurately). This applies to scenarios where sufficient flexibility is included in the mine plan so that no bottlenecks occur in the extraction or processing processes. In this case the degree of flexibility would require a commensurate degree of homogeneity and the rate and scale of mining would deviate very little from plan as a result of resource variabilities.

In contrast, mining operations that operate under strict reserve constraints or resource complexity/heterogeneity, such as geotechnical and hydrological constraints in environmentally sensitive areas, or operations that must deliver a product to specific purchaser contracts with penalties for non-compliance (e.g. some iron ore mines delivering off-take product to multiple steel mills) do not have the luxury of unlimited mining and treatment flexibilities. In some cases these mines cannot easily respond to changes in tonnages or grades as a function of resource variability. In the case of marginal operations with limited capital expenditure, the impact of this limited responsiveness is further exacerbated by the presumption of 'smoothed' ore horizons due to kriging with limited sampling data. The impact of this smoothing will be demonstrated in this case study.

4.2.3.1 Mining

A conventional room and pillar underground method (Hustrulid and Bullock, 2001) was used in this case study combined with the use of 'slashing and drifting' mining, depending on whether the dyke thickness is less than a specified mining threshold. Slashing and drifting was used as opposed to the conventional room and pillar method when dyke thickness narrowed and mining had to be more selective to try and minimise dilution (typically incurring a slower mining advance rate). An average extraction rate of 75 percent was imposed. Each mining block of size 250m by 250m was depleted based on a combination of rim tunnels, stope tunnels and stope slashing. An average daily call of 3150 treatment tonnes was imposed on the project by management. The mine plan and treatment plant were designed to meet this production requirement on average per year.

The tabular nature of this deposit and mining, hydrological and geotechnical restrictions severely limit the sequencing and optimization of extraction. Simplistic assumptions were made regarding the selection sequence of blocks based on the highest value blocks being extracted first to maximize the time value of money. While the author recognizes the work done by Dimitrakopoulos and Ramazan (2004); and Dowd and Dare-Bryan (2004) involving the optimization of blocks given resource and reserve uncertainties, the focus of this study was not on optimization but on attempting to model variability in a unique way that considered both the spatial and temporal scales. The mine plan provided an opportunity to understand the interaction of the spatial nature of the reserves with the temporal realization of their value.

A mining depletion programme was created in MS Excel software (using VBA programming) whereby mine blocks were depleted at a smallest mining unit (SMU) scale of 4 m by 4 m with a minimum mining height requirement of 2.0 m for equipment access into stope tunnels. The use of MS Excel software, instead of conventional underground mining scheduling software such as Earthworks Production Software (EPS), allowed the interaction between multiple resource models with selected reserve constraints. The maximum mining heights of stopes were constrained to 2.2m while rim tunnels were 3.5m high.

- Rim tunnels were 4m x 4m x 3.5m (height)
- Stope tunnels were 4m x 4m x minimum 2.0m (height)
- Stope blocks were 4m x 4m x minimum 1.0m (height)

Pillar dimensions varied depending on the support required but no span greater than 8m was created. Figure 12 shows the plan view of the mining depletion plan designed and created in MS Excel on the basis of an actual room and pillar mine design with 'slash and drift' mining to minimise dilution when depleting ore in narrow dykes – special low profile mining equipment would be used in this regard.

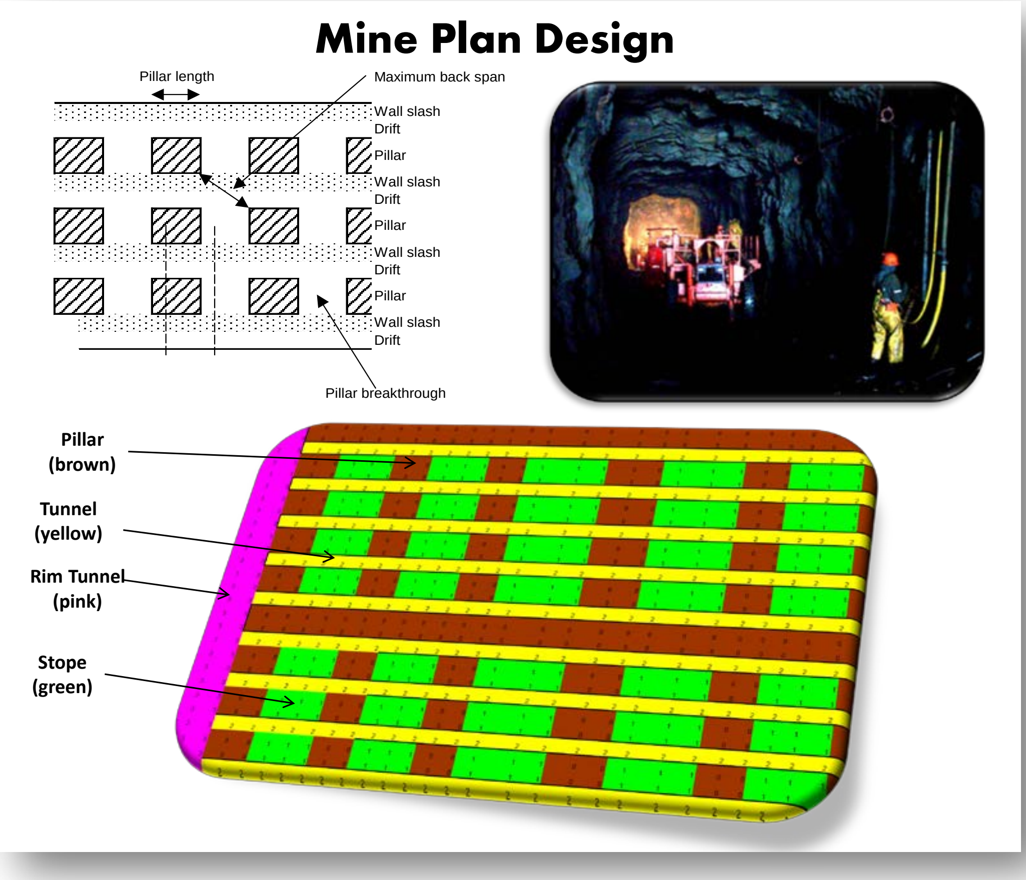


Figure 12. A 2-D plan view of the mining depletion programme created in MS Excel based on an actual mine plan incorporating a conventional room and pillar underground mining technique combined with 'slash and drift' mining techniques to deplete narrow dykes while minimizing dilution. The top right insert shows an actual photograph taken of an access tunnel in the underground mine.

4.2.3.2 Recovery Modelling

The estimation of the mean recovery factor and its variance is vital in determining the quantity of recovered material at a predetermined throughput treatment rate. The recovery factor for diamond projects depended largely on three key considerations:

- characteristics of the ore type;
- its liberation and separation properties; and
- the design and interaction of the treatment process in relation to this ore type.

The challenge of achieving efficient recoveries is to understand these complex three-way interactions. For this study, it was assumed that a linear relationship existed between the proportion of kimberlite ore and the waste.

The impact of the recovery factor on the recovered carats can be very marked especially if there are constraints on the system. For example, if the cut-off grade is close to the statistical mean, subtle variations in the mean cut-off grade could significantly impact the project NPV. If the cut-off grade is raised, the average grade above cut-off increases which may require mining that is too selective using the current mine design and equipment.

Plant design, by its nature, requires a best fit for the ‘average expected feed’ and hence cannot incorporate the daily feed variation that may occur over the project’s LOM. Conventional approaches to plant optimization (Parker, 1977) usually entail:

- adapting the plant to accept the variability;
- installing a stockpile blending system; and
- adapting the mining method to increase the number of faces or draw points and use smaller equipment to improve selectivity.

The example in this study is fairly fixed in terms of its mine design and equipment selection. In addition strict environmental policies regulated the creation of large stockpiles on the surface and limited space underground restricted the size of stockpiles below the surface. A total stockpile capacity of 3,000 tonnes was created, which included capacity from an underground storage bin. While some degree of flexibility was available to adapt the plant settings to the ore variability, this was more suited to weekly and monthly fluctuations but would not cater for daily variations in the system.

While dynamic simulations were considered as a possible means to estimate the short-scale variability in the recovery efficiency, this was beyond the scope of this study and not considered to be material at this stage. A simpler, pragmatic approach was sought to ascertain the impact. Depletions of the simulated 4m by 4m SMUs provided the ore-waste proportion information. A simplified linear relationship was imposed on treatment recoveries where the total grind and liberation of diamonds were a function of the proportion of waste and kimberlite, hence recovery efficiency improved as the proportion of kimberlite increased (see Figure 13). A plant surge capacity constraint was included to assess the impact of varying dyke thickness (on a 4m by 4m SMU scale) on the feed rate variability using an 'event-based' simulation.

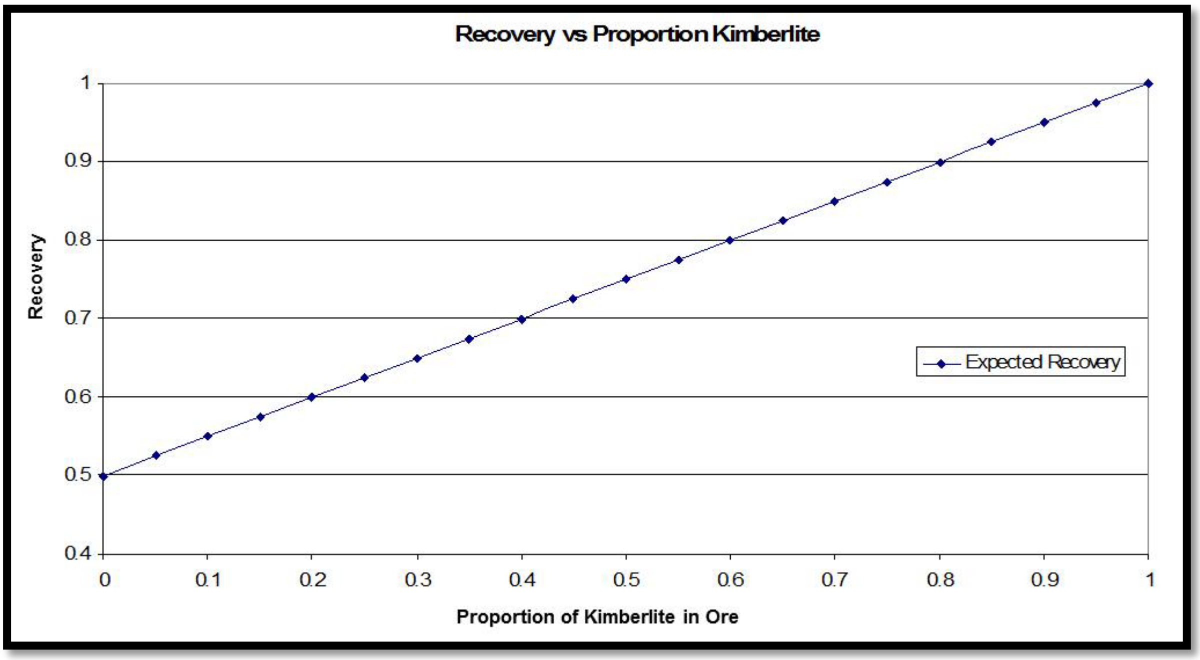


Figure 13 shows the linear relationship between the processing plant recovery and the proportion of kimberlite in the processed ore. Higher recoveries are associated with a higher proportion of kimberlite.

Principal strategic parameters that were considered for mining and treatment were:

- Annual mining rate in order to produce 3,150 tonnes per day;
- Total stockpile capacity with a maximum of 3,000 tonnes (comprising 1,500 tonnes in an underground storage bin plus a further 1,500 tonnes stockpile at surface);
- SMU selection (4m x 4m x height in metres);
- The maximum mining ramp angle (17 degrees); and

- A threshold imposed on the waste/kimberlite proportion (70/30); if any blasted block had more than 70 per cent waste then it was not sent to the treatment plant.

4.2.3.3 Mine Plan and Treatment Output

Daily production variations for scenario three (25m sampling campaign) are shown in Figure 14, which highlights the variability in the total tonnes mined (kimberlite ore plus waste) relative to the recovery factor for the first year. The correlation between the recovery factor and the proportion of kimberlite can also be seen due to the modelled linear relationship (depicted in Figure 13).

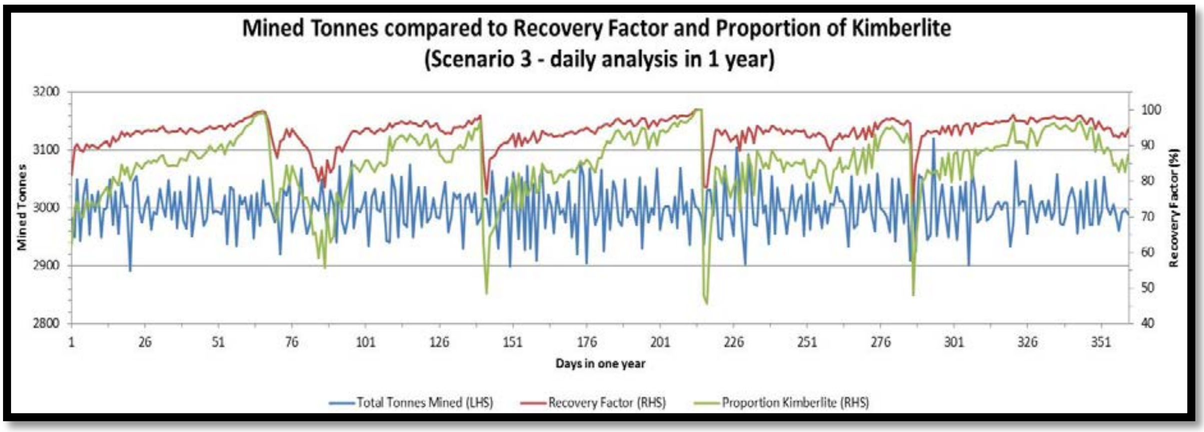
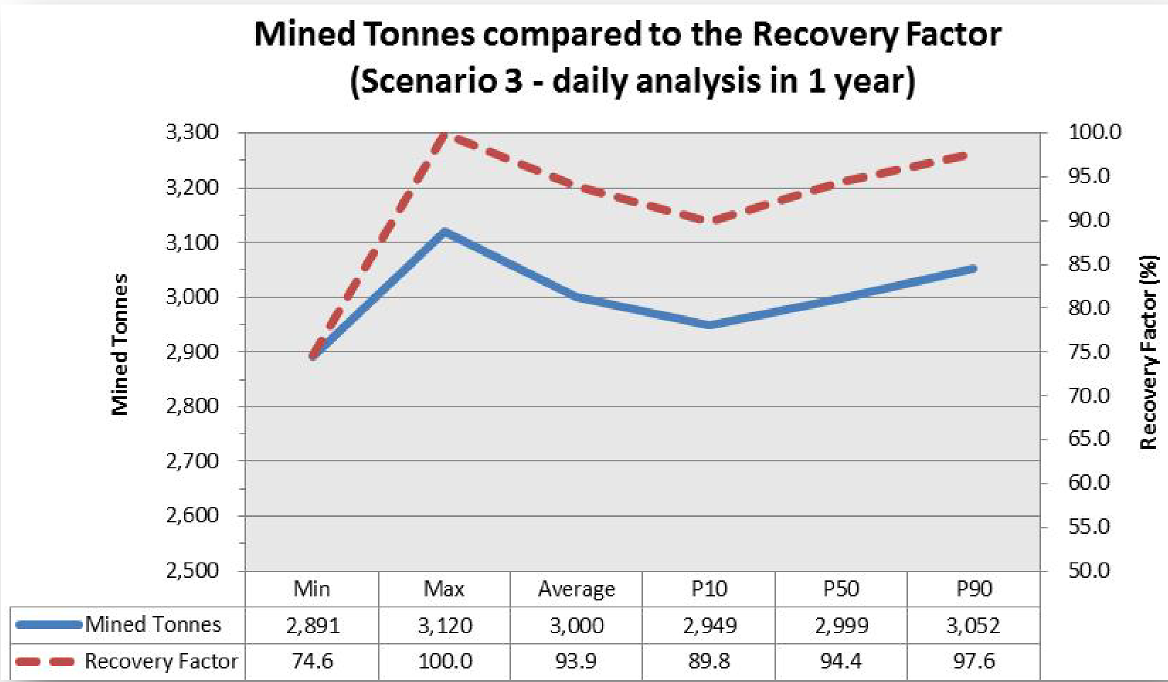


Figure 14. Graph depicting the relationship between total tonnes mined (ore plus waste) on a daily basis by depleting each mine block relative to the recovery factor which is influenced by the proportion of kimberlite (ore) sent to the plant on a daily basis. Results are shown over one year for scenario 3 (25m sampling programme).

It is important to distinguish between mining recovery and processing recovery. Mining recovery considerations included a 75% mining extraction rate that was imposed on each mine block. In addition if any blasted block had more than 70 per cent waste, it was not sent to the treatment plant but instead sent to waste. There is a further mining constraint in terms of the total storage capacity of 3,000 tonnes per day, which includes an underground storage bin of 1,500 tonnes and another 1,500 tonne stockpile at the surface. Processing recovery on the other hand was determined by the linear relationship shown in Figure 13 whereby recovery increased as a function of the proportion of kimberlite ore.

As shown in Figure 14 the daily tonnes mined varied as a function of the proportion of waste in each mining block combined with the impact of hitting the total stockpile capacity of 3,000 tonnes per day. In some cases additional ore is still mined on a daily basis as this ore has already been blasted and passed the 70% ore/waste criteria so is sent directly to the plant as the stockpile capacity threshold has already been reached. The recovery factor is determined by the proportion of kimberlite mined in each block, which in turn is influenced by the mining sequence. The lowest recoveries are realised when the mine plan depletes the end of a sequence of stopes and mines predominantly rim blocks comprising mostly waste material.

Figure 15 shows some of the daily statistics of total tonnes mined in relation to the recovery factor over the first year for scenario three (25m sampling campaign).



	Min	Max	Average	P10	P50	P90
Mined Tonnes	2,891	3,120	3,000	2,949	2,999	3,052
Recovery Factor	74.6	100.0	93.9	89.8	94.4	97.6

Figure 15. Figure shows the statistics between total tonnes mined (LHS) which is ore plus waste on a daily basis by depleting each mine block relative to the recovery factor (RHS), which is influenced by the proportion of kimberlite (ore) sent to the plant on a daily basis. Results are shown over one year for scenario 3 (25m sampling campaign).

On average 3,000 tonnes per day was mined which is expected given that the maximum stockpile capacity threshold was set at 3,000 tonnes. A total of 10% (P90) of the blocks exceeded 3,052 tonnes reaching up to a maximum of 3,120 tonnes mined per day. The plant utilization was 95.2% (name plate capacity was set at 3,150 tonnes per day) restricted mainly by the mining bottle-neck, due to there being insufficient mining flexibility (stopes) to get the ‘right ore tonnes to the plant. The P50 recovery factor was 94.4% with 10% (P90) of blocks exceeding 97.6% up to a maximum recovery of 100%, the latter was derived from one block which had 100% ore and no waste.

Over a ten year period the minimum total tonnes mined were 2,849 and the average was 3,000 tonnes reaching a maximum of 3,144 tonnes mined per day. Over this same period the P50 plant recovery was 93.8% with 10% (P90) of blocks exceeding 98.0% recovery. The minimum recovery over ten years was 63.8% and the plant utilization was 95.2%.

Output from the mining and treatment phase on an annual basis is tabulated in Table 4 for the VBod and each of the three scenarios.

		Kriged Results			Simulated Results		
	V-Bod	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
Total tons (milion)	10.8	10.8	10.8	10.8	10.8	10.8	10.8
Recovery Factor	92.5	93.6	93.3	93.1	92.1	92.5	93.0
Recovered carats (milion)	16.2	16.6	16.6	16.5	15.7	16.2	16.4
Recovered Grade	149.6	153.8	153.7	153.2	145.7	149.8	151.9

Table 4 shows annual production output for the VBod and three scenarios.

4.2.4 Financial Modelling

One of the most important considerations in the design and development of a financial model is temporal scale. This refers to the time interval in which cash flows are estimated, which should correspond with the time interval in which mining and treatment production data are measured and accumulated. This does not imply that that the financial model should be developed at a daily, weekly or monthly scale but rather that the short-term production outputs are correctly accumulated to form input into the financial model at an annual scale. Reserves depend on the mine plan’s ability to react to resource variability at the appropriate operational short scale.

This section of the study demonstrates that cash flow constituents derived from annual estimates in a top-down approach will not correctly reflect the asymmetries of operational variability on a local, daily basis. A more accurate way of deriving annual cash flow estimates required for project decision-making would be to accumulate the appropriate values from a bottom-up approach, i.e. daily, monthly, quarterly then derive annual estimates for NPV forecasts.

The bottom-up approach entailed estimation (via geostatistical kriging) of the main resource variables into a fine resolution grid (SMUs of 4 m by 4 m) based on sampling data from each campaign. Each SMU was analogous to a mining blast that was assessed to ascertain if it met the necessary mining and plant criteria, before either contributing to the daily plant call of 3,150 tonnes per day or being sent to the waste bin if it comprised more than 70 per cent waste. These daily accumulations were added together to form monthly, quarterly and annual production totals forming inputs into the cash flow models to derive NPVs for each scenario.

For the top-down approach, it was assumed that the mine plan only incorporated sufficient detail to deplete large-scale mine blocks of dimensions 250 m by 250 m. This implied that local mine plans (within each large-scale mine block) were not available to allow sequential depletion of the SMUs to accumulate tonnages and carats in a given year. Although the resource was modelled on a finer resolution (SMUs of 4m by 4m), these values were averaged into larger 250m by 250m mine blocks. The mine plan was designed to deplete on average 3.3 large-scale mine blocks per annum.

The average resource values for each year were run through this mine plan, assuming a fixed daily plant call of 3,150 tonnes per day could be attained. Total recovered carats were calculated as a function of depleting the average estimated tonnages (per large-scale mine block) at a fixed throughput rate of 3,150 tonnes per day, then multiplying the depleted carats with an average recovery factor per large-scale mine block. The carats per large-scale mine block were accumulated into annual cash flow models to produce global NPV estimates for each of the three kriged scenarios.

Table 5 presents local versus global NPVs (in CAD millions) calculated for each scenario using bottom-up and top-down approaches, respectively.

		Kriged	**Kriged**	**Kriged**
	V-Bod	**Scenario 1**	**Scenario 2**	**Scenario 3**
Global Annual NPV	-	91.6	80.1	73.9
Local Annual NPV	2.1	32.9	31.4	28.3
Differences	-	58.8	48.7	45.6

Table 5 shows differences between the global NPV using a top-down approach (called Global Annual NPV) compared to the NPV annual based on a bottom-up approach (or Local Annual NPV). All values were calculated using a flat FX rate and are shown in CAD millions.

NPV was used as a key metric to assess whether the project made a profit after all debts, invested capital and interests have been repaid. Once the NPV estimate was derived, the second step was to plot the annual DCFs as this shows when the major proportion of cash flows fall and whether there are any irregularities over the LOM. The annual, locally-derived NPVs using the kriged estimates for scenarios one and three are CAD 32.9 million and CAD 28.3 million, respectively. While the NPV for scenarios one, two and three show that the estimated NPV derived from the samples is getting closer to the V-Bod NPV as the sampling density increases, it is apparent that even scenario three is still significantly higher than the CAD 2.1 million NPV for the V-Bod. It suggests that the smoothing effect of kriging for the sampled scenarios is positively (but inaccurately) impacting the calculated cash flows.

Figure 16 compares the annual cash flows and DCF values for these two scenarios.

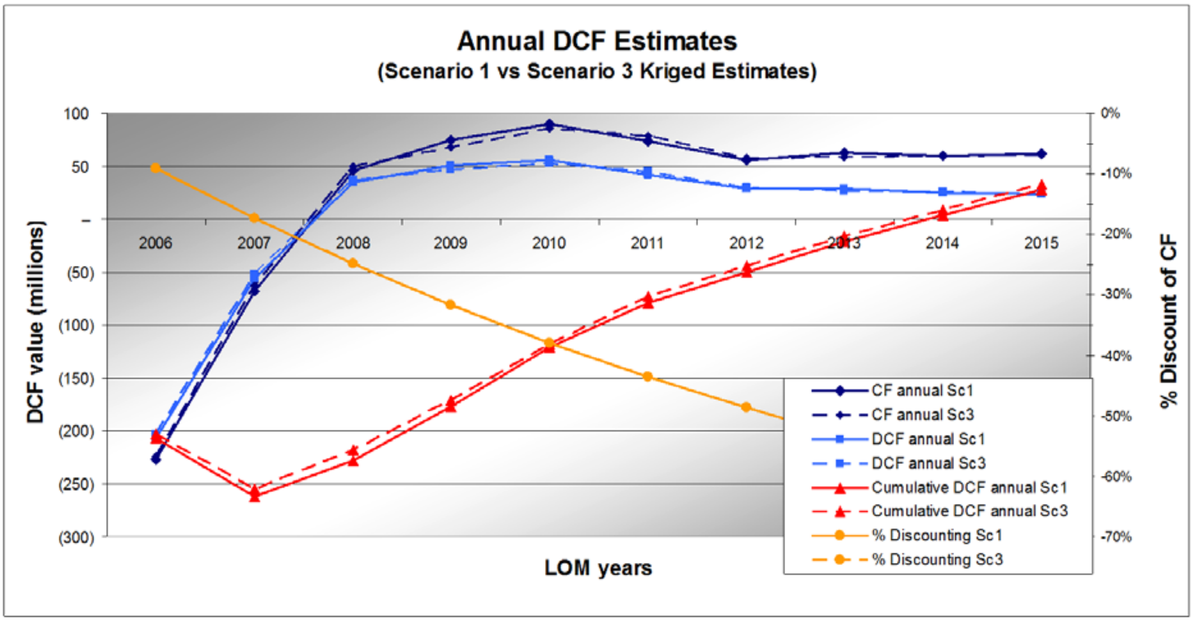


Figure 16 compares the net cash flows (CF), discounted cash flows (DCF), cumulative discounted cash flows in CAD million (LHS) and the percentage discounting (RHS) applied to net cash flows for scenarios 1 and 3.

Figure 16 shows that the period between 2008 and 2012 accounts for more than 60% of the project's positive annual cash flow and 70% of the DCF value. As cash flows generated after 2012 are discounted at values of 50% and higher, management would have to make significant operational changes in order to increase net cash flows beyond 2012. The challenge with this concept is that typically, mine managers run an operation for a few years only and they would need to spend considerable capex during their ‘watch’ that would have no immediate benefit to the way they are rewarded (w.r.t. performance bonuses etc.).

A more pragmatic way of thinking would be to plan the mine schedule in such a way that the operations have the capability of adapting to the resource/reserve variability over time – this may imply spending somewhat more capex in the initial design and construction of the mine and processing plants (perhaps adopting a modular approach) in order to provide financial benefits further on in the life of the mine. From a DCF NPV estimation perspective, the longer the LOM, the more difficult it will be to validate spending upfront capex during initial years to benefit the operation much later during its LOM. Management will more likely be inclined to spend capex on improving the net cash flows earlier on to maximize NPV.

4.2.5 Economic Modelling

In addition to the unsystematic risks, the financial model should also take due cognizance of systematic risks by incorporating these stochastic variables at the appropriate time scale (support size). In their paper, Brennan and Schwartz (1985) used a geometric Brownian motion based on Black and Scholes (1973) method with a convenience yield proportional to price in order to model the copper price. This was necessary to try and reproduce the natural variability of commodity prices over time. Diamond prices are not as volatile as other commodities. Factors such as oil price and exchange rates are often more volatile and have a material impact on project value; the oil price affects costs and the exchange rate influences the company’s revenue. The author has chosen to focus on the exchange rate for this study.

Many models have been developed for interest rate and foreign exchange rates, ranging from simple extensions of Black and Scholes (1973) through Vasicek (1997) and on to the latest models with stochastic volatility. The book edited by Hughston (1996) provides a good overview of the subject. For this study Nicholas et al. (2006) chose to use the Garman and Kohlhagen, 1983 (G-K) model, which is a simple extension of Black and Scholes’ original model. In this model the drift term is replaced by the difference between the domestic and foreign interest rates. If S_t denotes the spot exchange rate in terms of the domestic currency

(CDN) per US dollar, at time t and r_d and r_f are the domestic and foreign interest rates respectively, then:

$$dS_t = \left(r_d - r_f\right) S_t dt + \sigma_S S_t dW_t$$

Equation 18. Garman and Kohlhagen (1983) model for modelling foreign exchange rates.

where σ_S is the volatility of the exchange rate; dW_t is a Brownian element; and dS_t is the change in the exchange rate S going from period t to period t+1. A simple overview for the G-K model was provided in section 2.6 of this thesis.

Two scenarios considering FX uncertainty were integrated into the evaluation model. In both cases, the FX rate was applied only to the revenue component as sales from diamonds were in notional USD whereas all costs were assumed to be sourced locally. The first scenario assumed a flat rate of 1.21 CAD to a USD. This corresponds to a forward FX price. Transaction costs were ignored. The NPV results of the three scenarios relative to the VBod using the flat rate were shown in Table 5.

The second scenario assumed that the management team would expose the project to the FX rate volatility. FX stochasticity was modelled using a Garman and Kohlhagen (1983) model to incorporate mean reversion and volatility parameters. A total of 100 simulations were run over a 10-year period emulating the FX uncertainty (Figure 17).

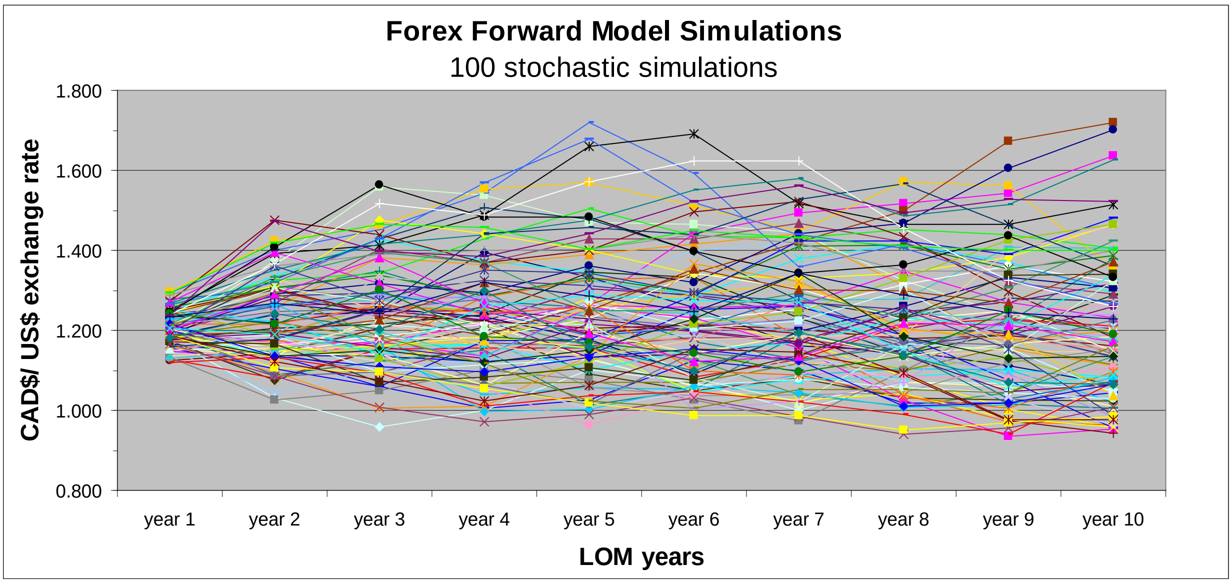


Figure 17 shows the FX rate stochastic output per year from 100 simulations.

Each of the 100 simulations was incorporated into the financial model to produce NPV estimates for each scenario and for the VBod. NPV histograms and cumulative probability plots for the VBod are shown in Figure 18 and Figure 19.

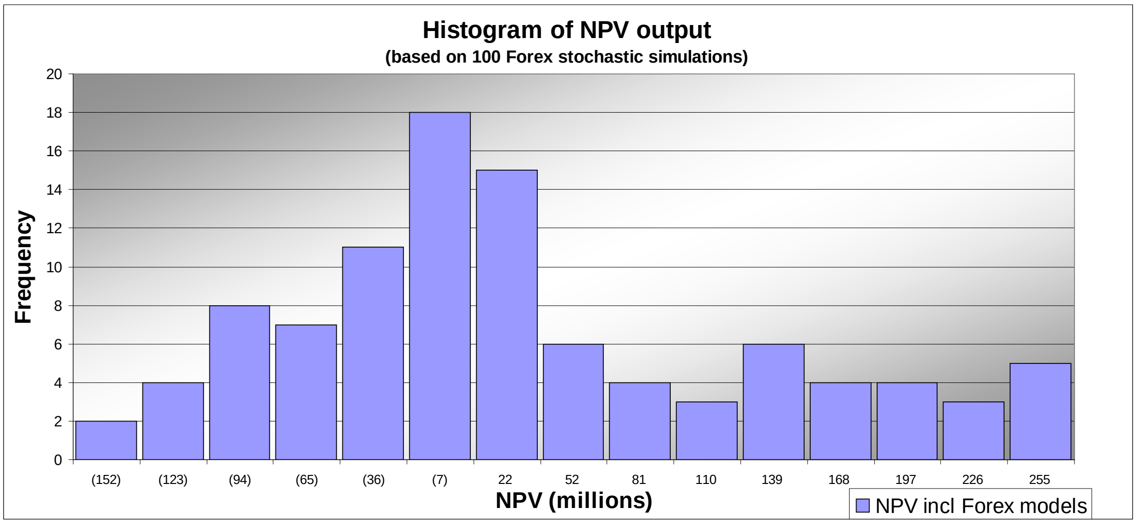


Figure 18 shows the NPV histogram (in CAD millions) for VBod after including 100 FX simulations.

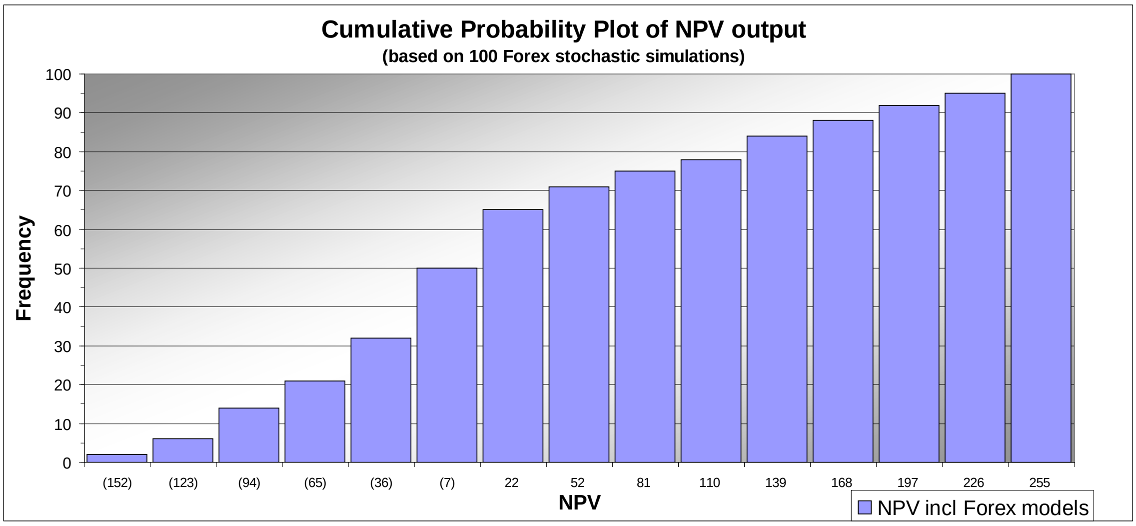


Figure 19 shows the cumulative probability plot of the NPV (in CAD millions) for VBod after including 100 FX simulations.

NPV comparisons incorporating the FX rate simulations are tabulated in Table 6 for each scenario and for the VBod. All values shown were calculated using the local estimation technique (bottom-up approach).

	V-bod	Scenario 1	Scenario 2	Scenario 3
Max. NPV (annual)	255. 2	292. 5	291. 5	287. 6
Min. NPV (annual)	(177.4)	(150.3)	(152.8)	(155.2)
NPV P50 (annual)	(6.7)	24. 5	22. 7	19. 6
P50 difference (%)	-	468%	440%	394%

Table 6 shows the maximum, minimum and 50th percentile NPV of the three scenarios relative to the VBod after including FX rate modelling (% differences are relative to the VBod P50 value).

4.2.6 Conclusions

This case study demonstrated the impact of resource and economic stochasticity on a project's NPV as a function of both sampling and temporal uncertainties. A virtual ore body (VBod) was created using a non-conditional geostatistical simulation where one simulation is assumed to be reality to provide a method of comparing scenarios. Three sampling campaign grids of 75m, 50m and 25m were conducted on the VBod to produce scenarios one, two and three, respectively. It was shown that global annual NPV estimates derived in a top-down fashion, markedly over-estimated the VBod NPV. Comparisons between scenarios showed material differences in the NPV estimates.

Global NPV derived from kriged estimates for the three scenarios (75m, 50m and 25m) are CAD 91.6 million, CAD 80.1 million and CAD 73.9 million, respectively. As drilling grid densities increased from 75m to 50m and 25m intervals, the uncertainty of v1 and dyke thickness decreases and the estimates improved relative to the actual VBod NPV (CAD 2.1 million). Nonetheless, all global estimates over-estimated the VBod NPV estimate by a magnitude of 43 to 35 times (75m to 25m scenarios).

Local NPV derived from kriged estimates for the three scenarios (75m, 50m and 25m) are CAD 32.9 million, CAD 31.4 million and CAD 28.3 million, respectively. Similarly, the NPV accuracy improved (relative to the VBod NPV) as more samples were taken. Local estimates over-estimated the VBod NPV estimate by a magnitude of 15 to 13 times (75m to 25m scenarios, respectively).

Note that the number of samples were significantly large (1,136 samples for the 75m scenario, 2,556 samples for the 50m scenario and 10,224 samples for the 25m scenario). The more complex a deposit (in terms of geological structures and mineralization dispersion), the more sample drill holes will be required to reduce uncertainty and produce more accurate estimates of the statistical means and variances of relevant variables. Greater NPV differences between sampling scenarios would be expected if fewer samples were taken.

While kriging produced the best unbiased estimates for key variables, it tended to provide 'smoothed' resource estimates based on limited data. Generally, kriging underestimates grade and overestimates tonnage, which produces the smoothing or smearing effect. NPV estimates would be over-estimated relative to the actual deposit. Contrary to kriging, spatial simulations provide a better indication of the range of variabilities to be expected. The scope of this study did not allow a range of simulated realizations for comparison purposes. Thus, only a single simulated realization was selected as an example of the expected differences in mean values.

Local NPV based on conditional simulated results for the three scenarios (75m, 50m and 25m) were CAD -26.1 million, CAD +3.6 million and CAD +18.1 million, respectively. These simulated outcomes were significantly lower than the kriged estimates and closer to the actual VBod NPV. This gave the impression that the conditional simulations provided more accurate NPV than the kriged results but these simulations represented only one extraction from a range of simulations. This could represent the tenth or ninetieth percentiles (P10 or P90) of the simulated distribution outputs. Further work is recommended to generate the E-type estimate from a complete range of conditional simulations to compare with the kriged result.

The use of a flat FX rate was compared with a stochastic forward model that considers FX rate volatility. A fixed FX rate of 1.21 was used (February 2006 USD:CAD rates) to derive a VBod NPV of CAD 2.1 million. Table 5 shows the probable range in NPV for the VBod and three kriged scenarios when each of the 100 FX models were run through the financial model. The medians (i.e. fiftieth percentile or P50) for scenarios one, two and three are CAD 24.5 million, CAD 22.7 million and CAD 19.6 million, respectively.

Using variable FX rates, the P50 of the VBod NPV reduces from CAD 2.1 million to negative CAD 6.7 million (4 times less). This implies that the project is materially susceptible to FX rate volatility. However, as shown in Figure 5, there is considerable upside opportunity when the fiftieth to ninetieth percentiles are considered. Projects that are particularly revenue or cost sensitive may benefit by conducting forward modelling of the FX rate as it allows management to gain an improved understanding of the range of probable NPV. The costs of hedging against downside risks of FX rate fluctuations should be weighed against the negative impact that it may have on project value.

The estimation of resources strives to create a view of the quantity of *in situ* material that can reasonably be mined. It is this ‘reasonable expectation’ of ‘mineability’ that implies it is impossible to estimate resources totally independently of all external factors. These factors include the economic and technological limits that have to be imposed, and the scale and rate of mining.

Lastly, on the basis that one simulation is taken as a reality rather than as a single realisation of a particular orebody, it can be concluded that the selection of the appropriate time measurement scale in which to evaluate a number of diverse variables in a mineral resource project is critical in attaining realistic NPV estimates. If the mine went ahead on the basis of the ‘top-down’ global evaluation method, material financial losses would be incurred. If the ‘bottom-up’ local evaluation method was used, management may have been more concerned about the overall economic viability of the project and may have elected to reject this project. Analysis of this model demonstrated that components of the evaluation model cannot be optimized individually; the synchronization of resource, mining and treatment, and financial components is required in order to achieve an optimal balance of the system.

4.3 CASE STUDY 2: ASSESSMENT OF RESOURCE VARIABILITY ON PROCESSING CONSTRAINTS FOR AN OPEN-PIT OPERATION

4.3.1 Technical Overview

In this study the impact of spatial resource variability on an existing business model was assessed using conditional simulations for grade, density, revenue per carat and yield variables on an open-pit diamond operation. It was deemed prudent to use spatial conditional simulations rather than try to reflect risks using Monte Carlo simulations (MCS) because the latter cannot easily incorporate the spatial covariance relationships of resource and reserve data for mineral projects.

Usually MCS (often using @Risk or Crystal Ball risk modelling software) is used to generate risk profiles of production outputs and financial parameters to produce a probability distribution of the NPV. While MCS may be useful to model variability around non-spatial variables, it is inadequate in the case of spatial resource variables for mineral projects because it does not consider the spatial distribution of variables nor the spatial covariances between variables. An alternative approach may use summarized statistics from spatial modelling of the resource as input parameters (e.g. the mean and variance) into probability distributions for MCS modelling. This method is not recommended because it can lead to scenarios where independent, random draws have been taken from the MCS but do not reproduce the covariance relationships between geological units in adjacent mine blocks.

The net result is that technical risks may be seriously over- or under-stated and lead to scenarios where the NPV risk probability profiles are either too broad or too narrow, misleading decision-makers (Nicholas et al., 2007).

A more pragmatic and statistically acceptable approach to evaluate the impact of technical risks in mining projects is to use spatial simulations to reflect the resource uncertainty and run these simulated outputs through various production (reserves) and financial models, as discussed by Dimitrakopoulos et al. (2002); and Dowd and Dare-Bryan, (2004). The differences between kriged estimates and conditional simulations have been well documented (Journel and Kyriakidis, 2004).

4.3.2 Methodology

A total of 25 spatial conditional simulations were produced for each variable using the geostatistical Turning Bands method by Isatis geostatistical software and incorporated into a block model with dimensions of 25m by 25m by12m (independent analysis verified that results stabilised after 25 simulations). It should be noted that simulations of the individual variables were generated independently of each other rather than co-simulated as there is negligible correlation among them. Each estimation unit of the block model comprised four resource variables (grade, density, revenue per carat and yield) of which there were 25 realizations for each variable. The estimation units and the selected mining units (SMUs) were the same size. The mine plan was imported into Datamine software and merged with the resource block model to produce a depletion volume for each estimation unit. This volume was assigned to a specific year according to the depletion sequence.

Geometallurgical considerations such as the impacts of dense media separation yield on throughput and the impacts of density on both hardness and liberation were incorporated into the model. The process plant was designed to accommodate surges in yield by including several stockpiles and a purge system. A 17% yield threshold (defined as the maximum percentage of heavy mineral concentrate passing through the plant) was incorporated into the risk model to quantify the impact of the high yield blocks.

The model assumed that for every 1% yield above 17% in a block, the process throughput will reduce by 0.5%. A plant recovery factor was calculated based on a quadratic relationship between the density of the block and the liberation that is achieved by the plant. Throughput was calculated by multiplying the depleted tonnes by the throughput factor, and then the grade of the reduced tonnage was used to calculate the carats fed to the process. These carats were modified by the plant recovery factor.

The depletion model was overlain onto the spatial realizations to generate an ore stream, translating the spatial data into a time-based framework. Outputs from the mining and mineral processing modelling were incorporated into the financial model. It is important to note that the mine and treatment plans were based on the kriged estimates. The study aimed to represent the impact of risks associated with the uncertainty of resource estimates given these ‘fixed’ mining and treatment processes and to demonstrate the uncertainty of the cash flows using conditional simulations.

Production outputs for each of the 25 simulations were imported into the financial model, which consisted of a sequence of Microsoft Excel spreadsheets. The resultant cash flow model for each of the 25 simulation outputs plus the cash flow model (based on the kriged estimates) were imported into a Risk Evaluation Model (using Visual Basic Applications code) and selected data analyses were carried out. The financial model was not altered in any way, other than importing the production outputs from each of the simulations and thereafter, exporting the estimated cash flows into the risk evaluation model.

4.3.3 Analysis of Results

Figure 20 plots the cash flows, discounted cash flows and cumulative discounted cash flows over the life of mine based on kriged estimates. A NPV of USD103 million was derived with an IRR equal to 11.35% at a discount rate of 10%. The kriged results show that the first 5 – 6 years (2008/9 to 2014/5) of the model were the main contributors of value to the NPV based on a discount rate of 10%. From 2016 onwards, less than 50% of the cash flow value contributes to the NPV (alternatively it could be stated that the cash flows beyond 2016 are discounted by more than 50%) implying that considerable time, money and effort would be needed to be expended during this time period to make an improvement to the NPV. As the discount rate was increased, the time window decreased placing more focus on the cash flows derived from the first few years. The ramp-up in production during 2010 - 2013 is pivotal in achieving tonnage throughput and ensuring positively contributing cash flows.

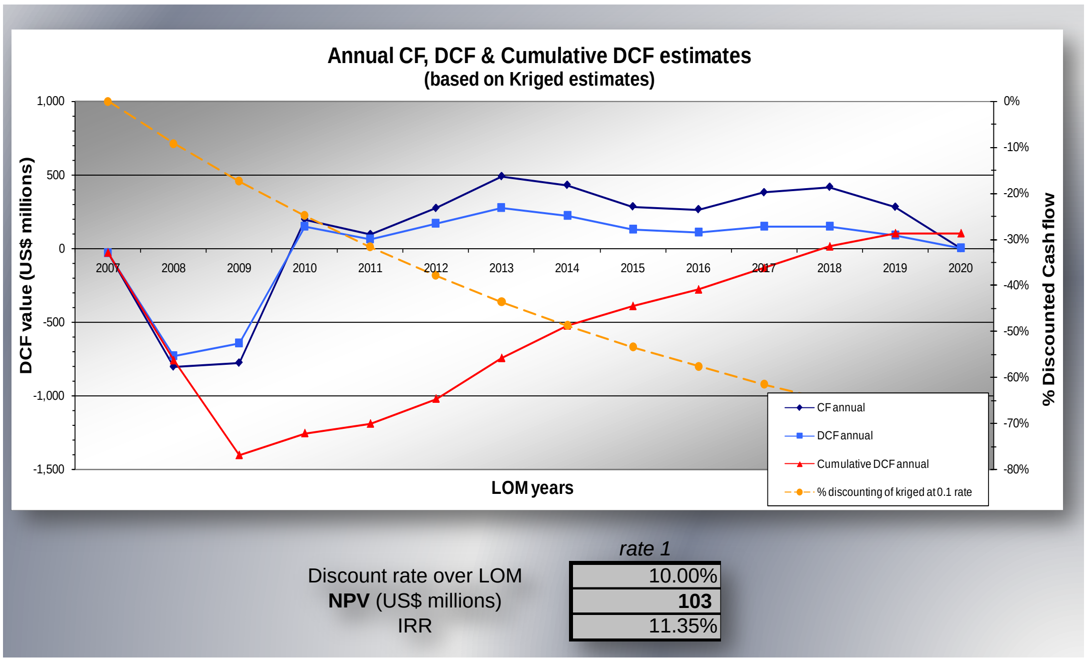
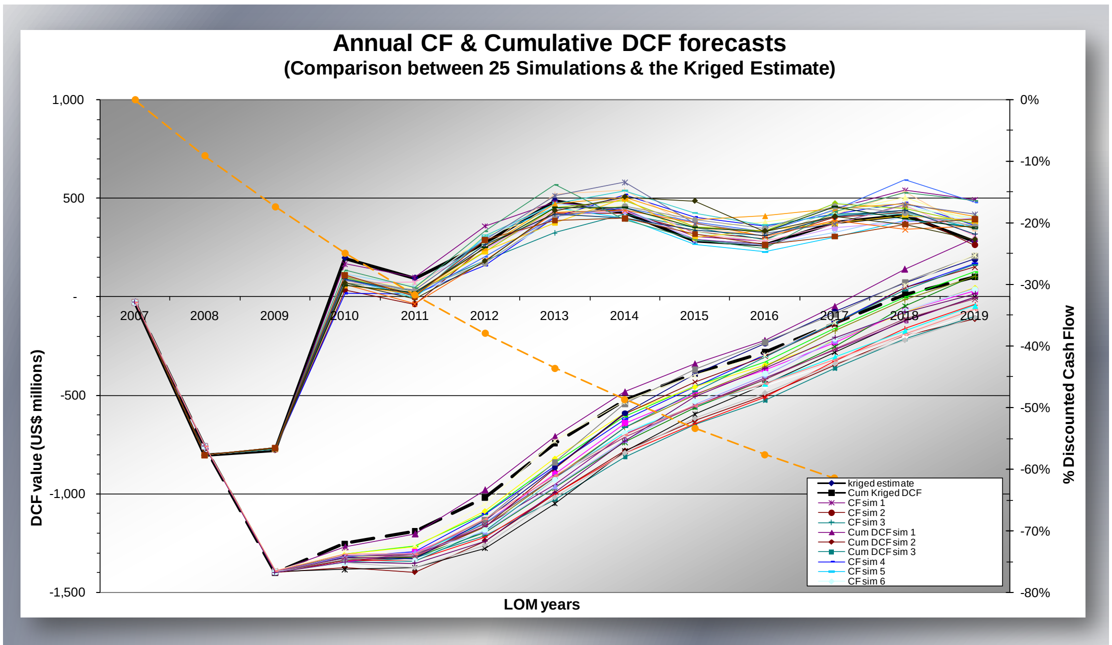


Figure 20 illustrates the cash flows (CF), discounted cash flows (DCF) and cumulative discounted cash flows (cum DCF) for the business model based on kriged estimates.

Figure 21 compares the kriged results (of Figure 20) with 25 conditional simulations plotted as cash flows, discounted cash flows and cumulative discounted cash flows over the project's life of mine. The solid black line represents the cash flow generated from the kriged estimates, while the dashed black line shows the corresponding cumulative discounted cash flow. The other lines represent 1 – 25 of the conditional simulated cumulative discounted cash flows. The P10 shows a 10% chance of achieving a NPV of USD -96 million or less; the P50 shows a NPV of USD 39 million while the P90 indicates a 10% chance of achieving more than USD 201 million. Note that the P50 simulated result of USD 39 million is materially less than the kriged business case of USD 102 million (62% less).



Percentiles	P10	P50	P90
NPV (US$ millions)	**-96**	**39**	**201**

Figure 21 compares the cash flows (CF), discounted cash flows (DCF) and cumulative discounted cash flows (cum DCF) between the kriged estimates and conditional simulations.

While it is expected that the block volume errors may have contributed to the reason why the kriged model lies notably higher than the conditionally simulated outputs, a more plausible explanation for the lower P50 NPV is that each of the conditional simulations was run through an integrated evaluation model (IEM) at the operational scale of planned depletion. Each block (25m x 25m x 12m) was run through the transfer function, i.e. the mining and treatment plan with fixed constraints.

A study by Nicholas et al. (2006) revealed that the financial impact of running kriged estimates through an IEM can be materially different to that generated by the traditional assumption of using annual, average mining and treatment constraints. An IEM considers the constraints that would be imposed on a block at an hourly or daily temporal scale rather than assuming average constraints, calculated over 12 months.

The business model based on the kriged estimates was not run through an IEM (it was beyond the scope of the study) and instead assumed annual average constraints based on the transfer function parameters. Where sampling data were widely spaced, the 'smoothing effect' of kriging would have the biggest impact. As a result, even if an IEM was used on the kriged estimates, a bias may exist in the results by understating the real variance. For this reason the use of conditional simulations using an IEM is recommended to provide a better reflection of the variability in cash flows.

Figure 22 illustrates how conditional simulations can be used to assist production planning (short and long-term). Histograms of the cash flow for each year can be produced to represent realizations that consider the impact of resource uncertainties over time. The aim of producing these cash flow probability plots is to show the range in expected values and compare them to forecasted values (based on the kriged model). The coefficient of variation (CV) can be calculated to highlight those specific years which have the greatest variability.

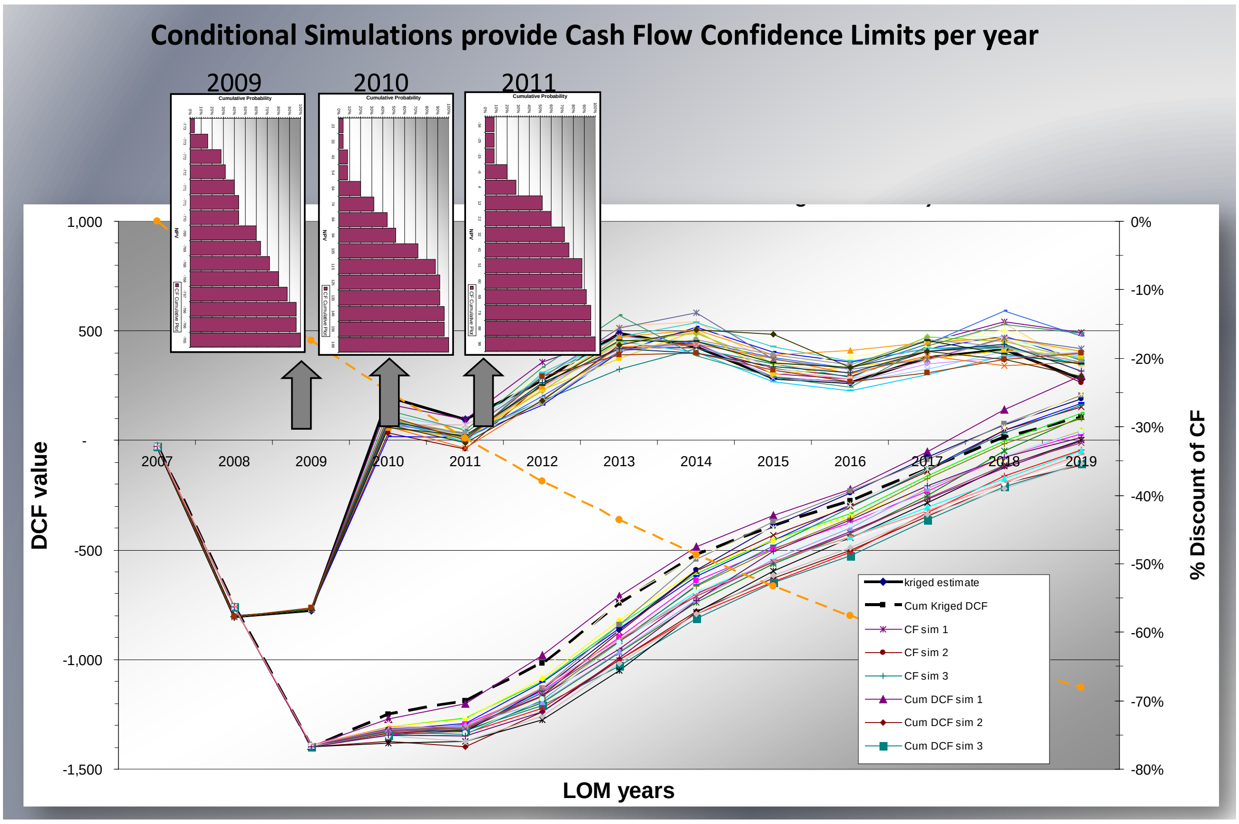
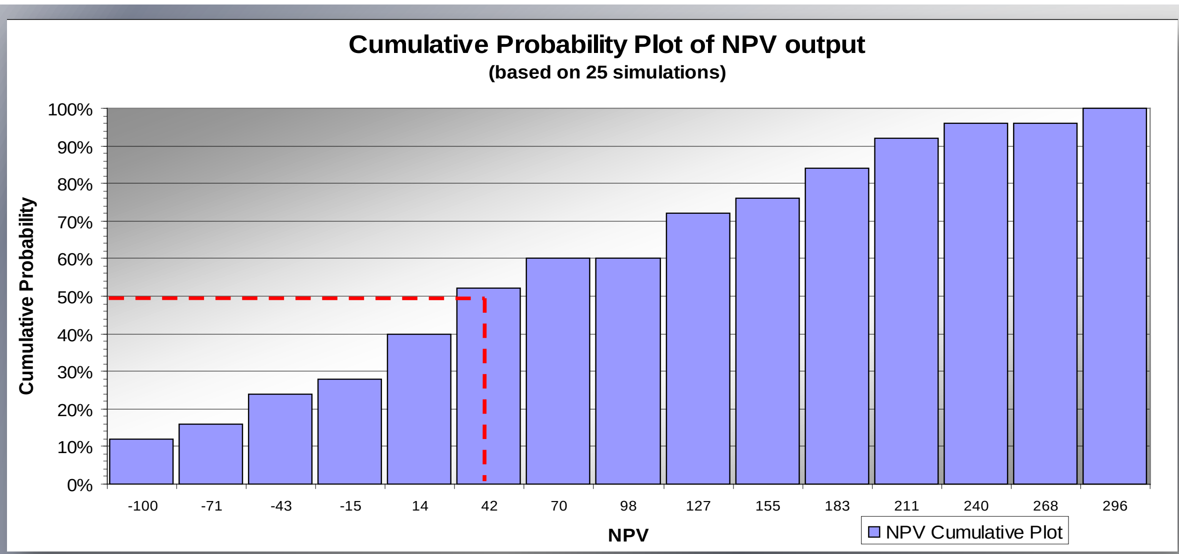


Figure 22 shows how histograms of the forecasted cash flows per year can be generated based on results from the conditional simulations.

The model consistently found that cash flows between years 2010 and 2012 had the highest CV. In the case where more than one resource variable is considered in the simulations, variables can be selectively excluded one at a time to identify that particular variable that has the greatest impact on the CV of the cash flows. Capital could be made available to mitigate the risks by providing adequate flexibility (in the mine plan or treatment plant) identified during these years – this should improve the process of capital budgeting. This approach should be compared with the prospect of additional drilling, which may be relatively less expensive and yield other advantages (confirmation of grades, densities etc.) before making mine or plant modifications.

Figure 23 shows the cumulative probability distribution for the NPV of this project, which provides a better representation of the risk profile for this project than simply quoting a single NPV figure or stating fixed percentiles (e.g. P10, P50 and P90).



P10	P50	P90
-96	39	201

Figure 23 shows the cumulative probability plot for the NPV of this project (in USD millions) based on results from 25 conditional simulations.

4.3.4 Conclusions

This case study demonstrates that conditional simulations can be used alongside kriged estimates to quantify the financial impact of resource uncertainties without adjusting the discount rate to compensate for technical risks. The financial impact of grade, density, yield and revenue per carat uncertainties were quantified. The findings of this study strongly suggest that an IEM should be an essential part of the business planning process. This is necessary to ensure that spatial resource uncertainties can correctly be translated into the time domain via depletion and treatment models, and compared to financial forecasts based on kriged estimates.

The use of an IEM is preferred to the approach of applying mining and treatment modifying factors (derived from annual averages) to production figures, which are likely to provide 'smoothed' perceptions of the actual variability that will be encountered on a daily basis. This could result in an over- or under-estimated financial value of the deposit as it fails to capture the short-scale effects of the mining and treatment constraints that are imposed on the production estimates on a block by block basis. It can also fail to capture upside opportunities where greater resource variability could result in increased production recoveries provided the mine and treatment processes are appropriately designed to provide this flexibility.

Depletion of simulated blocks in space and in time allows the financial impact of variability during each year to be accurately quantified. While volume, grade and density estimates show little variation in the simulations over the life of mine on an annual scale, it is the variability of these simulations within each year and the selection and sequencing of blocks over time that dictates the contribution to the cash flow model.

In this case study, the evaluation model shows that the highest variability in cash flows occurs early on in the life of mine (2010 to 2012) which has the biggest impact on the time value of money. This highlights the need for efficient operational execution to ensure that the 'right tonnes from the right areas were mined and treated during the right time'. The use of an IEM approach linked to financial modelling provides quantitative information about the expected variability of a deposit, which creates a basis for improved mine designing and operational planning.

4.4 CASE STUDY 3: FINANCIAL IMPACT OF RESOURCE VARIABILITY ON AN OPEN-PIT GOLD OPERATION

4.4.1 Technical Overview

The main objective of this case study was to assess the impact of resource grade variability on the reserve constraints of an open-pit gold operation to quantify the impact on metal production in financial terms. Part of achieving this objective entailed modelling and analysing the impact of gold grade variability on cut-off grade selection and stockpile management to maximise NPV (i.e. ensuring that the highest grade ore mined that met blending requirements was sent to the plant for processing).

In order to achieve this objective, an evaluation framework (an IEM) had to be developed that would appropriately link conditional simulations with the mining and processing schedules and the financial model. This was achieved by running 25 conditional simulations through an IEM approach, considering both mining and processing constraints, to generate multiple production outputs. These were run through a 'base case' financial model to generate a NPV output for each of the 25 conditionally simulated resource realisations.

All mining, processing and financial parameters used to develop an IEM for evaluation purposes were identical to those used to generate the kriged estimate. The company's financial model was used as a financial template to calculate cash flows per year for each realisation. A secondary objective of this case study was to demonstrate that an IEM approach could be readily adapted from diamond operations to other commodities, in this case an open-pit gold operation.

4.4.2 Modelling Parameters

The following 'physical' variables were included in the evaluation model by the author:

- ☑ Stochastic gold grades in the form of 25 conditional simulations;
- ☑ Density (S.G.) was assigned values per rock type;
- ☑ Recovery factors (95% for primary ore and 90% for oxide ore) were assigned to each SMU; and

☑ An average (global) cut-off grade policy was applied over the LOM (0.65 g/t for oxide material and 0.75 g/t for primary material), derived from the relationship between gold price, recovery factors and processing costs – see Equation 19.

$$Cut\_Off\_Grade = \frac{PC_t}{\left(\left(Au_{\$} \times \left(1 - RC_t\right) - C_r\right) \times \left(Met\,\mathrm{Re}c\right)\right) \times 31.1035}$$

= Cut-off grade in g/tonne
= Total Process cost in USD/tonne
= Gold Price in USD/ounce
= Royalty as % of revenue
= Refining cost total in USD/ounce

Equation 19. Cut-off grade calculation for gold.

The main geological units for the project were identified as per Table 7 below with densities assigned to each unit.

	Geological Unit Code	Density (S.G.)	Recovery Factor	Plant Rock Code
Oxidised	1	1.80	90.0%	1
Partially Oxidised	2	2.20	90.0%	1
Primary	3	2.80	95.0%	2
Other Clays	4	1.90	90.0%	1

Table 7. Assigned densities and recovery factors per met-code.

It is important to note that individual densities were assigned to each rock type, and consequently to each SMU as it was considered for reserve evaluation on a block by block basis – see Equation 20.

$$tonnes = volume * density$$

$$metal(g) = tonnes(t) * grade(g/t)$$

$$metal(oz) = \frac{metal(g)}{31.1035}$$

Equation 20. Relevance of incorporating density at a SMU scale.

This was relevant to how the total metal (initially in grams then converted to ounces) was calculated for each SMU as a function of its density multiplied by volume to generate tonnes. This was used in grade cut-off calculations to determine the total metal contributing to the production call each year.

Metallurgical recovery test work was not sufficiently detailed to assign a recovery or indicative value to each geological unit code at the time of conducting this study. Thus, geological unit codes 1, 2 and 4 were assigned the same plant rock code (i.e. 90% plant recovery) while geological unit 3 was assigned a value of 95% as shown in Table 7.

An E-type estimate was derived by averaging the 25 gold estimates (calculated from each of the conditional simulations) at each of the circa 600,000 SMU nodes. This E-type estimate represented a materially smoothed estimate in relation to the range of variation in cash flows derived from the 25 conditional simulations, but served its purpose as a comparison to the conditional simulations.

4.4.3 Programming Logic

Similar to the previous two case studies discussed in this chapter, it was necessary to develop a bespoke IEM approach for this open-pit gold operation to appropriately capture the unique resource and reserve characteristics of the project. An evaluation framework was developed that allowed each SMU (at a block scale of 10m x 10m x 4m) to be assigned grade and density values and year mined. A recovery factor (90% for oxide material and 95% for primary material) was applied per rock type to each SMU in the block model.

Each record (SMU) was then assessed in terms of whether it met the various average grade cut-off constraints and then either contributed to the metal processed at the plant or was sent to the stockpile or to waste. Ore tonnes from each SMU was accrued to form the annual mined and processed totals for that year, which provided input into the financial model to calculate forecasted cash flows and the project NPV.

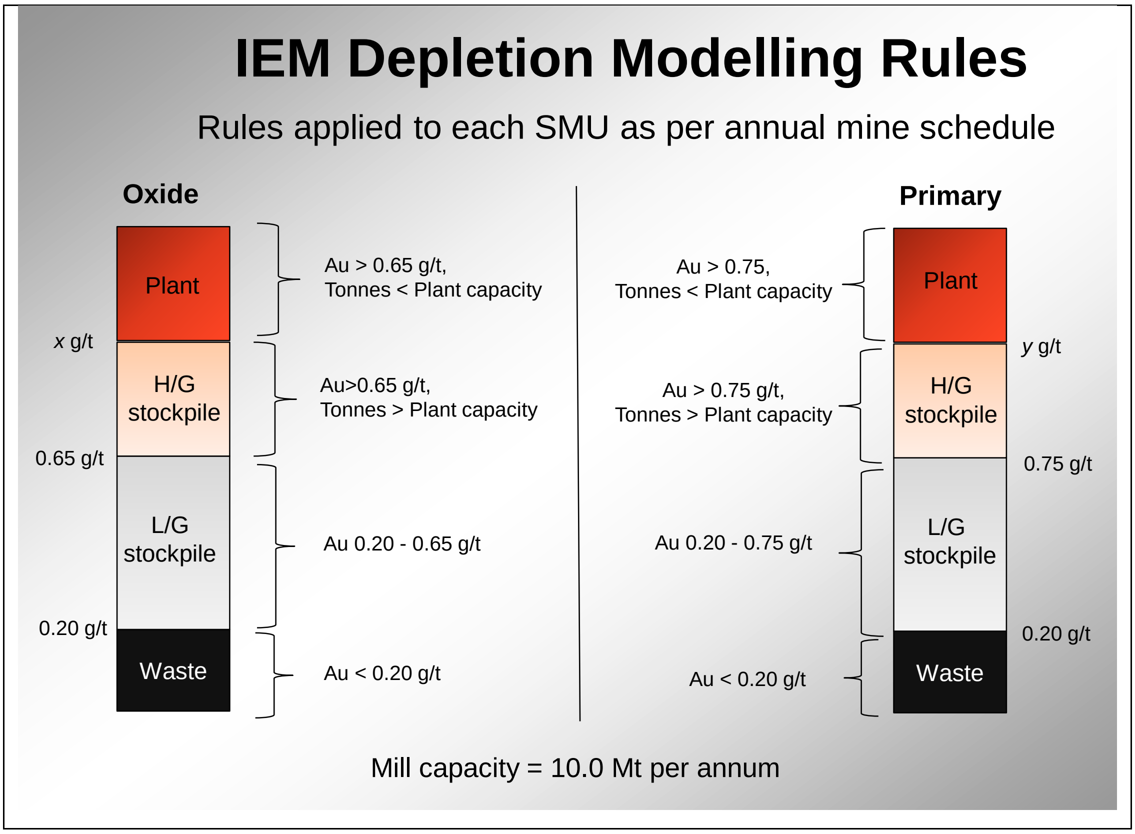


Figure 24. Depletion grade cut-off rules for oxide versus primary material.

As per Figure 24, each SMU in the resource block model was assessed in terms of certain mining criteria and consequently, either sent directly to the plant or to one of the stockpiles depending on whether the cut-off grade was met and whether the plant capacity of 10 million tonnes (Mt) had been exceeded. Each SMU was accrued (depending on its rock type and destination) to obtain production totals per year in the LOM schedule. These production outputs were fed into the financial model to calculate forecasted cash flows per year and derive the NPV. This process was repeated for each of the 25 conditional simulations until a range of cash flows and NPV were generated and confidence limits derived.

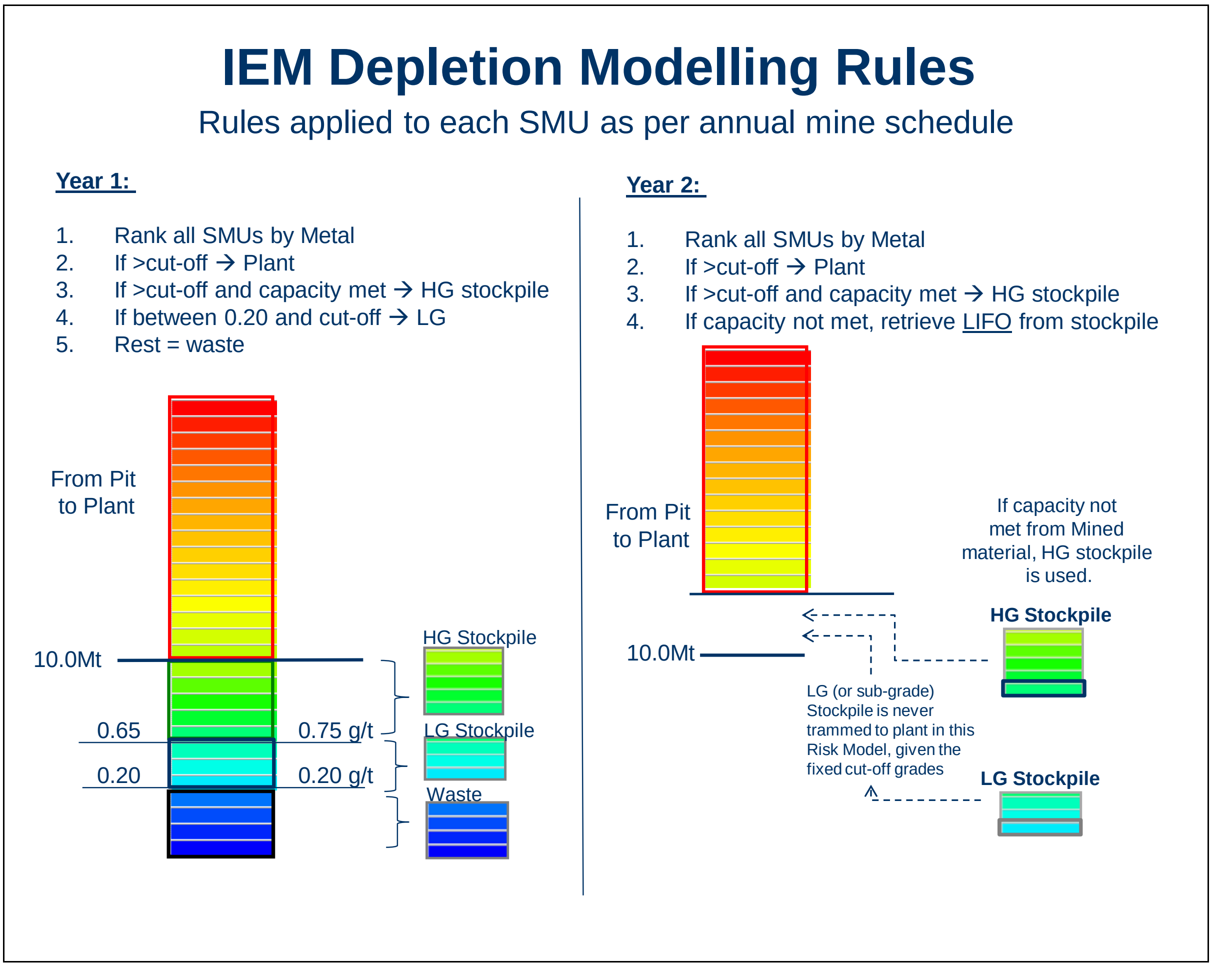


Figure 25. Depletion grade cut-off rules applied to each SMU on a block-by-block basis (with assistance acknowledged from Quantitative Group consultants). LIFO refers to Last-In-First-Out; HG refers to High Grade; and LG refers to Low Grade.

Figure 25, illustrates that the IEM depletion rules require that the "best" 10.0 million tonnes available in each year are fed to the plant. "Best" is deemed to be the blocks containing the highest total metal, based on the tonnes accrued from each SMU on a block-by-block basis depending on whether each SMU meets the specified grade cut-off criteria or not. The IEM can distinguish between SMUs selected on a purely grade-cut-off basis as opposed to metal content, i.e. the calculated metal in each SMU considers the density per geological unit.

If plant capacity is exceeded, the remaining tonnes above the cut-off grade are sent to a high grade stockpile. Tonnes that have a grade between the cut-off grade and the waste cut-off grade are sent to a sub-grade, or low grade stockpile, and the tonnage below the waste threshold is discarded (programming logic depicted in Figure 26).

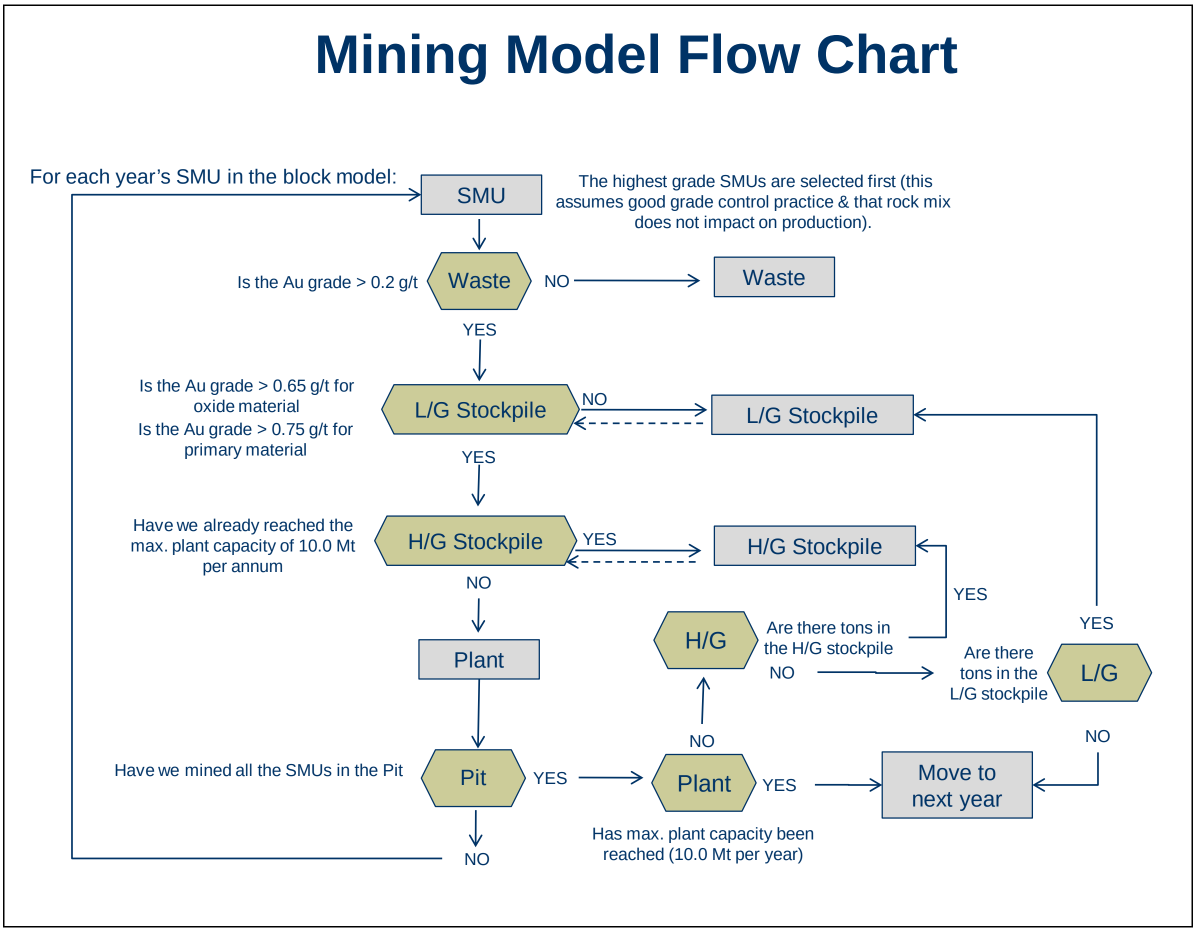


Figure 26. Programming logic for evaluating each SMU according to the reserve constraints.

In years where tonnes mined from the pit do not meet the available plant, the shortfall is made up by recovering material first from the high grade stockpile and then from the low grade stockpile. Stockpiles are managed in a last in first out (LIFO) manner, i.e. if there is a shortfall of tonnes (above cut-off grade) from the mine, the last block sent to the stockpile, will be sent to the plant. This means that the actual grade and metal content in each block can be accounted for and the gold balance in the system will be correct.

4.4.4 Analysis of Results

Figure 27 provides an illustration of the variability around metal (gold grams) produced after plant recovery over the LOM. The chart was generated by running each of the 25 conditional simulations through the IEM for the same selected reserve parameters (i.e. mining and processing constraints). Thus, the variation in output shown is purely as a result of the spatial resource simulation inputs into the evaluation model.

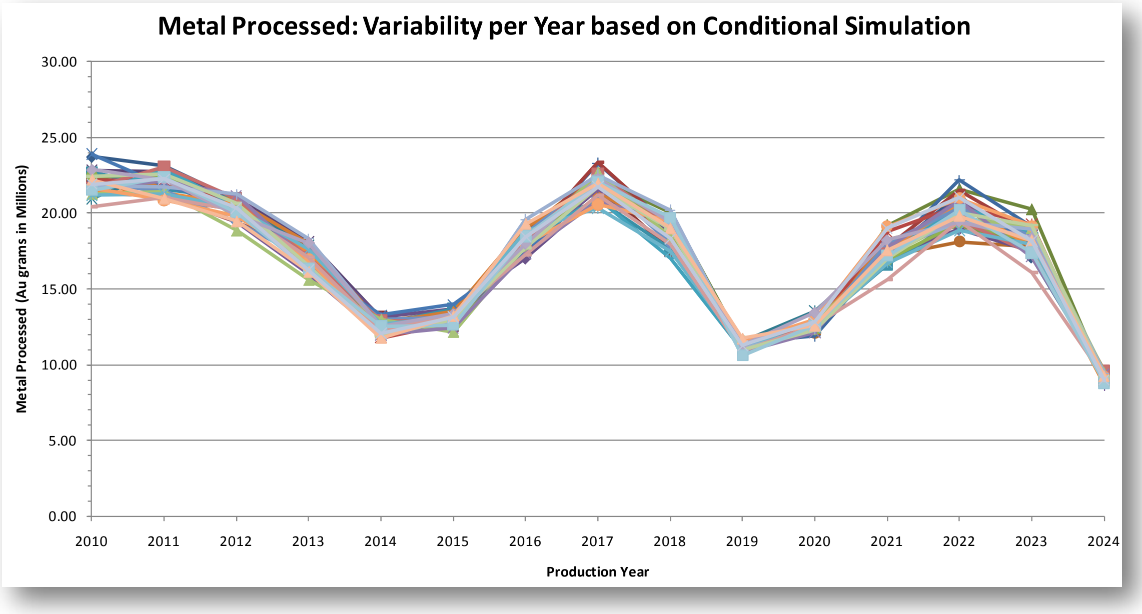


Figure 27. Graphical depiction of the metal (gold) produced over the LOM based on the 25 conditional simulations of the resource.

The 'tightness' in the spread of the simulated metal output indicates that there is not a huge amount of variability between realisations, although in latter years, 2021 onwards, (due mainly to limited resource drill holes in the resource during this period) the spread in metal produced does increase somewhat.

A histogram and cumulative probability plot (based on the 25 conditional simulations) of total metal produced over the entire LOM (2010 to 2024) is shown in Figure 28. The probability of achieving an estimated metal produced in any particular year can be derived from the cumulative probability plot. Similarly, as depicted in Figure 29, cumulative probability plots for metal produced in each year can be compared with each other to assess the relative variability in relation to production targets.

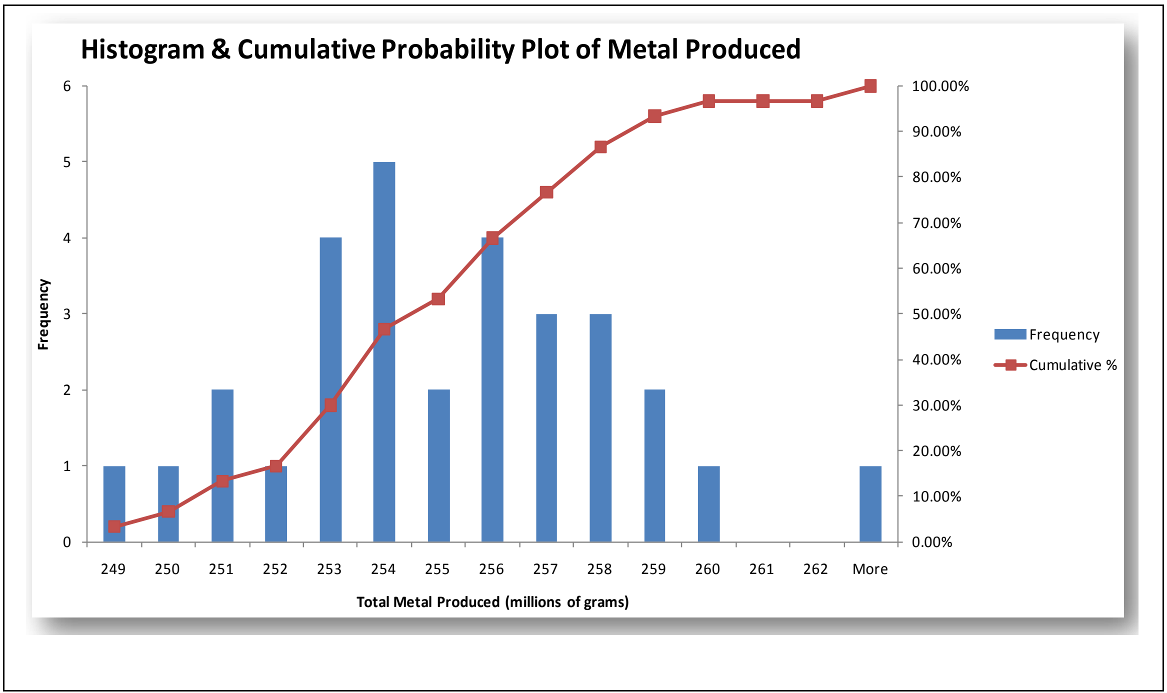


Figure 28. Histogram and cumulative probability plot of the metal produced over the entire LOM.

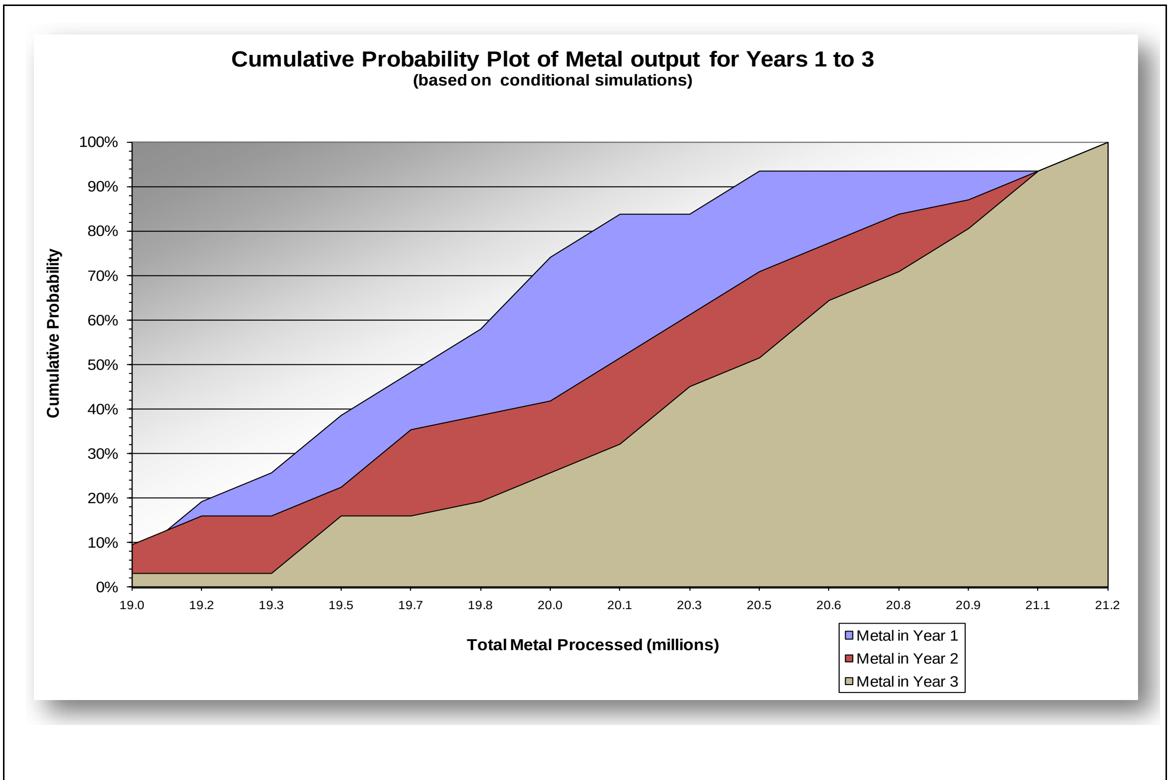


Figure 29. Cumulative probability comparison plots of metal produced for years 1 to 3.

These cumulative probability plots for metal produced directly influenced the cash flow model used to derive financial metrics such as NPV and IRR.

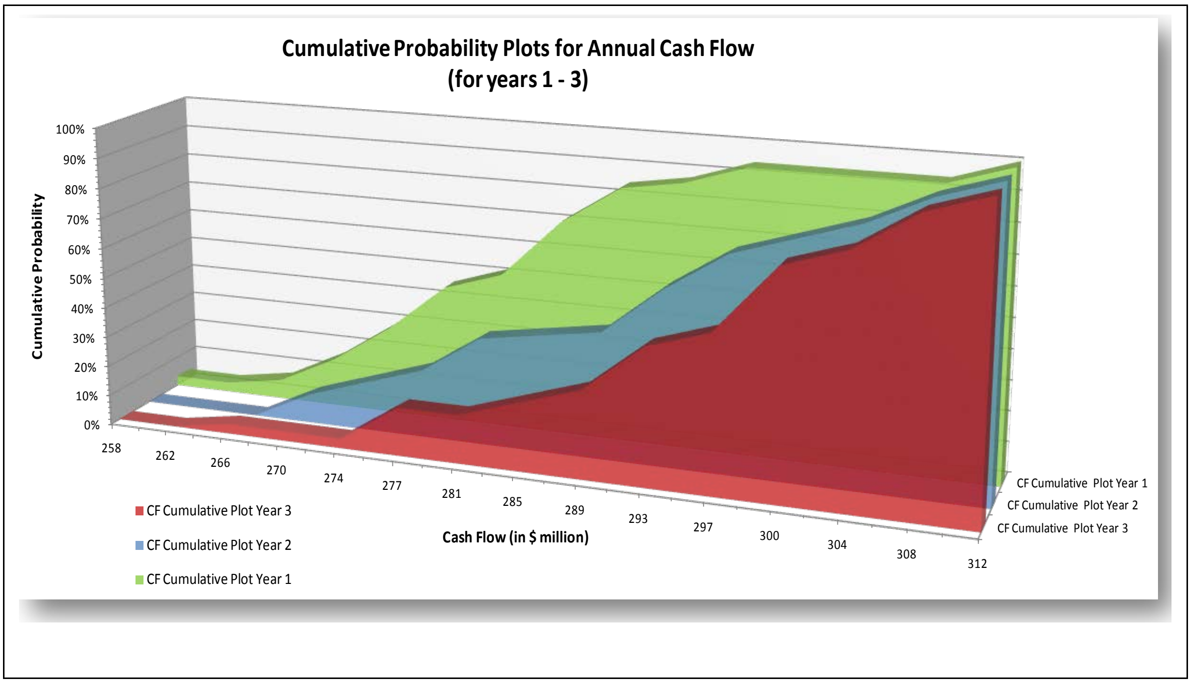


Figure 30. Cumulative probability plots of cash flow comparing years 1 to 3, which were derived from processing of each of the 25 conditional simulations through the IEM.

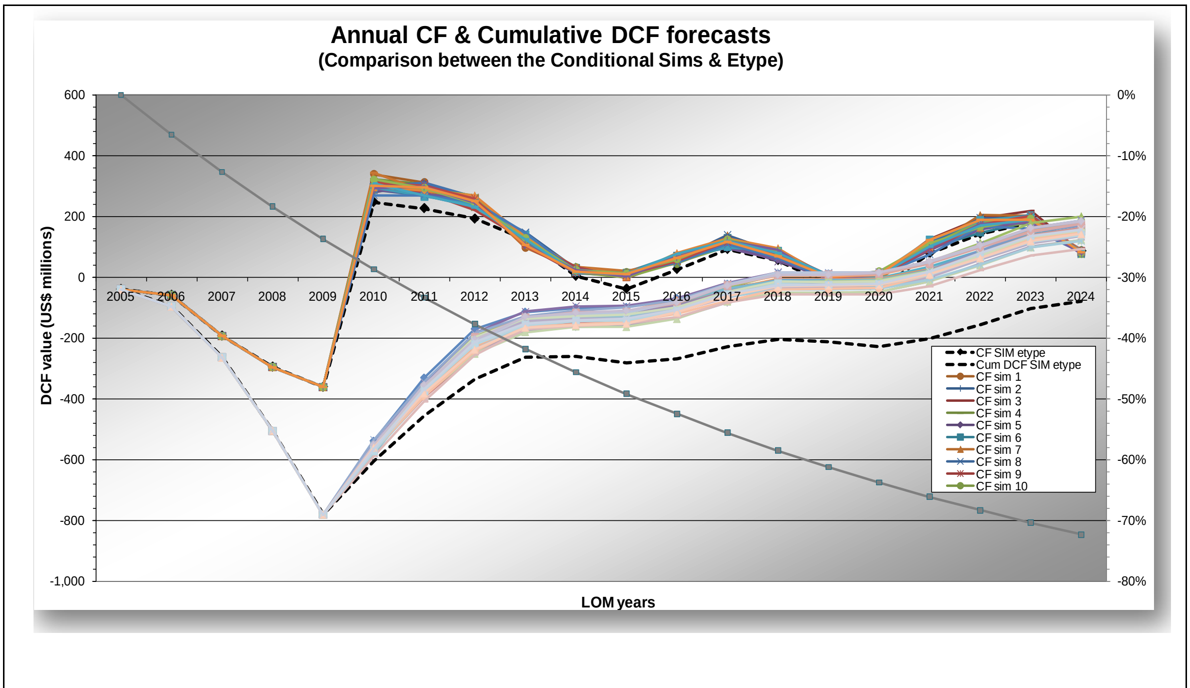


Figure 31. Annual cash flows and cumulative DCF plots. The proportional effect of the discount rate against the cash flows is plotted (grey line with square turquoise fill) along the secondary y-axis.

Figure 31 plots both the cash flows and cumulative discounted cash flows of each of the conditional simulations relative to the E-type (shown as a black dashed line). The proportion of cash flow discounting (based on a 7% discount rate) provides a graphical illustration of the discounting effect on cash flows due to the time value of money. The cash flow distributions for the conditional simulations and E-type models show three distinct periods where the cash inflow is high, of which the most important from a time value of money is the 'risk in time' period 2010 – 2014. The irregular cash flow distributions are mainly due to the interaction of *in situ* gold grade variability in each of the conditionally simulated realisations with the annual production derived from the mine plan schedule, the capped 10 Mt plant throughput per year and the effect of stockpiling.

It is immediately apparent that the E-type estimate lies materially below the conditionally simulated outputs. Firstly, it should be reiterated that the E-type estimate was directly derived from each of the 25 conditional resource simulations. Secondly, it needs to be highlighted that the outputs shown in this figure have each been subjected to the specified reserve constraints, then run through the financial model, resulting in these differences. The cumulative DCF values are higher than the E-type estimate because the E-type represents a 'smoother' reflection of the *in situ* grade variability compared to the conditional simulations.

As a function of selectivity based on highest to lowest grade material fed from the ROM stockpile to the plant, the E-type estimate contains materially lower grades than the conditional simulations, resulting in lower cash flows. This implies that it is important that SMU estimates designated for 'optimal' scheduling between the mine and processing plant, must have the correct variability, and can result in significant cash flow benefits as shown in Figure 31. NPV for the 25 conditional simulations range from USD59 million to USD165 million, with percentiles are shown in Table 8. A comparison is also provided between the E-type estimate and P10, P50 and P90 percentiles for the conditional simulations.

NPV Results:	Conditional Sims	
10%	50%	90%
P10	**P50**	**P90**
88	119	150
at a discount rate =	7.00%	

NPV Results:	Etype estimate		
Discount Rate	**7.00%**	**5.00%**	**10.00%**
NPV	**-114**	**-43**	**-187**

Table 8. 10th, 50th and 90th Percentiles for NPV results based on outputs from all conditional simulations compared to the E-type estimate (at various discount rates).

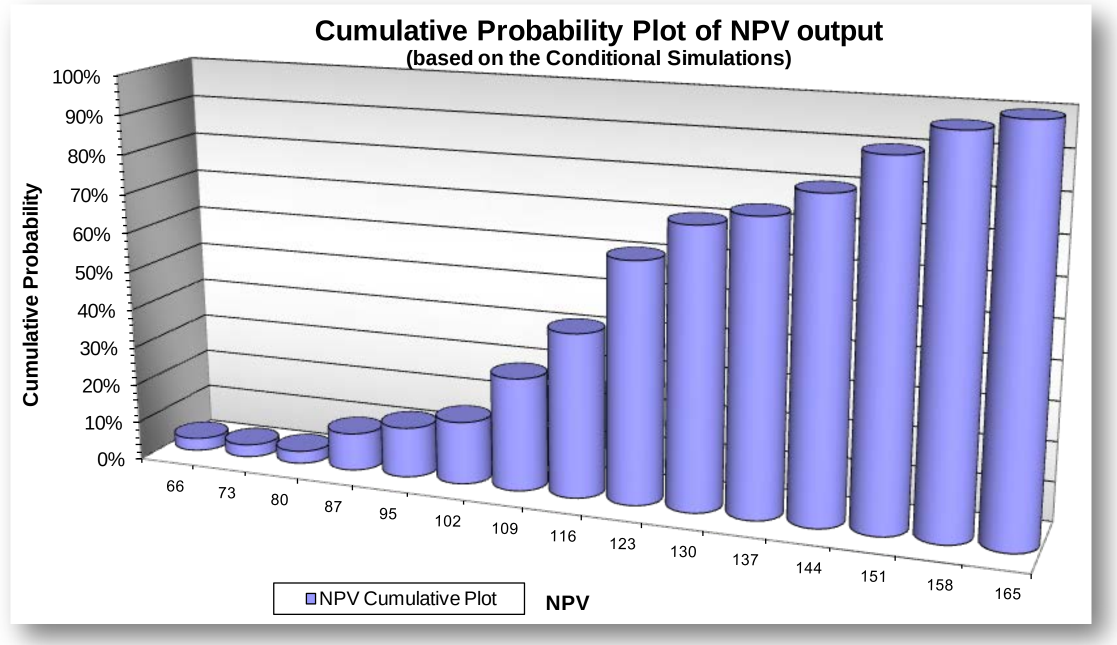


Figure 32. Cumulative probability plot of NPV for the project based on running all conditional simulations through the same reserve and financial models. The fiftieth percentile NPV is USD119 million (based on a 7% discount rate).

The cumulative probability chart for the NPV output based on running all the conditional simulations through the reserve constraints and financial model is shown in Figure 32. The P50 NPV is USD119 million, which is considerably more than the -USD114 million based on the E-type estimate. There are two main reasons for this difference in NPV which when combined, resulted in the material gap. Firstly, each of the conditional simulations has a significant variance in the *in situ* resource gold grades (coefficient of variations ranged from 112% to 118%), while the E-type estimate (coefficient of variation of 80%) represents a 'smoother' average of these conditionally simulated values at each SMU node. Secondly, the processing plant is capable of treating only 10 Mtpa while the mine can generate on average, approximately 12 Mtpa. Thus, only those blocks with the best grades (or metal content) were selected from the pit (and stockpiles, where necessary) to process through the plant.

The E-type mean grade and the average of all the conditional simulations are the same, however, when the 'best' 10 Mtpa is selected in each year, the conditional simulations have a greater variability range in the top end of the grade distribution than the E-type estimate, which results in a higher mean grade for that year and contributed to more metal tonnes. This results in greater cash flow contributions and higher NPV for each of the conditional simulations.

It was identified that the LIFO strategy, combined with ranking each SMU grade from highest to lowest imposed on selecting blocks from the stockpiles, theoretically implies that the grade distribution on the respective high and low grade stockpiles is perfectly known – this enables a matrix of blocks ranked from highest to lowest grade with only the best ranking SMUs contributing to the 10 Mtpa production target fed to the processing plant. Furthermore, it assumed that grade control in the mine was at such a representative spacing and high quality that the grades (and densities) of each SMU mined from the pit was perfectly known and sent to the stockpiles with perfect knowledge of its location.

While this strategy is likely the best possible scenario for optimal extraction and selection of blocks being sent to the plant, it is highly unlikely that it would be achieved in reality. Hence, it was deemed necessary to run two alternative stockpile selection scenarios. Test-1 involved modifying the original E-type estimate so that the same SMU grades were ranked in ascending order from lowest to highest grades (instead of highest to lowest). When the best 10 Mtpa for sending ore to the processing plant took place, the lowest grades were selected first, leaving many of the higher grade SMUs on the ROM stockpile. Test-2 involved modifying the original E-type estimate such that an average grade for all SMUs (per mining period) was calculated and blocks were sent to the processing plant at the average SMU grade above cut-off (rather than their inherent resource grade). Figure 33 depicts the cumulative discounted cash flows of these two scenarios in relation to the original E-type estimate (black dashed line).

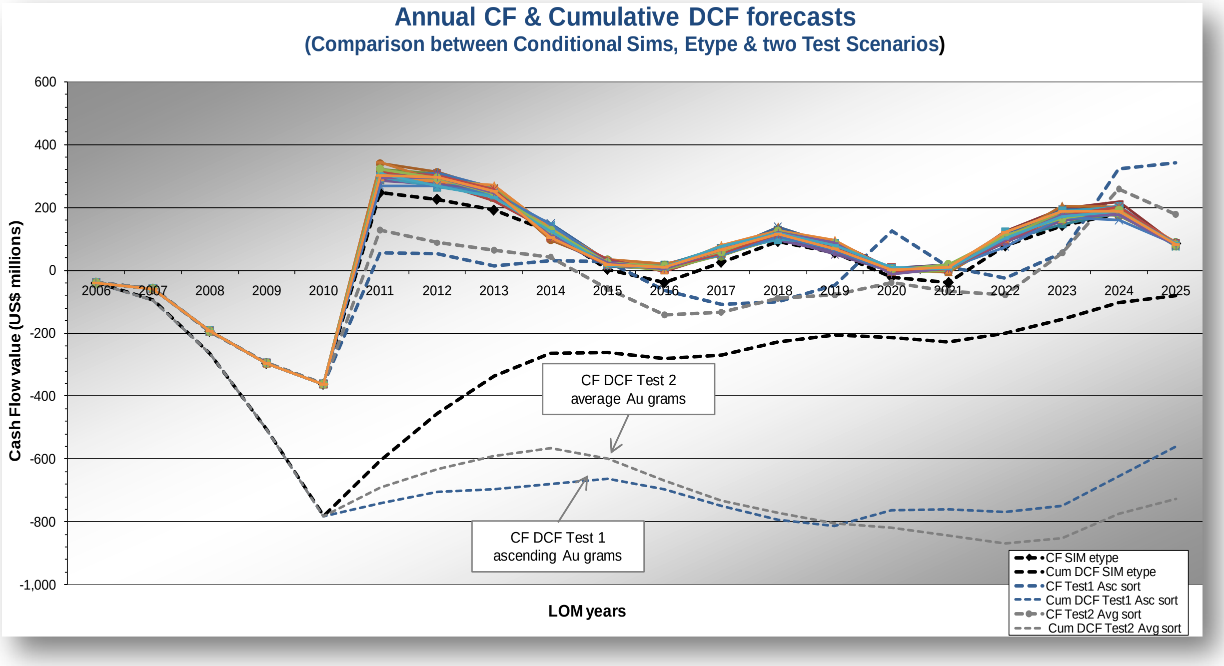


Figure 33. Annual cash flows (conditional simulations shown in multi-colours) and cumulative DCF plots of conventional E-type estimates sorted from highest-to-lowest grades (black dashed line), E-type Test 1 estimate (stockpile facility) sorted from lowest to highest grades (blue dashed line), and E-type Test 2 estimate (stockpile facility) with no sorting and grades set to the average grade per period (grey dashed line).

Figure 29 shows that both E-type tests (one and two) lie materially below the original E-type estimate. The original E-type estimate represents the theoretical best scenario where blocks are sorted in descending order from highest to lowest grades to select the best 10 Mtpa for processing at the plant, while the other two tests are sub-optimal scenarios. Test 1 may be perceived as the worst case scenario (from a time value of money perspective) where grade control, scheduling and blending practices in the pit and on stockpile management are minimal to non-existent and the lowest grade blocks are sent to the plant first. Test 1 ‘crosses over’ Test 2 at year 2019 due mainly to the fact that there is insufficient ore at that stage from the pit and ore needs to be sourced from the stockpiles. Ore from Test 1 has higher grades sitting on the stockpile than Test 2, which results in a higher cumulative DCF value.

These two E-type tests demonstrate that it may be worthwhile to implement an effective grade control, blending and stockpile management programme, especially where the bottleneck is the plant and the ‘best’ tonnes are selected from mining production. Test 1 showed a difference of USD481 million while Test 2 showed a difference of USD647 million that could contribute to funding grade control and stockpile management programs to improve the project's NPV.

4.4.5 Conclusions

This case study demonstrates that an IEM could be effectively used to create a simulation study to quantify the effects of resource variability for a gold operation on production and financial outputs. Additional ore tonnes from the mine creates greater mining flexibility to select those blocks first that had the highest ore tonnes for processing to meet the 'best' 10 Mt plant capacity in that year. This is crucial for management in deciding the value (e.g. tightness of sampling grid) and quality (e.g. blast holes versus reverse-circulation drill holes) of a grade control programme to better understand resource variability and how best to use this information to schedule mined blocks in relation to the optimal use of stockpiles, thereby ensuring that the best ore is processed first through the plant.

The study demonstrated that the financial benefit of grade control systems and stockpile management can be quantified in financial terms by using an IEM. Perfect knowledge of grades within each block and its subsequent location on the stockpile implies that an increase in NPV of USD481 million (compared to Test-1) and USD647 million (compared to Test-2) is possible. While perfect knowledge of the ore body is acknowledged as being impractical, this IEM framework does provide a method to quantify the cost/benefit scenarios implement grade control and stockpile management systems.

This IEM framework provides a way of studying the planned constraints on the size/capacity, quantity, and layout of stockpiles. It was unrealistically assumed in this risk model that stockpile capacities were unconstrained. It is likely that as the number of stockpiles increase, the total maintenance, trucking and handling costs would also increase, which should be reflected in the financial model. In the event that the number, and size, of stockpiles on surface needs to be evaluated for environmental purposes, the IEM provides an opportunity to evaluate different risk scenarios (which is distinct from conventional optimization studies) to provide more confidence and accuracy with mine planning and financial costing exercises.

Given the non-linear relationships between resource, mining, processing and financial constraints, this particular problem could not have been solved through any form of closed-form mathematical model – an IEM approach was necessary.

Chapter 5 : Risk Analysis

5.1 INTRODUCTION

The objective of this chapter is to demonstrate that sensitivity analysis and Monte Carlo simulations can provide an improved understanding of project risks but there are limitations of using these techniques that need to be understood more clearly. A virtual ore body (VBod), as discussed in chapters three and four, is used to evaluate the accuracy of a mineral project based on different risk analysis methodologies. Case studies discussed in chapter four are expanded to include sensitivity analysis and Monte Carlo simulation techniques with results compared to an integrated evaluation modelling (IEM) approach. Finally, financial variance reduction techniques are explored in more detail to ascertain whether they represent a viable alternative to an IEM approach.

5.1.1 Modelling of Uncertainty and Variance

Before modelling risk in an evaluation model for a mineral project, it is important to understand the inherent correlations among key resource variables, and the system linkages (or dependencies) among the various stages in the evaluation pipeline and the temporal scale at which to evaluate risks.

Variables such as nickel, iron and sulphur may be spatially correlated with each other and their covariance relationships should be appropriately considered in the selected risk analysis technique. Relationships between the drill hole spacing, selection of the estimation unit size (EUS), desired accuracy (globally and locally) in the selected estimation/modelling technique, selected mining unit (SMU) size and associated mining method, management of risk mitigation/variance reduction strategies related to mining and processing flexibility options, and capital and operating cost expenditures should all be considered as 'system linkages' in the evaluation model (except for the IEM approach, most conventional risk analysis techniques do not). Temporal scale and the required level of detail/depth in the evaluation model must be weighed up in terms of the accuracy of the output results versus available resolution of the data and time (model set-up and processing) requirements.

There is a multitude of risk analysis techniques, ranging from subjective matrix-style assessments to objective spatial stochastic modelling, with each technique producing outputs that should be considered in terms of the representivity, accuracy and comprehensiveness versus detailed characteristics of the proposed solution. The risk analyst/modeller must define the question clearly (i.e. the objective of the risk study) and evaluate the suite of risk modelling techniques available to ensure that the selected risk analysis technique is the most appropriate once all the advantages, disadvantages and assumptions have been considered.

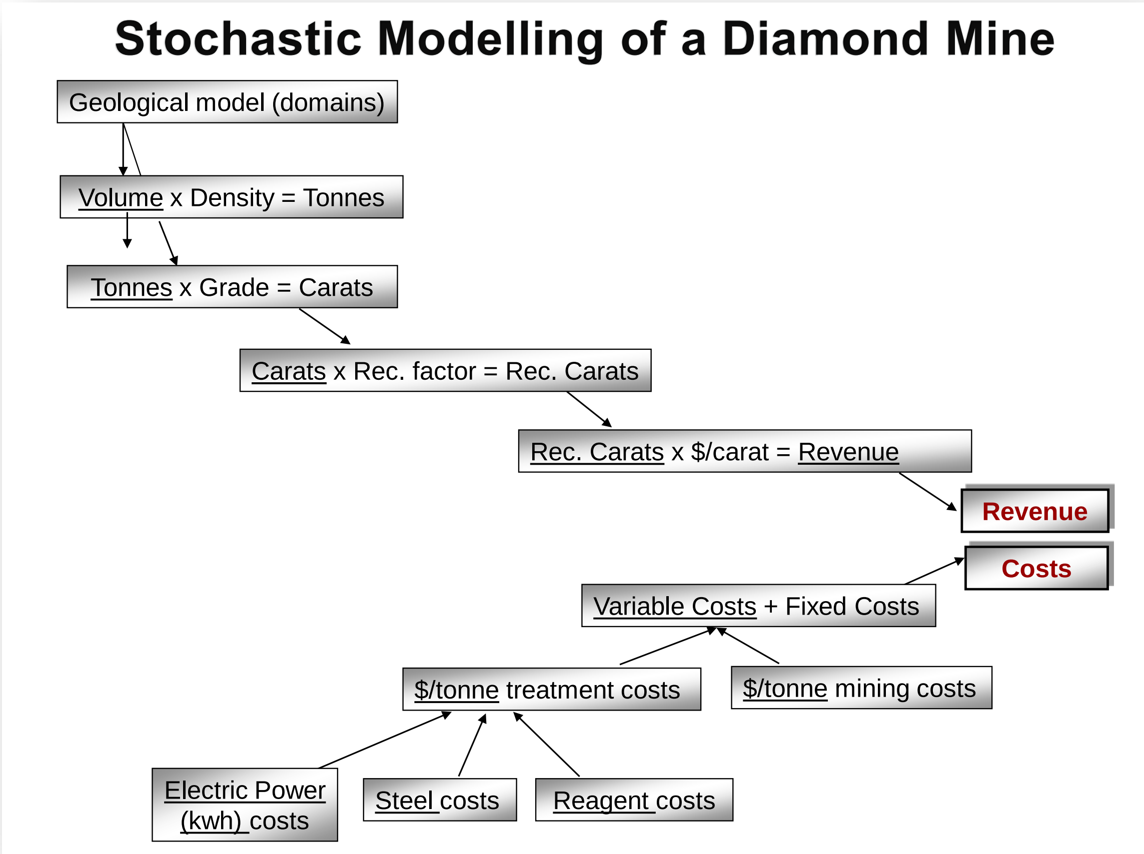


Figure 34. Simplistic overview of calculating the economic contribution (i.e. revenue less costs) of a diamond mine – each parameter has a stochastic modelling component that should be considered in the overall contribution calculation.

Calculation of the contribution derived from operational cash flows is an important component in determining the NPV of any mining project. Figure 34 provides a simplistic overview of the different components of an evaluation model required to estimate the revenue and costs of a diamond project. It can be observed that each component, starting from the geological model to the estimation of revenue, has a stochastic component. The delineation of hard and/or soft boundaries for geological domains is usually a subjective decision by the modeller/geostatistician that includes the consideration of geology, mineralisation, weathering profiles and available data. Similarly, deciding on the appropriate EUS to estimate into will determine the block volumes and should consider (usually a multiple of) the SMU size of the deposit that the mining engineer will allocate.

In Figure 34 density, grade, the recovery factor and dollar per carat (USD per carat) are all stochastic variables in the estimation of diamond revenue. Calculation of costs also comprises stochastic variables in terms of both variable and fixed costs. Inputs into variable costs are uncertain as they contain estimated usages and costs for electricity (kilowatt hours), diesel costs for plant and mobile equipment, steel for construction, reagent costs etc. The final costs (variable and fixed) will incorporate these uncertainties and hence, the net contribution will typically have a large spread in value.

Consideration of all spatial and non-spatial stochastic variables is not always possible in a single evaluation model due to the number of permutations that would need to be modelled if every conceivable scenario was contemplated. It is important to recognise where in the evaluation model the risk modelling is conducted (i.e. from resources to reserves to financial and economic parameters) as there is usually a compromise between the level of detail and the required processing time. Computer power has improved processing time dramatically over the last decade but is still relatively slow to process simultaneously a large number of stochastic variables for medium to large mining projects; e.g. block models of circa 500,000 nodes comprising more than six to seven variables may take several hours to process. The use of Monte Carlo Simulations (MCS), and to a lesser extent, sensitivity analysis, offers risk analysts an opportunity to 'simply' and 'speedily' model risk around some of these parameters but due caution should be exercised because there are various limitations, elaborated upon later in this chapter.

5.1.2 Sensitivity Analysis Overview

Sensitivity Analysis (SA) is a technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. SA can add value when attempting to assess the impact of the uncertainty in the variable. By creating a given set of scenarios, an analyst can determine how changes in one variable(s) will impact the target variable (Investopedia, 2011). The parameter values and assumptions of any model are subject to change and error. Sensitivity analysis can be described as the investigation of these potential changes and errors and their impacts on conclusions to be drawn from the model (Baird, 1989).

In very simplistic terms, sensitivity analysis is 'what if' analysis, which is an important notion at the core of any application of decision tools and may be applied to a wide range of uses. Pannell (1997) lists four broad categories of SA applications: decision making or development of recommendations for decision makers, communication, increased understanding or quantification of the system, and model development. According to him, uncertainty is one of the primary reasons why sensitivity analysis is helpful in making decisions or recommendations. If parameters are uncertain, sensitivity analysis can provide information such as:

a. How robust the optimal solution is in the face of different parameter values;
b. Under what circumstances the optimal solution would change;
c. How the optimal solution changes in different circumstances; and
d. How much worse off would the decision makers be if they ignored the changed circumstances and stayed with the original optimal strategy or some other strategy.

One of the most important decisions in setting up the experimental design for the sensitivity analysis is the consideration of:

a. the contribution of an activity to the objective (e.g. which calculation parameters to include in the sensitivity analysis for estimation of the DCF);
b. the objective (e.g. minimise risk of failure instead of maximising profit);
c. constraint limits (e.g. the maximum availability of a resource);
d. the number of constraints (e.g. add or remove a constraint designed to express personal preferences of the decision maker for or against a particular activity);
e. the number of activities (e.g. add or remove an activity); and
f. which technical parameters to vary.

A crucial consideration in setting the experimental design is whether the parameters will be varied one at a time or in various combinations of inter-dependencies. An important issue in this decision is the relative likelihood of combinations of changes. ‘One-way sensitivity analysis’ is the simplest form of sensitivity analysis where one value in the model is varied by a given amount to examine the impact that the change has on the model’s results (Taylor, 2009).

If two parameters tend to be positively correlated (e.g. prices of two similar outputs), the possibility that they will both take on relatively high values at the same time is worth considering. Conversely, if two parameters are negatively correlated, the modeller should examine high values of one in combination with low values of the other. If there is no systematic relationship between parameters, it may be reasonable to ignore the low risk that they will both differ substantially from their base values at the same time, especially if they are not expected to vary widely.

If combinations of changes to two or more parameters are being analysed, a potential approach is to use a "complete factorial" experimental design, in which the model is solved for all possible combinations of the parameters. While this provides a wealth of information, if there are a number of parameters to analyse, the number of model solutions which must be obtained can be enormous, and time consuming to process and interpret (Pannell, 1997).

Sensitivity analysis is most commonly applied to the mean parameters used to determine a defined output (e.g. NPV) by varying one parameter at a time while keeping all the others constant to ascertain which parameter has the biggest effect on the calculated output.

Figure 35 depicts an example in which the mean grades were estimated by ordinary kriging (centre figure) and then adjusted by -15% (left figure) and +15% (right figure) by shifting the mean estimated kriged grade in each estimation unit of the resource block model by a factor, *k*, in this case first by -15% then by +15% to calculate the response on the cumulative DCF output.

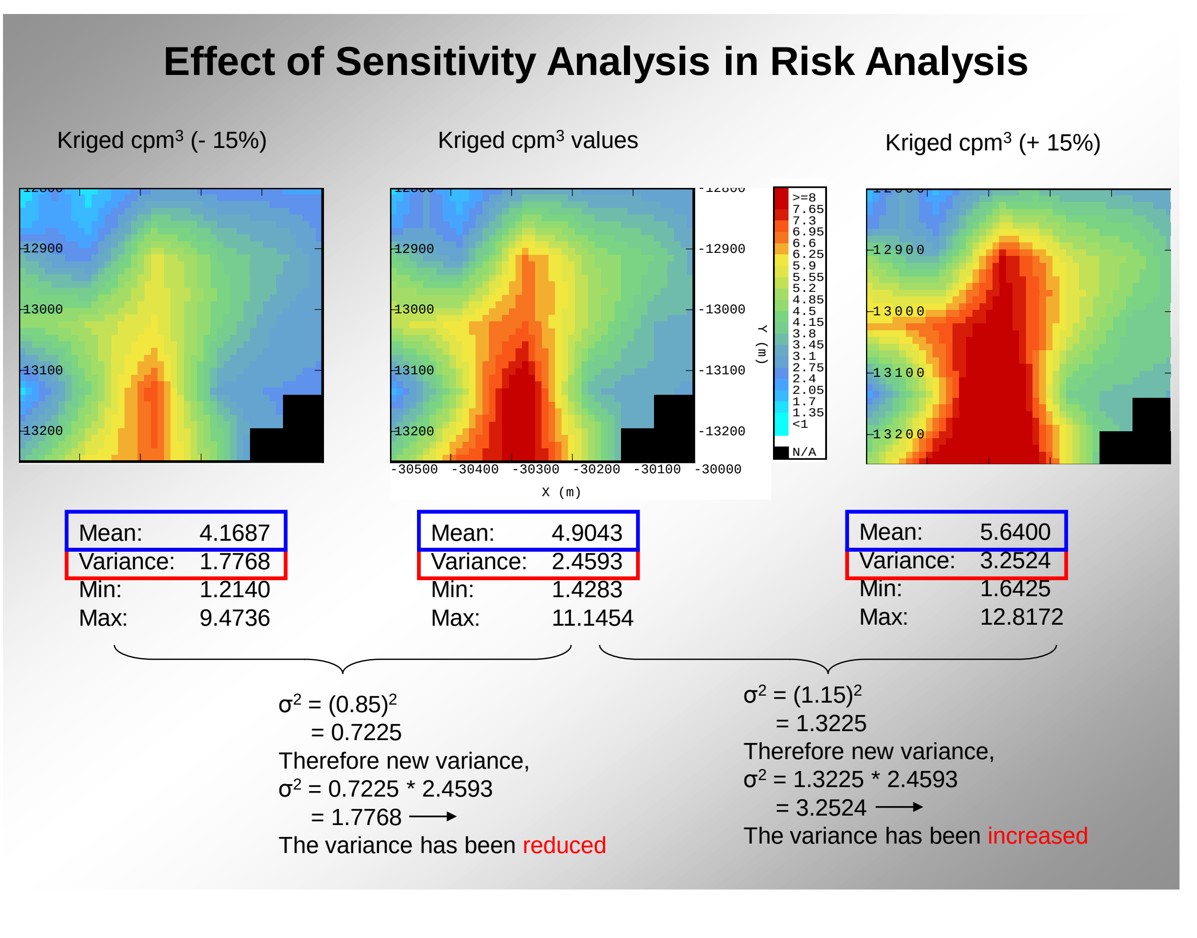


Figure 35. Effect of sensitivity analysis on kriged diamond grades (centre picture) is shown in the figure above by modifying the both the mean and the variance by a factor, k = -15% (left picture) and +15% (right picture). Note that warmer colours represent higher grades and cooler colours represent lower grades.

In the author's experience, not enough thought is given to the effect on the variance when sensitivity analyses are conducted; i.e. when the mean is shifted by a factor, k, the variance is simultaneously adjusted by a factor k^2. The net effect of this variance adjustment can be observed in Figure 36 by the dashed red lines representing the +15% and -15% adjustment of the estimated kriged grades.

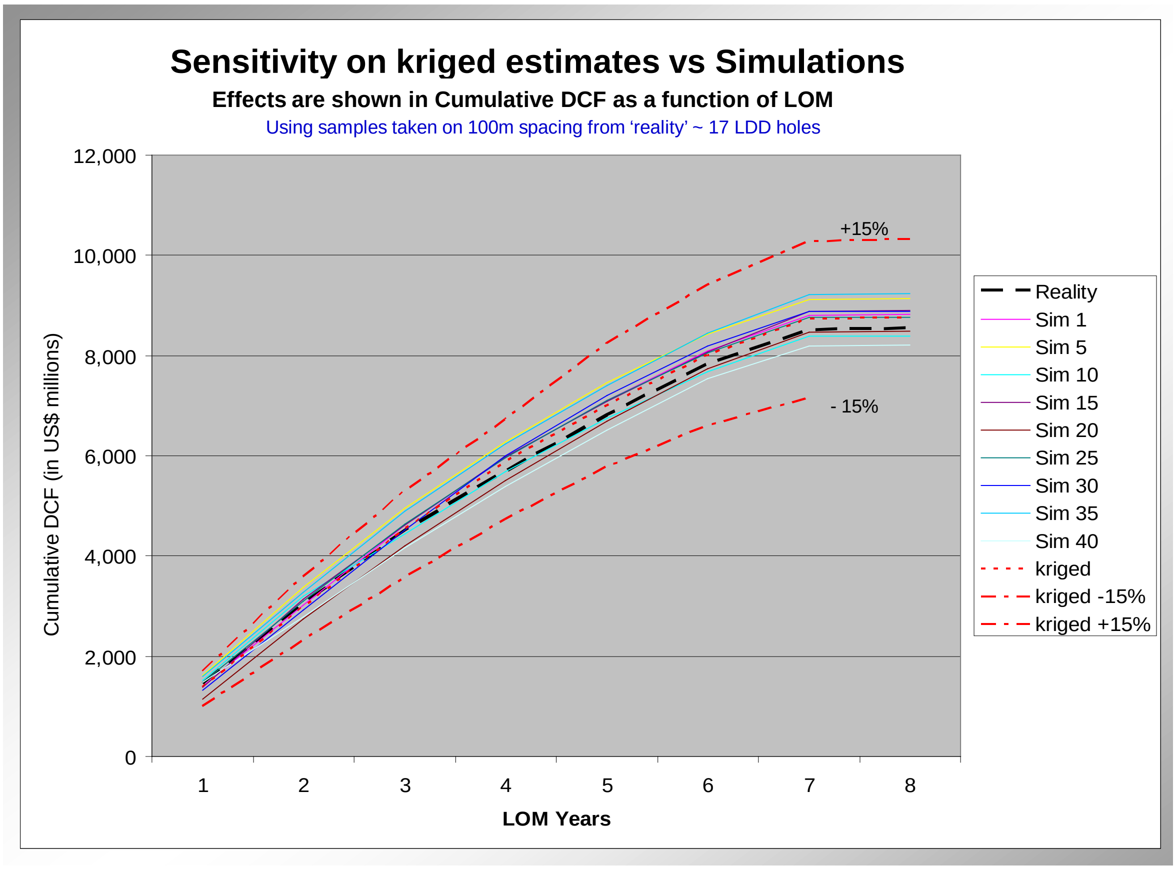


Figure 36. Effect of sensitivity analysis on kriged estimated grades, ±15% (shown in dashed red lines) relative to conditional simulations (multi-coloured lines) and the VBod reality (black dashed line).

From Figure 36 it is clear that the sensitivity analysis range lies outside the range provided by the spatially conditionally simulated realisations (multi-coloured lines). This is also evidenced by the distance of both the +15% and -15% sensitivity scenarios away from 'reality' (represented by the black dashed lines of the VBod). The author selected a sensitivity range of ±15% to replicate financial analysis conducted by corporate finance and banks (typically in the ±10% to ±15% sensitivity range). So in this scenario the benefit of using sensitivity analysis needs to be carefully assessed in terms of the original question asked. Its use to determine the 'break-even' value of a specific parameter may be of more value, assuming that the parameter is largely independent of the other parameters used to calculate the specified output. In this example sensitivity analysis does not provide an accurate reflection of the plausible spread in DCF outputs – this is far better represented by conditional simulations or kriging, which takes into consideration the spatial characteristics of the deposit based on the available drill hole data.

Figure 36 also shows the value of plotting the kriged estimate (and its sensitized ±15% values) relative to the VBod reality. In year 1 of the LOM schedule the kriged estimate under-estimated the cumulative DCF VBod reality by 5% (circa USD70 million) while in year 8, the kriged estimate was over-estimated by 3% (circa USD230 million). While kriging represents the best, unbiased linear estimate, it is acknowledged that in certain scenarios there will be greater degree of accuracy associated with global rather than local estimates. If it could be readily measured, a 3% - 5% level in the accuracy of the kriged estimate relative to reality would usually be deemed acceptable but there are scenarios when the degree of smoothing associated with ordinary kriging of the resource estimate will result in greater discrepancies between global and local estimates due to complex geology/mineralisation/structural issues coupled with widely spaced drill hole data (e.g. as described in case study one of chapter four).

Because sensitivity analyses are usually conducted on a single, 'best estimate' of the resource model, it will also reflect that level of local inaccuracy and could be misleading to decision-makers; especially where banks provide funding to companies and depend on the confidence around 'local' production and cash flow estimates within short periods (1 – 5 years tenure) of the overall LOM schedule. It is suggested that a far better alternative to sensitivity analysis is to use spatially representative conditional simulations to reflect the variance of spatial parameters. While it may take longer to set-up, it is recommended that an IEM approach (discussed in chapters three and four of this thesis) is a more robust solution than traditional sensitivity analysis to quantify the risk of spatial attributes and correlations between technical parameters within an evaluation model. Sensitivity analyses could then be conducted on the non-spatial parameters of the evaluation model.

5.2 A COMPARISON BETWEEN A BOTTOM-UP IEM APPROACH, SENSITIVITY ANALYSIS AND MONTE CARLO SIMULATIONS

5.2.1 Background

Case study one in chapter four of this thesis discussed the impact of scale of measurement, in NPV terms, on the evaluation of an underground diamond mine. This case study forms a background to the following section to compare the accuracy in calculating the NPV of a project using various evaluation and risk analysis methods. Differences between the use of a detailed 'bottom-up' evaluation approach using an IEM versus a conventional 'top-down' evaluation methodology is further elaborated upon from case study one. Furthermore, sensitivity analysis and Monte Carlo (MC) simulation methods are compared with the 'bottom-up' IEM method to highlight improvements in accuracy of this risk assessment technique.

To assess the impact of geological variability on project valuation, the author simplified a real-life problem by assuming that dyke thickness (analogous to an ore vein) and shape variability derived from face-mapping in development tunnels of an underground, diamond mine were representative of the entire deposit. Three approaches were adopted; a conventional sensitivity analysis where the variables, dilution, tonnage throughput and recovery were changed by ±5%, ±10% and ±15% from their expected values; MC simulations were run on the same variables using expert opinion to parameterize the input variables; and finally an IEM was developed to allow both bottom-up and 'top down' evaluation methodologies. Results of these three approaches were compared with each other and against a virtual ore body (VBod).

A virtual ore body (VBod) was created using a non-conditional geostatistical simulation (by using Turning Bands in Isatis software) based on data gathered from a combination of drilling information, bulk-samples and face mapping from an exposed part of the dyke – this is the same VBod described in Chapter four of this thesis. It was assumed to be the 'reality' on which the various sampling campaigns were conducted to generate the sample data. Two variables were considered in the evaluation model, viz. geometrical variability of the top surface of the dyke (v1) and thickness related to the volume of the dyke. Grade was not deemed to have any significant variability between scenarios and thus, a single sampling campaign on a 50m grid sufficed.

5.2.2 Effect of Information on Project Evaluation

Three sampling campaigns were used to sample the VBod at point support using the geostatistical software, Isatis from Geovariances. Vertical core drilling campaigns were designed on a 75m, 50m and 25m grid to sample for v1 and thickness variables, creating resource scenarios one, two and three respectively.

Resource models for v1 and thickness were generated for each scenario based on sampling data from each campaign. Ordinary kriging was used to generate estimates for each scenario for the selective mining units (SMUs) of 4m by 4m. The same grades were applied to each scenario in order to keep grade constant and only assess the impact of geological variability.

Figure 37 graphically demonstrates the effect of sampling information (drill holes) in a kimberlite dyke and its corresponding impact on reserve evaluation as a function of the interpreted geological and grade complexity.

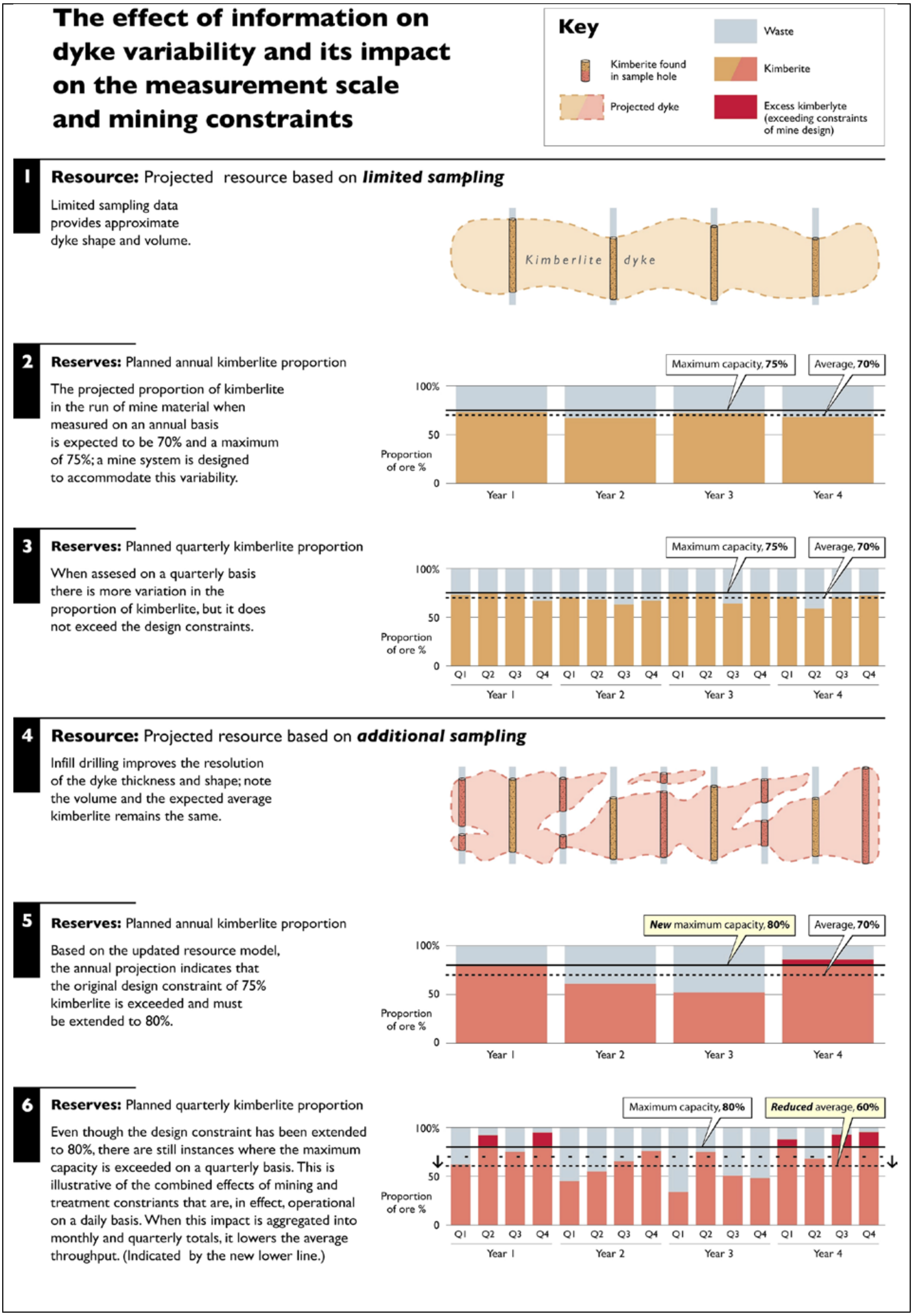


Figure 37. The effect of information (drill holes) on assessing the variability within a kimberlite dyke (Nicholas et al., 2007).

5.2.3 Bottom-up versus Top-Down Evaluation Approaches

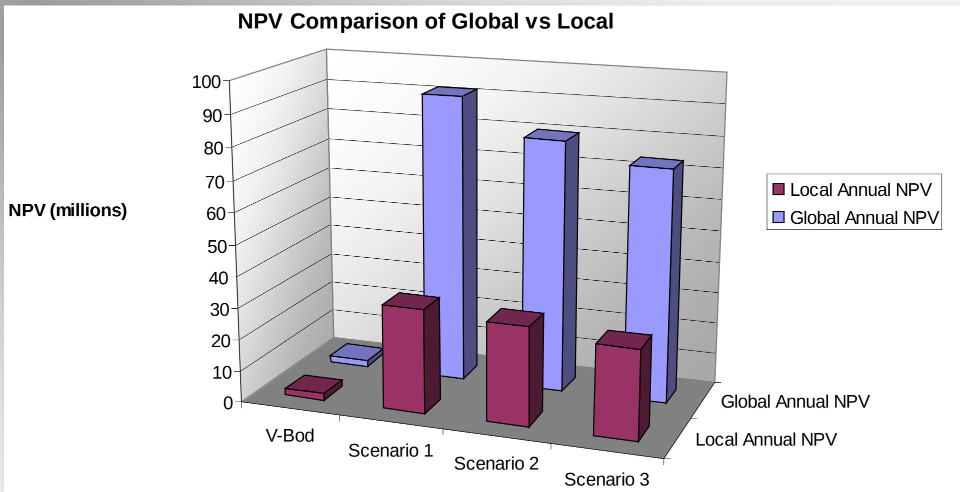
The IEM used a bottom-up evaluation technique that was based on depletions of the ore body at a local SMU scale of 4m by 4m. The SMU grades were seeded directly from kriged estimates (discussed in Chapter four of this thesis) and sampled at intervals of 75m x 75m (Scenario 1), 50m x 50m (Scenario 2) and 25m x 25m (Scenario 3). Production tonnages and grades were calculated from these blasts on a daily basis and accumulated monthly, quarterly and annually. Each SMU was analogous to a mine blast that was assessed whether it met the necessary mining and plant criteria before either contributing to the call of 3,150 tonnes per day or being sent to the waste bin if it comprised more than 70% waste. The increased short-scale variability of the dyke resulted in the mining and treatment constraints being hit more often than estimated in the top-down approach.

The IEM was designed to correlate directly the financial model with the mining and treatment databases enabling all production estimates, revenues and costs to be accumulated from a blast by blast basis to a daily basis and collected quarterly and annually. These production outputs formed inputs into the cash flow model. Conventional DCF valuation was used to calculate NPVs at an initial discount rate of 10%. NPVs were calculated in real money terms (after royalties and tax deductions, allowing for inflation).

The top-down evaluation approach refers to annual forecasts that were calculated from depleting resource estimates through a global mine plan. It was assumed that the mine plan only incorporated sufficient detail to deplete large-scale mine blocks of dimensions 250m by 250m. This implied that local mine plans (within each large-scale mine block) were not available to allow sequential depletion of the SMU to accumulate tonnages and carats in a given year. The average resource values for each year were run through the same mining and treatment constraints as imposed on the bottom-up approach.

However, instead of accumulating actual tonnages from short-scale depletions, the top-down approach assumed a fixed daily plant call of 3,150 tonnes per day would be achieved, then multiplied depleted carats with an average recovery factor per large-scale mine block. The carats per large-scale mine block were accumulated into annual cash flow models to produce global NPV estimates for each of the three kriged scenarios.

Global ‘top-down’ evaluation method compared to a Local ‘bottom-up’ method



	VBod	Scenario 1 (75m)	Scenario 2 (50m)	Scenario 3 (25m)
Global NPV (CAD million)	-	91.6	80.1	73.9
Local NPV (CAD million)	2.1	32.9	31.4	28.3

Figure 38. Comparison between the local (bottom-up) and global (top-down) evaluation approaches.

Figure 38 highlights differences in NPV between the ‘bottom-up’ (or Local) evaluation method and the ‘top-down’ (or Global) evaluation method. While the actual NPV for the VBod was CAD2.1 million, it is clear from the figure above that the local (bottom-up) evaluation method more closely approximated the actual project NPV than the global (top-down) method. It is also apparent that as the drilling grid density increased (from 75m to 25m), i.e. more holes were drilled, the accuracy in estimating the project NPV improved, albeit somewhat marginally. Given that the actual short-scale variability of the dyke operates at a scale of less than 10m, there is still significant improvement in the evaluation of the project’s NPV to be had by increasing the drilling grid density (closer than 25m spacing) – although this would not likely be practical from a logistical and cost-sensitive perspective.

This section of the study demonstrated that cash flow constituents derived from annual estimates in a top-down approach will not correctly reflect the asymmetries due to operational variability on a local, daily basis. The ‘bottom-up’ evaluation method represented a more accurate way of deriving annual cash flow estimates needed to make decisions on projects by accumulating the appropriate values from a bottom-up approach, i.e. daily, monthly, quarterly then derive annual estimates for NPV forecasts.

The use of sensitivity analysis to highlight the main parameters affecting project value (e.g. NPV) is standard practice but results can be misleading if variables are conventionally assumed to be totally independent of each other and plotted relative to each other in a typical ‘spider diagram’ sensitivity chart. In this case study dilution loss (i.e. excessive waste material mined), plant throughput and recovery loss (i.e. lower plant recoveries) have a significant influence on NPV by affecting the number of carats produced. Figure 39 portrays sensitivities for the 75m sampling campaign (scenario 1 - top-down evaluation method). One of the three selected variables is varied at a time while all other input variables are held constant to isolate the impact of the chosen variable.

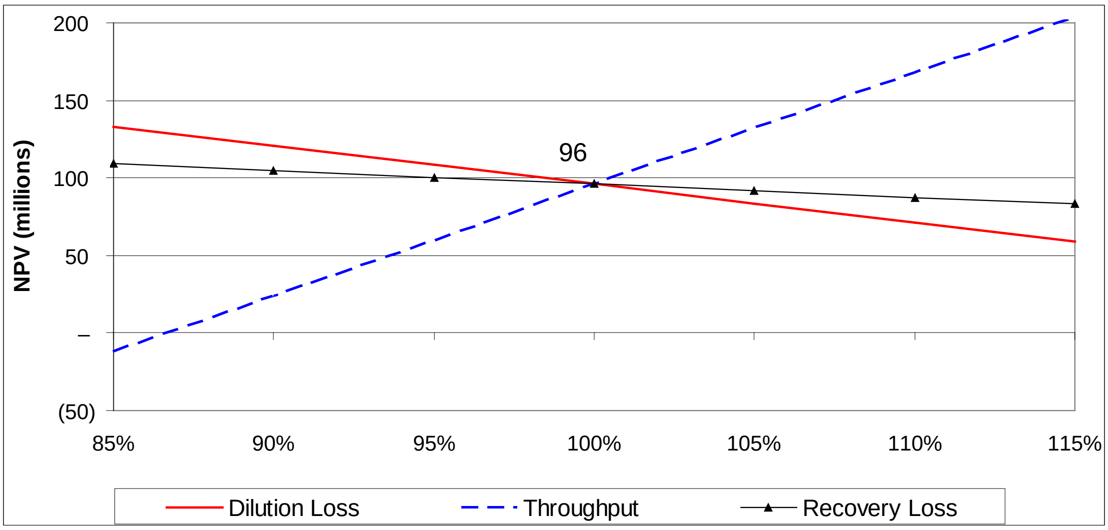


Figure 39. Graphic plot (‘spider diagram’) of sensitivity around NPV (in CAD millions) within a range of ±15%.

Results from the sensitivity analyses in the figure above show a large range in NPV from negative CAD5 million to positive CAD200 million by varying ‘dilution loss’, ‘throughput’ and ‘recovery loss’ in the order of ± 5%, 10% and 15%.

Traditionally, the rate of change (the slope) of an activity level of the objective function (e.g. cumulative DCF) is compared to changes in a parameter (e.g. dilution loss, throughput or recovery loss). An issue is the need to compare slopes for different parameters. The units of measure of different parameters are not necessarily comparable, so neither are absolute slopes with respect to changes in different parameters. To get around this problem, 'elasticities', can be calculated, which are measures of the percentage change in a dependent variable (e.g. an activity level) divided by the percentage change in an independent variable (e.g. a parameter) – see Equation 21.

$$e = \frac{\%\Delta Y}{\%\Delta X} \ or \ e = \frac{\partial Y}{\partial X} . \frac{X}{Y}$$

Equation 21. Elasticities (*e*) can be calculated to compare the rate of change (the slope).

For the purposes of the example in Figure 39, it is clear that 'Throughput' has the steepest slope (or elasticity value), however, the sensitivity range (-15% to +15%) is not based on any probabilistic data and does not provide any information on the actual chances of an event occurring. Therefore, a 15% decrease in throughput has a big impact on the NPV but the probability of that occurring is actually a lot less than a 5% increase in dilution occurring in reality. Typically, most sensitivity analyses assume independence between variables, and it is highly likely in this instance that an increase in dilution (i.e. harder waste rock mined instead of ore) will require additional crushing and grinding, which will slow plant throughput and may also negatively impact plant recovery, due to particle lock-up, to achieve the required ore-waste blend. Correlation between these parameters needs to be assessed carefully before placing too much value on the outcome of this sensitivity analysis, which can mislead decision-makers.

Monte Carlo simulations (MCS) were used to provide confidence intervals around the expected NPV output for this case study. MCS is a mathematical method used to model uncertainty in one or more parameters of a model that calculates the expected, probability outcome. The user specifies the input probability distributions for each parameter, defines the correlations (if any) between parameters, and then runs the MCS to produce multiple realisations (draws) from each defined probability density/mass function (pdf) to calculate the expected output.

Expert opinions (derived from project engineers and metallurgists) were used to define the probabilistic ranges for dilution loss, throughput and recovery loss. Triangular distributions were used to parameterize the input probability distributions. To convey the typical MCS approach used in the diamond industry, no statistical correlations were included between the three variables, which are recognized as a limitation of this type of analysis.

Figure 40 demonstrates the cumulative probability plot for NPV based on simultaneous random draws from dilution, throughput and recovery probability distributions. Scenario 1 (75m sampling campaign) is shown, as the variance around the NPV was similar for all three scenarios. The MCS results produced NPV ranging from CAD50 million to CAD154 million, noticeably outside the NPV from the IEM (CAD28million - CAD33 million). The respective NPV for the three sampling campaigns (75m, 50m and 25m) conducted on the top-down evaluation method over-estimated the NPV derived from the IEM method in the order of 160% to 180%.

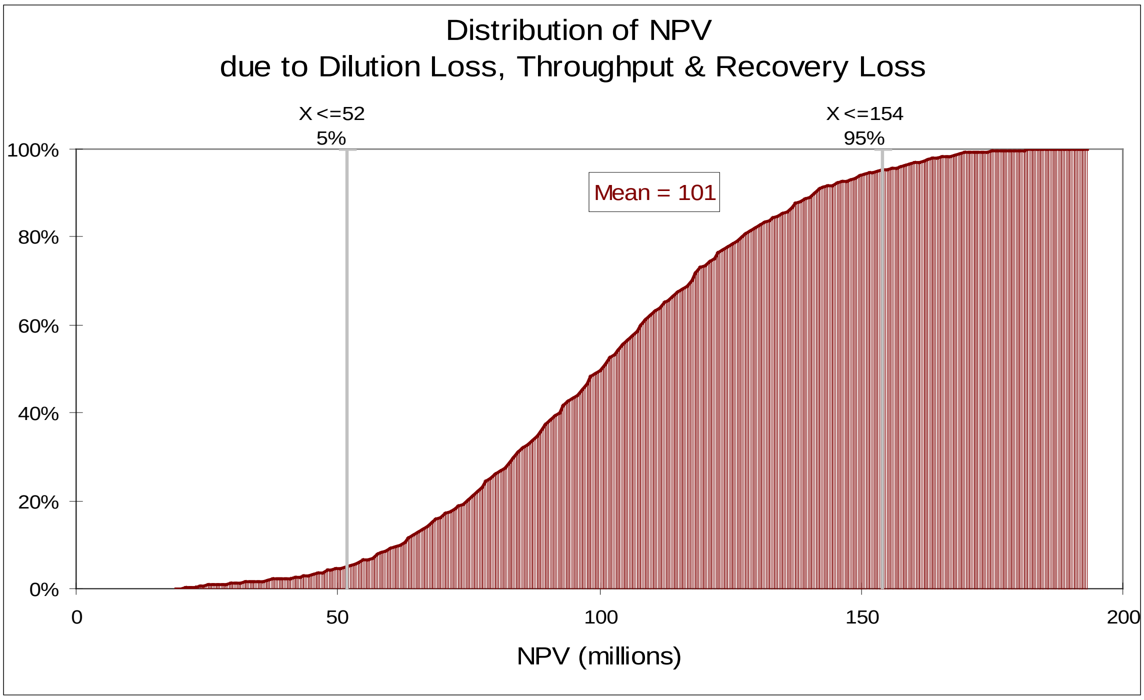


Figure 40 shows Monte Carlo Simulation for the NPV (in CAD millions) based on assuming independence between dilution loss, plant throughput and recovery loss.

The difficulty of valuing managerial flexibility in a mineral project usually results in mineral assets being undervalued using a traditional DCF approach, (Davis, 1995). However, in this study, limited sampling information resulted in a smoother, more continuous estimate of dyke thickness for both the top-down and bottom-up evaluation methods relative to the VBod ‘reality’. The acquisition of additional sampling data is expected further to reduce deviations between estimated NPV and the VBod. However, the likelihood that closer spaced sampling grids will actually be drilled is low due to high sampling costs, project delays and other practical limitations. The objective of applying an IEM approach is not to recommend closer spaced sampling grids but to find a balance between the required sampling resolution and the derivation of quantitative estimation errors in NPV.

5.2.4 A Further Analysis of Monte Carlo Simulations

Case study three in chapter four of this thesis assessed the impact of resource variability (in terms of gold grades) on the reserve constraints of an open-pit gold operation to quantify the impact on metal content in production and financial terms. The evaluation framework used was an IEM ‘bottom-up’ evaluation approach. The author recognised that Monte Carlo Simulations (MCS) are often used as a means to express the risk in a project and initiated a research project, unpublished work (Boardman and Nicholas, 2009) to compare different evaluation approaches using MCS.

There are several ways to include MCS modelling in a project in an attempt to quantify physical risks. For this case study it was decided to select two MCS approaches that were based on logic derived from statistical analyses of 25 conditionally simulated spatial data (discussed previously in case study three). A global and local evaluation method using MCS (@Risk from Palisade software) were investigated. The global evaluation method refers to an approach using MCS to generate a grade distribution for each year of the life of mine (LOM) schedule for the two main ore types, oxide and primary material, i.e. each year comprised a sequence of blocks (SMUs) mined and processed to meet scheduled production targets.

Inputs into the probability distribution for each respective year were the mean and standard deviation derived from the production statistics calculated from all records falling within each respective year – the average mean and variance was calculated, then the square root of the variance was used to derive the standard deviation. This is in contrast to the local method that used MCS to generate a grade distribution for each record within each year (i.e. *n*-number of records, each with a MCS distribution), then summed up to derive the total output for each year of the LOM schedule.

Grade was the only stochastic variable as assigned densities were used per ore type (described in case study three) to calculate total gold grams, as per the formula below:

$$tonnes = volume * density$$
$$metal(g) = tonnes(t) * grade(g / t)$$
$$metal(oz) = \frac{metal(g)}{31.1035}$$

Equation 22. Calculation of metal (gold grams and ounces) as a function of density.

A lognormal probability distribution model was fitted to grade data after analyzing stochastic data outputs from the spatially simulated resource conditional simulations (described in case study three of chapter four). An Anderson Darling fitting provided the best 'goodness-of-fit' results compared to Chi Square and Kolmogorov-Smirnoff fittings (Vose, 2002).

Goodness-of-fit statistics are not necessarily easy to interpret. They do not provide a true measure of the probability that the data actually comes from the fitted distribution, but instead provide a probability that that random data generated from the fitted distribution will produce a goodness-of-fit statistic value as low as that calculated for the observed data (Vose, 2002). The Chi Squared (X^2) statistic measures how well the expected frequency of the fitted distribution compares with the observed frequency of a histogram of the observed data.

$$X^2 = \sum_i \frac{[O(i) - E(i)]^2}{E(i)}$$

where $O(i)$ is the observed frequency of the *i*th histogram class or bar; and $E(i)$ is the expected frequency from the fitted distribution of *x*-values falling within the *x*-range of the *i*th histogram bar

$$E(i) = [F(i_{\max}) - F(i_{\min})] * n$$

where $F(x)$ is the distribution function of the fitted distribution

$(i_{\max})$ is the *x*-value upper bound of the *i*th histogram bar

$(i_{\min})$ is the *x*-value lower bound of the *i*th histogram bar

Equation 23. Chi Squared (X^2) goodness of fit statistic.

Because the Chi Squared statistic sums the squares of all the errors [$O(i) - E(i)$], it can be disproportionately sensitive to large errors. For example if the error of one bar is twice that of another bar, it will contribute four times more to the statistic (assuming the same $E(i)$ for both). Hence, the Chi Squared statistic is very dependent on the number of bars (N) that are used and by simply changing the number (N) of histogram bars, the goodness-of-fit can easily switch between different probability distribution types.

The Kolmogorov-Smirnoff (K-S) statistic (D_n) is an alternative goodness-of-fit method that is only concerned with the maximum vertical distance between the cumulative distribution function of the fitted distribution and the cumulative distribution of the data. The K-S statistic is defined as:

$$D_n = \max\left[\left|F_n(x) - F(x)\right|\right]$$

where D_n is known as the K-S distance

n = total number of data points

$F(x)$ = distribution function of the fitted distribution

.

$F_n(\mathrm{x}) = \dfrac{i}{n}$

and $i =$ the cumulative rank of the data point

Equation 24. Kolmogorov Smirnoff (K-S) goodness-of-fit statistic.

The Kolmogorov-Smirnoff statistic is usually more useful than the Chi Squared statistic because data are assessed at all data points and it avoids the problem of having to determine the number of histogram bars (or bands) to split the data into. However, its value is only determined by the one largest discrepancy and does not take into account the lack of fit across the rest of the distribution. Usually, the greater the standard deviation of the data distribution, the more chance that (D_n) will fall into that part of the range causing the K-S statistic to focus on the degree of fit at x-values away from a distribution's tails.

The Anderson-Darling statistic is the preferred alternative to the Kolmogorov-Smirnoff statistic to cater for the fit at the extremes of distributions. The Anderson-Darling statistic is preferred to the K-S statistic for the following reasons:

- $\psi(x)$ allows for the increased variance of the vertical distances between distributions (σ^2_{K-S});

- *f(x)* weights the observed distances by the probability that a value will be generated at that *x*-value.
- Vertical distances are integrated over all values of *x* to make maximum use of the observed data (whereas the K-S statistic only looks at the maximum vertical distance).

$$A_n^2 = \int_{-\infty}^{\infty} \left| F_n(x) - F(x) \right|^2 \psi(x) f(x) dx$$

where $\psi(x) = \dfrac{n}{F(x)\left[1 - F(x)\right]}$

$n =$ total number of data points

$F(x) =$ distribution function of the fitted distribution

$f(x) =$ density function of the fitted distribution

$F_n(x) = \dfrac{i}{n}$

$i =$ the cumulative rank of the data point

Equation 25. Anderson Darling goodness-of-fit statistic.

In the case of grade data that have significantly positively skewed tails, the Anderson-Darling is a more useful goodness-of-fit statistic than both the Chi Squared and Kolmogorov-Smirnoff statistics. For this reason the author used the Anderson-Darling technique to fit the appropriate distribution to the relevant data.

The mean and standard deviations required as inputs into each MCS probability distribution were calculated from the spread in grade data derived from the conditional simulations.

Table 9 describes the output from the two MCS local versus global evaluation methods for the first three years (2010 – 2012).

Year	Ore type	CV	Method	Total Metal Expected (g)	% difference (local - global)
2010	Oxide	72%	Local	10,800,00	-1.9%
			Global	11,000,000	
	Primary	67%	Local	16,000,000	3.4%
			Global	15,500,000	
2011	Oxide	92%	Local	3,500,000	0.0%
			Global	3,500,000	
	Primary	72%	Local	24,000,000	2.1%
			Global	23,500,000	
2012	Oxide	83%	Local	2,100,000	2.1%
			Global	2,100,000	
	Primary	75%	Local	23,900,000	1.4%
			Global	23,600,000	

Table 9. Differences in the calculated expected metal (gold grams) between the Local versus Global evaluation methods for a gold operation with oxide and primary ore types (Boardman and Nicholas, 2009). Note that these production figures are all before plant recoveries are considered.

Table 9 shows that differences between the local versus global method were relatively immaterial for the first three years considered in the analysis (maximum difference was 3.4%). In general the local evaluation method tended to provide a marginally higher result (in terms of total metal in grammes) than the global evaluation method. Given the amount of work and computer processing power (for several hundred thousand records) required to generate a MCS probability distribution for each record of the local evaluation method, it is deemed unnecessary and the global evaluation method is more practical given the small loss in accuracy.

Overall, the MCS method does reasonably well in terms of modelling uncertainties but it is unable to deal with the benefits of scheduling by ranking the grades of SMUs and using stockpiles. It is recommended that an IEM approach should be the tool of choice in this regard.

		2010 Totals	2011 Totals	2012 Totals
IEM total gold grams processed	**Average**	21,974,336	21,954,699	20,326,995
	Max	23,906,984	23,090,792	21,246,444
	Min	20,434,150	20,855,435	18,852,474
MCS Local Evaluation	**Local total gold grams processed**	24,929,689	25,964,137	24,612,886
MCS Global Evaluation	**Global total gold grams processed**	24,608,978	25,490,343	24,264,021
Difference (Local - Global)		*1.3%*	*1.8%*	*1.4%*
Difference (Global - IEM)		*10.7%*	*13.9%*	*16.2%*

Table 10. Comparison between the Integrated Evaluation Model (IEM) and Monte Carlo Simulation (MCS) methods for local and global evaluation, showing total gold grams (oxides plus primary ore) processed for the first three years. These figures include plant recovery factors (90% for oxides and 95% for primary ore).

Table 10 lists the differences in metal (grams) processed between the local and global evaluation methods relative to the IEM approach for years 2010 – 2012. It can be observed that the global evaluation method consistently exceeds results obtained from the IEM method for 2010 (+11%), 2011 (+14%) and 2012 (+16%). This trend was investigated further for the remaining years of the LOM schedule comparing the global MCS evaluation method against the IEM results (see Table 11). Over the LOM, the global MCS evaluation method estimated 10% more metal than the IEM method with an average of 20% more metal for the global MCS method in years 2010 – 2017. The latter years of the LOM schedule (2018 – 2024) showed an average of 10% less metal for the global MCS results compared to the IEM.

		Global MCS Method				IEM Method
Year	Ore Type	Total expected metal (g)	Plant Recovery Factors %	Processed Au metal (g)	Total Processed Au metal (g)	Total Processed Au metal (g)
10	Oxide	11,012,098	90%	9,910,888	24,608,978	21,974,336
	primary	15,471,673	95%	14,698,089		
11	Oxide	3,481,220	90%	3,133,098	25,490,343	21,954,699
	primary	23,533,942	95%	22,357,245		
12	Oxide	2,100,652	90%	1,890,587	24,264,021	20,326,995
	primary	23,550,983	95%	22,373,434		
13	Oxide	1,178,371	90%	1,060,534	18,605,409	17,052,058
	primary	18,468,290	95%	17,544,876		
14	Oxide	2,501,958	90%	2,251,762	13,787,687	12,496,722
	Primary	12,143,079	95%	11,535,925		
15	Oxide	292,517	90%	263,265	16,193,929	12,988,089
	Primary	16,769,120	95%	15,930,664		
16	Oxide	120,655	90%	108,590	24,391,027	18,226,379
	primary	25,560,461	95%	24,282,438		
17	Oxide	9,149	90%	8,234	29,743,537	21,813,875
	primary	31,300,319	95%	29,735,303		
18	primary	22,121,889	95%	21,015,795	19,965,005	18,835,744
19	primary	10,339,277	95%	9,822,313	9,331,197	11,193,875
20	primary	13,629,376	95%	12,947,907	12,300,512	12,577,882
21	primary	24,242,606	95%	23,030,476	21,878,952	17,577,835
22	primary	24,088,713	95%	22,884,277	21,740,063	20,128,844
23	primary	17,413,345	95%	16,542,678	15,715,544	18,217,493
24	primary	2,123,175	95%	2,017,016	1,916,165	9,175,276
				Totals (g)	**279,932,371**	**254,540,101**
				total difference (g)		25,392,270
				percentage difference		10%

Table 11. Comparison between the Integrated Evaluation Model (IEM) and global evaluation method using Monte Carlo Simulation (MCS) for years 2010 to 2024, showing total gold grams processed (oxides plus primary ore).

It can be deduced from Table 11 that oxide ore (together with primary ore) is mined from years 2010 – 2017, while from 2018 – 2024 only primary ore exists. While the precise reason for the differences (on average 10% more in the MCS results than the IEM) is not known, it is worthwhile noting that the average coefficient of variation (CV) for the oxides expressed as a percentage for the period 2010 – 2017 is 95% and only 79% for the primary ore. Thus, there is a greater spread of data values for the oxides (about 20% more) than the primary ore.

While the CVs between the oxides and primary ore will not necessarily drive differences in the MCS results, it is likely the shape of the probability distribution specified in the input parameters of the IEM that will impact the outputs, while the skewness and kurtosis parameters are not usually specified as inputs in a MCS probability distribution.

Further investigation as to the differences between the IEM and MCS methods are beyond the scope of this thesis but is recommended in order to determine whether a MCS solution could provide an appropriate approximation to the more detailed (but time consuming) IEM approach.

5.3 VARIANCE REDUCTION

This section examines variance and elaborates upon the effects of reducing it versus managing it. Previous literature, Knight (1921), Dasgupta and Pearce (1972), Vose (2002) and Kleingeld and Nicholas (2004) distinguished between risk (or variability) and uncertainty, where variability was defined as the ‘inherent stochastic nature of a mineral deposit’ while uncertainty refers to a lack of information, usually related to sampling data. Other authors, such as Levy and Sarnat (1984) often use the terms risk (or variability) and uncertainty interchangeably so while there is a general appreciation of the concept of risk, semantics around the usage and nomenclature are sometimes inconsistent. The concept of risk typically includes both undesirable consequences and likelihoods of the risk event occurring. A common definition of risk represents it as an asset of scenarios, likelihoods and consequences (NASA, 2011).

Resource companies often conduct resource (infill) drilling campaigns to drill closer-spaced holes within an existing drilling grid in order better to delineate resources, naively expecting to reduce the overall variance while the mean is expected to remain constant (as illustrated in Figure 41A). However, Figure 41B illustrates a scenario whereby additional drilling delineates a different geological and/or mineralisation structure that fundamentally results in the mean shifting over time (and may also result in a change in variance). This particular scenario was illustrated in more detail in Figure 37 where additional drilling delineated a shorter-scale structural variability that had not been visible before based on ‘assumed’ geological continuity from wide-spaced drill holes results.

The Information Effect & Variance Reduction

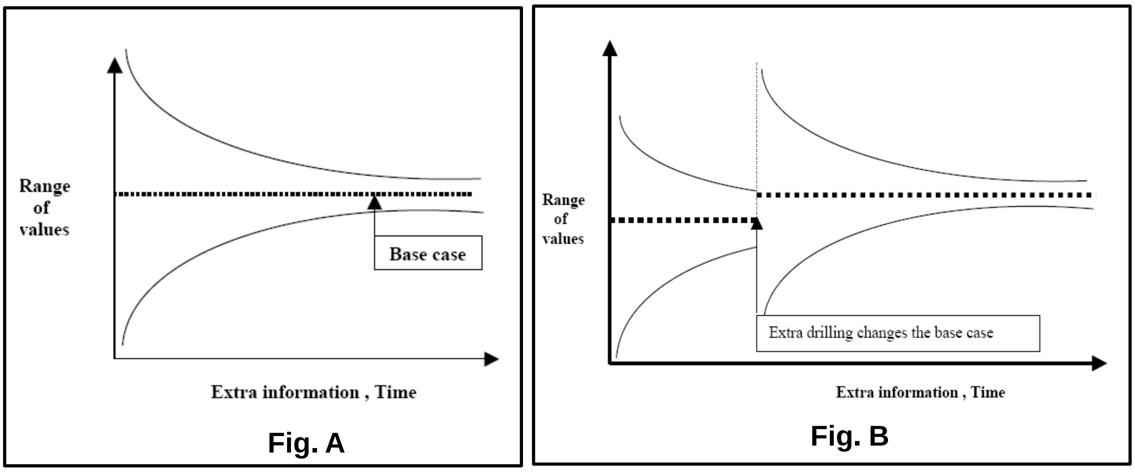


Figure 41. It is intuitively expected that uncertainty should decrease over time as more information becomes available (in Fig. A above). However, in reality Fig B may be realized, after Goria (2006).

To provide a better understanding of the relationship between the mean and the variance and its relationship in regards to the information effect (i.e. the quantity of drill holes available) for a diamond project, case study one of chapter four was further expanded using the virtual orebody (VBod) concept. Construction of this VBod was discussed in case study one of chapter four and included grade and (ore) thickness variables. This VBod was systematically ‘virtually’ sampled for both grade and thickness starting at a drilling grid spacing of 1,500m by 1,500m and eventually reduced to a 1m by 1m grid (see Table 12). In each sampling scenario the mean and variance were recorded.

Grade (cpht)

Grid	Samples	Min	Max	Mean	Variance
1,500	8	190.8	283.7	234.1	1,254.1
1,450	8	175.5	283.7	226.5	1,576.9
1,400	8	144.8	283.7	212.2	1,447.0
1,350	8	131.4	283.7	208.9	2,350.2
1,300	8	163.6	283.7	213.7	1,851.7
1,250	8	92.1	283.7	230.7	3,349.5
1,200	8	107.2	283.7	196.0	2,401.7
1,150	10	86.7	283.7	187.2	2,619.8
1,100	10	63.4	287.0	231.6	4,379.3
1,050	10	27.0	283.7	171.5	4,585.3
1,000	15	157.8	283.7	199.7	1,590.2
950	15	98.6	283.7	186.3	2,109.0
900	18	109.0	283.7	203.5	2,102.5
850	18	141.5	283.7	206.0	1,516.7
800	18	51.1	287.0	207.3	3,105.1
750	21	92.1	287.0	216.0	2,802.2
700	21	104.2	283.7	206.8	1,698.2
650	21	82.0	283.7	200.7	2,461.0
600	32	107.2	287.0	204.5	1,486.3
550	36	63.4	287.0	217.3	2,313.5
500	36	30.5	283.7	205.2	2,541.8
450	40	81.3	287.0	201.6	2,405.8
400	55	51.1	287.0	204.9	2,407.4
350	65	-	287.0	189.4	3,791.8
300	90	14.2	287.0	204.3	2,061.2
250	126	30.5	287.0	197.5	2,573.3
200	198	-	287.0	200.0	2,739.8
150	319	-	287.0	195.9	2,719.1
100	688	-	287.0	198.3	2,971.8
75	1,176	-	287.0	189.1	3,172.3
50	2,688	-	287.0	195.6	2,768.5
25	10,354	-	287.0	192.7	2,822.7
10	64,218	-	287.0	190.0	2,836.4
5	256,564	-	287.0	190.7	2,833.5
1	6,389,760	-	287.0	191.0	2,854.6

Ore Thickness (m)

Grid	Samples	Min	Max	Mean	Variance
1,500	8	-	2.30	1.61	0.45
1,450	8	0.84	2.62	1.75	0.25
1,400	8	0.93	2.07	1.65	0.11
1,350	8	1.45	2.25	1.79	0.08
1,300	8	0.13	2.07	1.53	0.34
1,250	8	0.56	2.54	1.63	0.42
1,200	8	1.45	2.52	2.03	0.14
1,150	10	1.47	2.10	1.78	0.04
1,100	10	1.56	2.57	1.91	0.09
1,050	10	1.42	2.58	1.82	0.13
1,000	15	0.89	2.29	1.72	0.10
950	15	1.35	2.80	2.01	0.15
900	18	0.24	2.55	1.90	0.25
850	18	1.56	2.33	1.92	0.07
800	18	1.04	2.44	1.94	0.14
750	21	-	2.30	1.68	0.36
700	21	0.93	2.38	1.72	0.12
650	21	0.13	2.45	1.67	0.20
600	32	-	2.80	1.68	0.51
550	36	-	2.57	1.74	0.26
500	36	-	2.80	1.75	0.33
450	40	0.31	2.41	1.83	0.15
400	55	-	2.80	1.76	0.23
350	65	-	2.80	1.78	0.22
300	90	-	2.61	1.74	0.33
250	126	-	2.80	1.68	0.32
200	198	-	2.80	1.66	0.29
150	319	-	2.80	1.73	0.26
100	688	-	2.80	1.66	0.31
75	1,176	-	2.80	1.70	0.29
50	2,688	-	2.80	1.69	0.31
25	10,354	-	2.80	1.70	0.31
10	64,218	-	2.80	1.71	0.30
5	256,564	-	2.80	1.69	0.31
1	6,389,760	-	2.80	1.70	0.31

Table 12. Information effect on grade and ore thickness showing changes in the mean and variance.

Figure 42 and Figure 43 show the change between the mean and variance for both grade and (ore) thickness, respectively. The *y*-axis was normalized to reflect the change in percentage difference between the VBod and the virtual sampling scenarios, e.g. a difference of 0% implies that the VBod and the specified sampling scenario (in number of samples) plotted on the *x*-axis were identical. Hermite polynomial trends (maximum of six polynomials) were plotted for both the mean and variance. By observing the smoother polynomial trend in both figures (for grade and thickness) it can be seen that the mean stabilized a lot sooner than the variance.

It should also be observed from the short-scale variations (i.e. not the smoother trends) in both figures that even though the number of samples increased (on the *x*-axis), the percentage difference (on the *y*-axis) occasionally increased before reducing further as the overall number of samples increased. This implies that there are short-scale variations as illustrated in Figure 41 while the overall trend shows a decrease in variance, which is not uncommon in structurally complex diamond projects.

Importantly, the observations from the above-mentioned figures support the logic that it is easier to define the mean of a distribution (for both diamond grade and thickness in these examples) for a specified number of samples at an acceptable confidence limit, while substantially more samples are required to gain confidence around the variance of the same distribution.

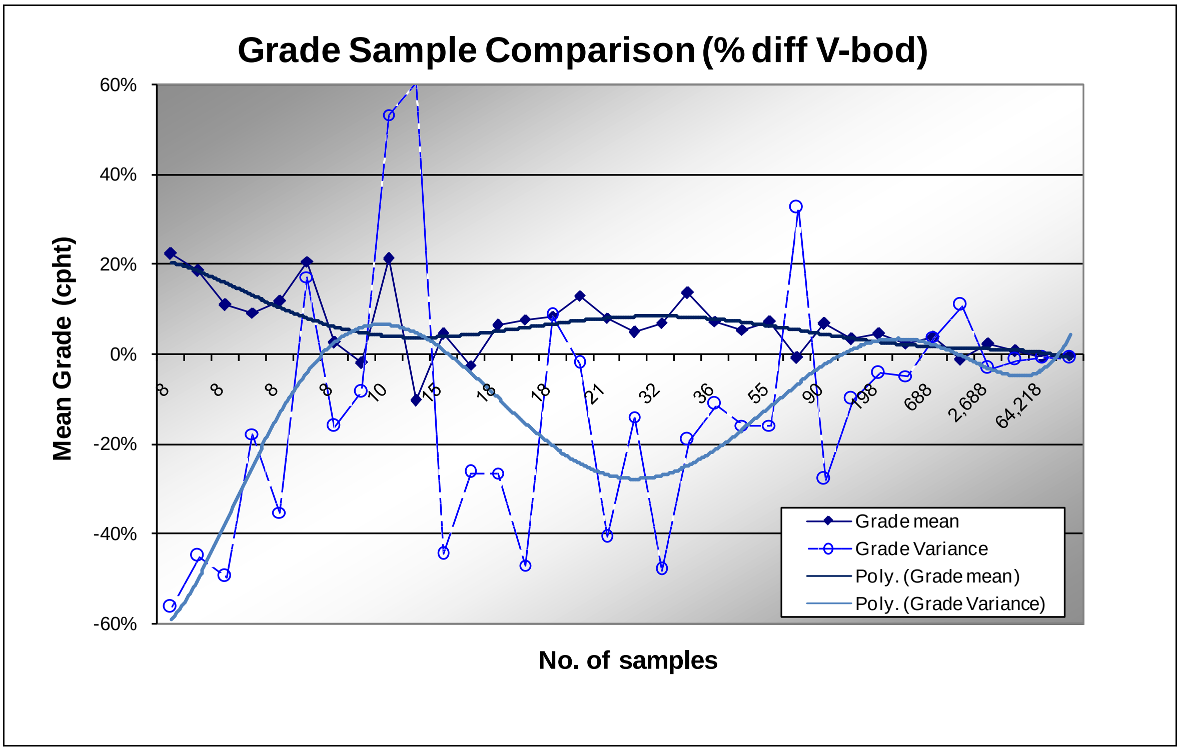


Figure 42. Demonstration of the information effect as variance in grade is reduced by 'virtually drilling' more holes into the deposit and measuring the change in grade for the mean and variance. The figure shows that as the number of samples (drill holes) increases from left to right (x-axis), the mean grade stabilizes quicker than the variance, i.e. more samples are needed to stabilize the variance than mean grade. Hermite polynomials are fitted to the data to model the trends of both the mean grade and variance.

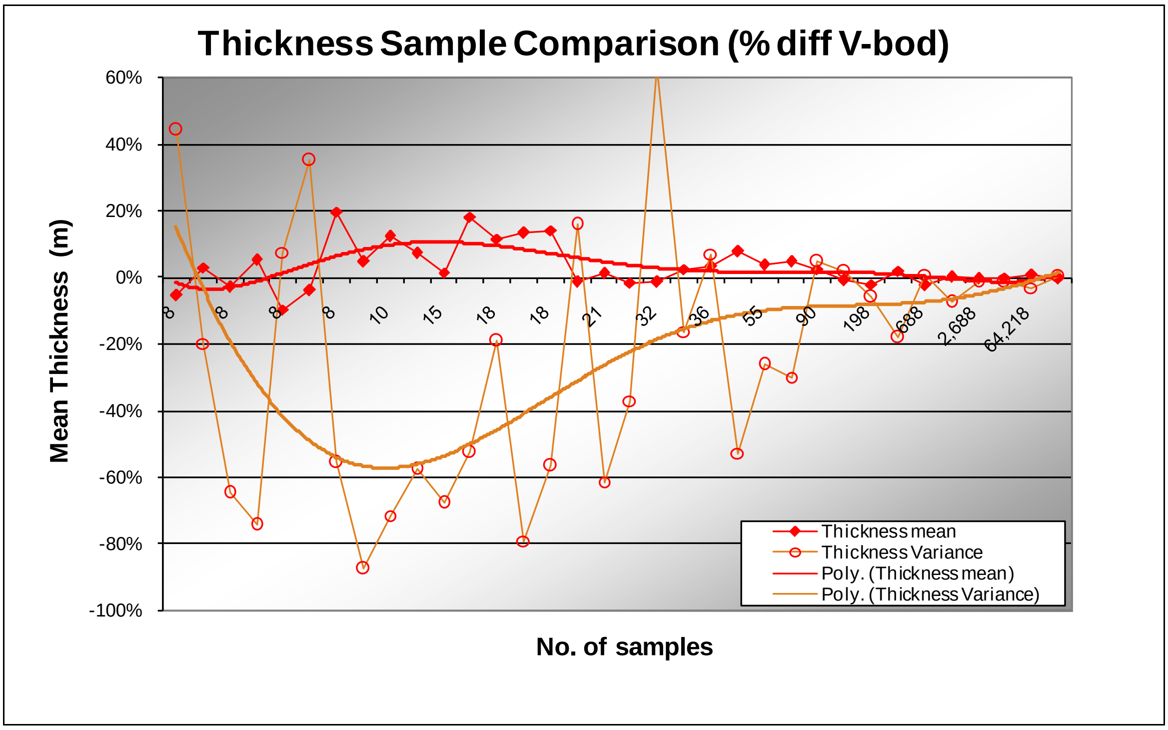


Figure 43. Demonstration of the information effect as variance in ore thickness is reduced by 'virtually drilling' more holes into the deposit and measuring the change in thickness for the mean and variance.

This exercise demonstrates that the mean of a distribution is usually determined first before the variance, as additional holes are drilled into a diamond resource. This is true for grade and ore thickness, and by inference, could also be applied to other stochastic variables such as stone size, stone quality and density. Thus, total project variance will include several stochastic variables used to determine the total value of the resource and its selected reserves. As information is made available (usually through drilling more holes, geophysical surveys, bulk sampling of mining faces, etc.) the overall project variance will reduce as a function of gaining more information and reducing the uncertainty. However, the true inherent variance is that random component of the distribution for each variable, which cannot be reduced further by gaining more sampling information (see Figure 44).

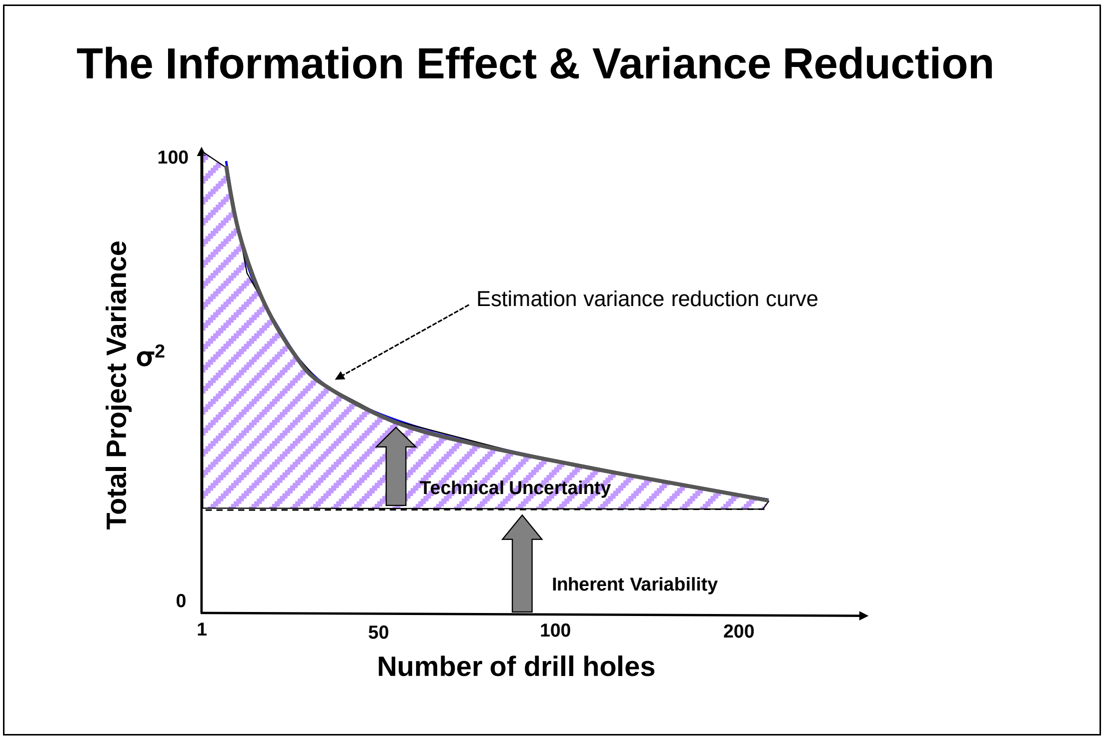


Figure 44. Impact of reduction in variance due to gaining more information (number of drill holes).

This inherent variability needs to be managed through operational and management flexibility options, such as blending of ore from stockpiles either in the pit/underground and/or on surface, ensuring that a sufficient number of mining faces are available at any one time, or that there are multiple processing streams, to name but a few. The fewer the number of sample data, the more the likelihood increases of under-estimating the true variance and assuming a smoother profile with less variability. This has a compounding effect in that management may not understand the degree of resource variability or its potential impacts on how the inherent variability could exceed design specifications of the mine and process plans on production performance. Consequently, they may be unaware of the importance of incorporating sufficient flexibility in the mine and/or processing designs, resulting in inadequate capital and time allocation.

The IEM approach introduced and discussed in this thesis provides an innovative means to evaluate the cost/benefits of operational and management flexibility options, using a real options evaluation approach (to be elaborated upon in chapter 6). Variance related to technical risk and its role in conventional DCF financial evaluations are discussed below.

5.4 FINANCIAL VARIANCE ANALYSIS

The relationship of the weighted average cost of capital (WACC) and the discount rate in the calculation of DCF NPV estimates was discussed in the literature review (chapter two) of this thesis. Smith (1982) described how the discount rate is a fundamental way of reflecting risk in discounted cash flow evaluations. He identified the main constituents of the discount rate as the real interest rate, mineral project risks and country risks. He also highlighted that the beta factors in the calculation of the cost of equity, based on the capital asset pricing model (CAPM) from Sharpe (1964), Lintner (1965) and Treynor (unpublished), actually measure the performance of company stocks relative to the stock market, but do not address the risks and characteristics of individual projects.

In order to better estimate the risk of an individual project, Smith (1982) proposed that the discount rate can be related to these three components by the equation:

$$\boxed{d_{discount\,rate} = I_{risk\ free\,rate} + R_{portion\,rate} + R_{country\,risk}}$$

where $d_{discount\ rate}$ = project specific, constant dollar, 100% equity, discount rate

$I_{risk\ free\ rate}$ = real, risk free, long term interest rate

$R_{portion\ rate}$ = risk portion of the project discount rate

$R_{country\ risk}$ = risk increment for country risk

Equation 26. Calculation of the discount rate as a component of the cost of equity in the WACC.

While economic risks such as commodity prices, foreign exchange rates and even country risks etc, are diversifiable and can be systematically reduced or managed through portfolio diversification (Markowitz, 1952), technical risks related specifically to a project are usually classified as unsystematic risks and are not be diversifiable. This view is not shared by Samis et al. (2005) who believe that project-specific risks (also known as unsystematic or ‘unpriced’ risks) such as geological and technical uncertainties are not correlated with the overall economy and can be completely diversified through the use of an investment portfolio. However, decision-makers ‘within’ a company that only have a small number of project investments may not have the luxury of having a ‘well-diversified’ portfolio of projects to offset individual project risks against each other.

Project managers usually have to compete against other ‘in house’ company projects on a ‘stand alone’ risk versus return basis for capital funding while also competing with corporate external acquisition targets. Additionally, diamond projects are all related to the same commodity within a portfolio and primarily distinguished from each other in terms of risk-returns that each individual project represents. Capital expenditure costs often run in the several hundreds of millions of dollars to billions of dollars, hence, project risks associated with a single project could ‘blow out’ and affect the capital and investment fund availability for the rest of a company’s project portfolio.

The author recognised that the calculation of project risks ($R_{portion\ rate}$) in Equation 26 was not easily quantifiable using a scientific approach. Based on the concept of the VBod (discussed in Chapter four and five of this thesis) the author devised a method to back-calculate each stochastic component (such as grade, geology and density) of the technical portion of the discount rate to ascertain its contribution to the overall project risk ($R_{portion\ rate}$). The case study discussed earlier in this chapter (described in Figure 35 and Figure 36) was expanded to estimate the effect of stochastic grades on the determination of the project risk portion of the discount rate.

The aim of this study was to find an appropriate discount rate (forming part of the DCF NPV technical risk premium) that would allow the NPV derived from the kriged estimate to be reconciled with the VBod NPV. Samples were virtually taken from the VBod at 100m sample spacing, which was equivalent to drilling a total of 17 large diameter holes into the VBod, and used to generate kriged estimates and 40 conditional simulations for grades of a major open-pit diamond project over an eight year LOM (independent analysis confirmed that results stabilized after 40 simulations).

Cash flows for each of the eight years were calculated by running each of the 40 simulations plus the kriged estimate (and ±15% sensitivity scenarios) through a pre-determined, open-pit mine plan. The mine plan was set-up in NPV Scheduler (using Datamine software) and overlain on each of the resource scenarios. For each year of the LOM, these cash flows were compared to the VBod cash flows, which was the original source data for both the conditional simulations and kriged estimates.

The upper-most table in Figure 45 compares the difference (in percentage) in cash flows for each of the eight years of the LOM for five selected scenarios (on the left). ‘Highest Cash’ refers to the conditional simulation that, on average, returned the highest cash flows per year in relation to the VBod. Conversely, ‘Lowest Cash’ refers to the conditional simulation that, on average, returned the lowest cash flows per year in relation to the VBod. ‘Kriged Cash’ is simply the cash flows returned from the base case kriged resource grades. ‘Kriged (-15%) Cash’ and ‘Kriged (+15%) Cash’ refer to the sensitivity scenarios whereby the grades in the base kriged scenario were adjusted by -15% and +15%, respectively.

Technical Discount Rates

GRAPHS COMPARING TECHNICAL DISCOUNT RATE WITH VBOD 'REALITY'

Cash Comparison

		1	2	3	4	5	6	7	8
Lowest Cash	Sim No.	-5%	8%	-4%	15%	4%	-9%	7%	1%
Highest Cash	15	12%	8%	9%	6%	-4%	-6%	1%	9%
Kriged Cash	1	-1%	-13%	-6%	21%	13%	-2%	2%	-140%
Kriged (-15%) Cash	krige	-44%	-22%	-17%	-2%	-6%	-23%	-26%	
Kriged (+15%) Cash	1	14%	15%	14%	18%	26%	13%	20%	27%
	1								

Weighted avg rate Cash

	rate								
Lowest Cash (avg. disc. rate)	**0.9%**	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%
Highest Cash (avg. disc. rate)	**1.7%**	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%
Kriged Cash (avg. disc. rate)	**0.6%**	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%
Kriged (-15%) Cash (avg. disc. rate)	-5.1%								
Kriged (+15%) Cash (avg. disc. rate)	6.1%								

Discount Rate

	Discount Rate	
Geology	0.5%	Technical risks
Grade	0.6%	
Density (S.G)	0.4%	
Other	1.0%	
	2.5%	

.

Figure 45. Alternative method to estimate a technical risk premium for the project risk portion of the discount rate.

The differences (in percentages) in cash flows for each of the five scenarios were back-calculated to the present value in ‘today’s money’ terms based on the discount rate (10%) for each of the eight years. The bottom-table in Figure 45 shows the average value of the ‘back-discounted’ differences between the five scenarios. Of specific importance is the 0.6% project risk portion derived from the differences in grade estimates between the VBod and the ordinary kriged estimates. Note that this 0.6% risk portion is the average difference over all eight years and, if required, an individual risk portion per year could be calculated for inclusion into the total discount rate on an annual basis.

Using a similar approach, the other risk portions for geology, density, etc. could also be estimated (by repeating the estimation process derived from virtual samples taken from the VBod) to derive the overall technical project discount rate portion. In this example an additional 2.5% should be added to the risk free rate and country risk as per Equation 26.

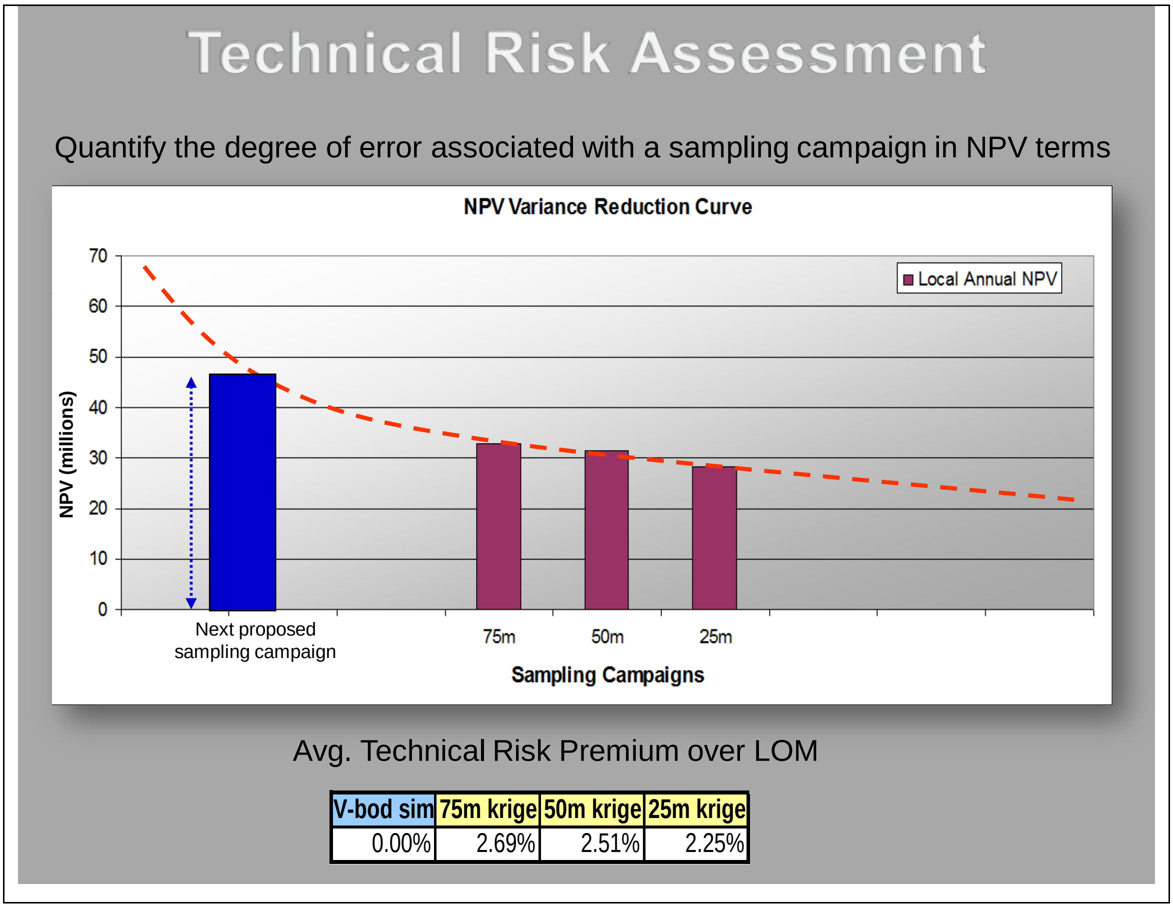


Figure 46. Graph modelling the trend (red dashed line) for the Technical Risk Premium (TRP) to be added to the basic discount rate. Note how the TRP marginally reduces from 2.69% to 2.25% as the number of drill holes increase from 75m to 25m spacing.

The project risk approach to estimate the project risk portion of a discount rate can be further expanded to calculate a technical risk premium based on several virtual sampling campaigns derived from one or more VBod. Case study one in chapter four of this thesis discussed the impact of the scale of measurement in NPV terms for project evaluation based on three virtual sampling campaigns (75m, 50m and 25m).

Figure 46 shows an illustration where an algebraic equation (shown by the dashed red line) is estimated by modelling a trend based on the three virtual sampling campaigns derived from the VBod (see Table 13). The blue bar in the figure illustrates a sampling campaign at wider-spaced drilling than the modelled 75m, 50m and 25m grids. Based on the modelled trend, the appropriate technical risk premium can be calculated for that drill hole spacing that should be applied to the project.

	V-bod	Scenario 1	Scenario 2	Scenario 3
Description	reality	wide-spaced	moderate	detailed
Grid Dimensions	4m x 4m	75m x 75m	50m x 50m	25m x 25m
No. of samples/ nodes	399 360	1 136	2 556	10 224
Sample % of V-Bod	-	0.28%	0.64%	2.56%

Table 13. Three main virtual sampling scenarios (75m, 50m and 25m) derived from the VBod.

The author feels strongly that there are at least two pragmatic arguments against using the approaches described above to estimate a risk portion and modelling the average trend between the selected virtual sampling campaigns.

Firstly, the use of a technical risk portion that is added to the risk free rate and country risk, according to Smith (1982) assumes that technical risk increases exponentially over the LOM. In Equation 27 *CF* refers to the cash flow in each period *i* and *r* is the discount rate for the standard DCF NPV formula. This equation can be rewritten as a weighted sum to illustrate the impact of the discount rate on the variance of the DCF. The equation shows that if a technical risk portion is added to the risk free rate to derive a total discount rate, *r*, then the discounting effect, *w*, will increase exponentially each year.

$$NPV = \frac{\sum CF_i}{(1+r)^i} - I_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$DCF = \sum CF_i * \left(\frac{1}{(1+r)^i}\right) \quad \text{or} \quad DCF = \sum CF_i * w_i \quad \ldots\ldots\ldots\ldots(2)$$

Equation 27. The effect of the discount rate, *r*, over each year, *i*, in the LOM.

In reality this is not true for technical risks. For most mid-cap and major mining companies it is usual practice to gather and process information derived from close-spaced blast holes and drilling (as part of grade control practices). These practices together with actual mining activities provide insight for operational management into the behaviour of project risks related to the ore body over time. Management gains more information and reduces uncertainty in project risk issues related to geology, grade, density, mining and processing rates, mining and processing efficiencies etc. The only caveat to these statements is where historical mining of an ore body is not related to planned future areas of the same ore body, possibly due to a change in mining methods or different geological/mineralisation domains.

Secondly, it is probably true for most ore bodies that no two mineral projects are identical to each other and each project must be uniquely assessed in terms of its project risks and overall risk versus returns. This is even more applicable to diamond projects due to the extreme value nature of the commodity in relation to its size, distribution, quality and dollar per carat value characteristics. An open-pit mining operation may have many unique characteristics associated with the resource (geology, grade, density etc) that need to be considered and modelled stochastically. An open-pit project is likely to have vastly different risk-return value characteristics to that of an underground operation. Consequently, the trend derived from modelling technical risk premiums calculated from virtual sampling campaigns is likely to be applicable to that project only and is not directly transferrable to another mineral project.

Furthermore, results derived from virtual sampling campaigns are based only on a single VBod realisation. In order to make the estimation of a technical risk premium (TRP) more representative it will be necessary to model several VBod realisations (via spatial conditional simulations or some form of appropriate non-linear technique) and re-generate the virtual sampling campaigns for each VBod. Then the TRP for each VBod realisation will need to be modelled and the average trend calculated. This will be particularly time consuming and require significant processing power. It will also include subjective assumptions in estimating and modelling the stochastic variables, which will influence the expected objective outcomes of the exercise.

Based on the above arguments, the author does not advocate the use of a technical risk premium as a viable method to better predict the value of a mineral project by considering its project physicals.

5.5 SUMMARY AND CONCLUSIONS

The objective of this chapter was to compare the advantages and limitations of conventional sensitivity analysis and Monte Carlo simulations with an integrated evaluation model (IEM) approach for mineral project evaluation. The risk analyst/modeller has to clearly define 'the foremost question' (i.e. the objective of the risk study) and select the appropriate risk analysis technique to provide a solution to that question while weighing up the benefits versus limitations of:

- the desired accuracy of the risk model output and set-up time for the model;
- computer processing capabilities and speed of the hardware and software;
- which parameters and constraints to include in the model and which to exclude;
- the available information supporting parameterization of risk model inputs;
- modelling correlations between parameters and capturing the system dependencies between key stages of the evaluation pipeline;
- whether parameters will be modelled (or varied) one at a time or simultaneously considered in the risk model;
- the appropriate level of detail to include in the risk model;
- where to include risk modelling along the 'evaluation pipeline' (i.e. within resources, reserves and/or financial modelling); and
- the core intended use of the risk model outputs and the ability to effectively communicate results to decision-makers.

While sensitivity analysis offers many benefits in understanding how different values of an independent variable will impact a particular dependent variable under a given set of assumptions, there are limitations to this technique as a risk analysis tool that needs to be understood. Sensitivity analysis is not recommended for spatial (physical) resource parameters in a risk model as it cannot correctly capture the correlations nor variance between variables, whereas conditional (spatial) resource simulations are preferred for this reason. The plotting of several scenarios on a single sensitivity analysis 'spider diagram' chart should be interpreted carefully as it can easily mislead interpretations assuming equal probabilities between each scenario, which is untrue in reality.

Similar to sensitivity analysis, the use of Monte Carlo simulation (MCS) can be beneficial in modelling uncertain parameters in a risk model to determine the expected probabilistic output. MCS is not recommended for spatial (physical) resource parameters in a risk model as it is common practice for the user to define the input probability distributions and correlations between variables, which is often not straight forward and typically results in the risk modeller assuming independence of parameters. Based on the author's experience MCS can mislead decision-makers as they may assume that they have correctly captured the range of possible outcomes with the expected outcome ('the mean') safely lying within the modelled variability range. This is not always true, as demonstrated by the case study within this chapter and can result in material evaluation errors (in the range of 160% - 180%).

Different levels of detailed MCS modelling (global versus local) were also investigated as part of a gold mine study to determine whether they were material. The case study showed that the results were relatively immaterial for the first three years considered in the analysis (maximum difference was 3.4%). In general the local (more detailed) evaluation method provided a marginally higher result than the global evaluation method but was considerably more time consuming, and deemed unnecessary for the small loss of accuracy.

It was also demonstrated that as additional sampling information is acquired, the overall project variance reduces as a function of gaining more information and reducing uncertainty. The fewer the number of sample data, the greater the likelihood of under-estimating the true variance and assuming a smoother profile with less variability where linear estimation techniques are used. The objective of implementing an IEM philosophy for mineral project evaluation is not to recommend closer spaced sampling grids but to find a balance between the required sampling resolution and the derivation of quantitative estimation errors in NPV.

While it may take longer to set-up the IEM model and establish the appropriate correlations between parameters and system linkages, it provides an innovative means to evaluate the cost/benefits of acquiring more sampling data and quantifying the benefits in financial terms.

Chapter 6 : Hedging Strategies using Real Options Valuation (ROV) in an Integrated Evaluation Model (IEM)

6.1 INTRODUCTION

"In a constantly changing and uncertain world marketplace, managerial flexibility and strategic adaptability have become crucial to capitalizing successfully on favourable future investment opportunities and to limiting losses from adverse market developments or competitive moves. Corporate capabilities that can enhance adaptability and strategic positioning provide the infrastructure for the creation, preservation and exercise of corporate real options", Trigeorgis (2002).

The objective of this chapter is to quantify the financial impact of managerial flexibilities by evaluating different hedging strategies that simultaneously consider production and economic uncertainties using an integrated evaluation modelling framework. The importance of linkages within an integrated evaluation modelling (IEM) framework are demonstrated between unsystematic (project specific) risks related to the resource/reserve parameters and systematic (economic) risks, viz. the foreign exchange rate, to evaluate the most appropriate management hedging strategy for a diamond mining company.

Instead of conventionally evaluating multiple hedging strategies for foreign exchange rate uncertainty using a single production scenario as a basis that is typically generated from a kriged resource estimate, each hedging strategy is run against multiple realisations of the ore body generated from conditional simulations. Actual foreign exchange rates for the period 2006 to 2009 are used to compare hedging strategies. This provides a unique opportunity to compare selected hedging strategies with reality, i.e. stochastic modelled foreign exchange rates are compared with actual rates for that same period, generated for each conditionally simulated realisation of the ore body, and then compared to a simulated resource reality, referred to as the virtual ore body – see Figure 47.

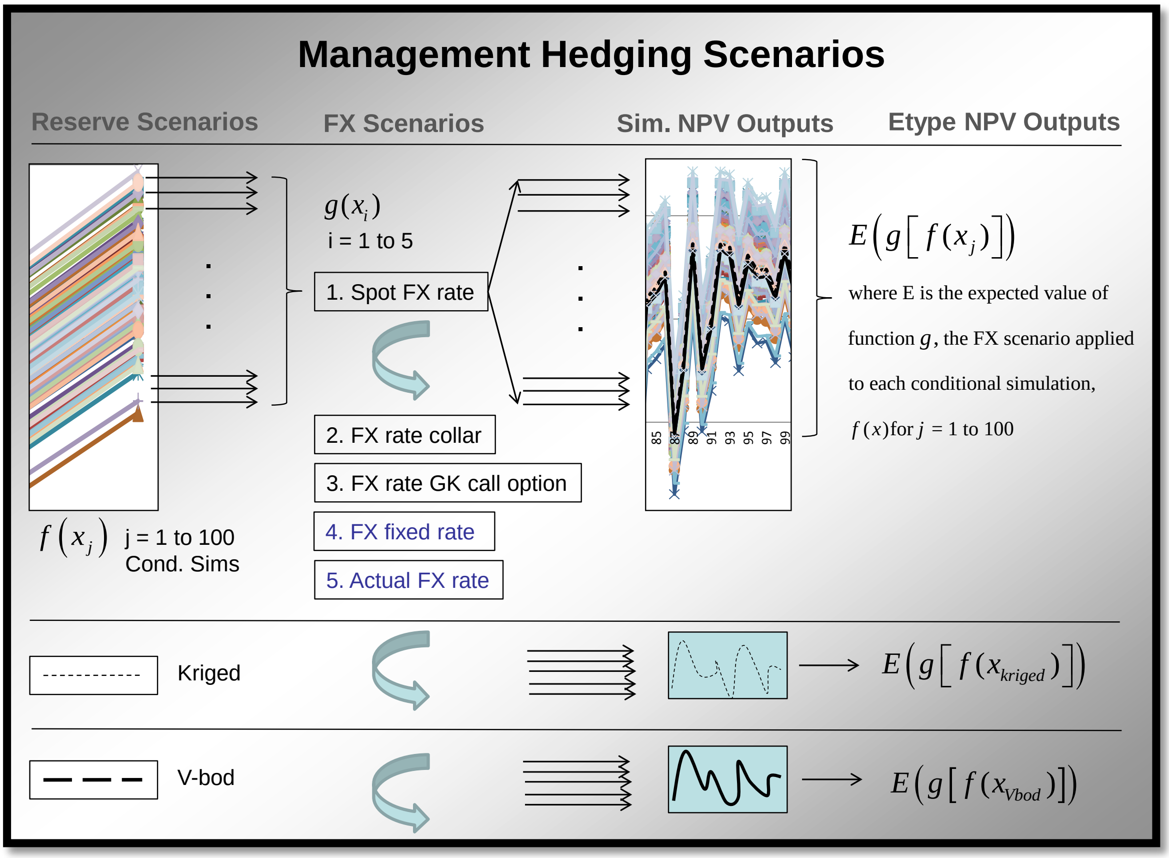


Figure 47 - Overview of model depicting combined reserve uncertainty and economic (FX) rate scenarios to generate outputs expressed in NPV terms. A total of 100 conditional simulations reflecting resource uncertainty was run through a production model to generate 100 reserve scenarios, and then run through a financial model where each reserve scenario was run through five different FX scenarios to compare NPV outputs.

The IEM provides a unique platform to incorporate key technical linkages between resource uncertainties, reserve parameters and the cash flow model at the appropriate temporal scale. Complex resource estimation problems are often expressed through simplified mathematical equations that solve a global or local geostatistical problem. However, the production and financial impacts of non-linear resource-to-reserve relationships cannot be approximated using a closed-form mathematical solution as each project has its own set of resource and reserve variables, which interacts with mining and processing constraints in a sequential, non-linear and unique way.

It is important to understand that in the modelling approach depicted in Figure 47, each of the 100 "conditionally simulated" production scenarios is used to evaluate five different hedging scenarios. A mean estimate for each hedging scenario is produced in NPV terms once all of the 100 simulated production scenarios have been evaluated. Only then are statistical means, percentiles and coefficient of variation statistics calculated on the data set. This is to ensure that the modelled hedging outputs capture the full variance of the input data set modelled via the conditional simulations. In the context of this approach, it would be incorrect to calculate a mean of the 100 conditionally simulated production scenarios and then use it to determine the impact on the five different hedging scenarios as the variance would be greatly understated in that case.

The economic parameters considered in this chapter are specific to foreign exchange (FX) rates as this uncertainty can have a material impact on the profitability of projects and diamond companies as a whole. When reserve uncertainty is combined with stochastic FX rates, it can be perplexing for management to decide whether to hedge against the domestic FX rate and if so, what level of FX exposure the project (or company as a whole) can accept before margins are materially negatively impacted. The importance of the FX rate impact in calculating the NPV of a mining project as a function of the operational cash flows can be explained using the simplified equation:

$$NPV = \sum_{j=1}^{n} \frac{CF_j}{(1+r_{dr})^j}$$

$$where\ CF_j \quad = \quad \sum_{i \in nb_j} \left(\left[V_i * SG_i * \frac{G_i}{100} \right] * R * P * g\left(r_{fx}\right) \right) \quad - \quad \sum_{i \in nb_j} \left(C_f \ + C_{V_i} \right)$$

and nbj is the set of blocks contributing to the cash flow in period j

and $g\left(r_{fx}\right)$ is the selected FX rate (as a function of the hedging strategy) in USD:CAD terms

and $C_v = \left(c / t_m * \left[V * SG\right]\right) + \left(c / t_p * \left[V * SG\right]\right)$

and r_{dr} is the discount rate equal to the weighted average cost of capital (r_{WACC}),

with $r_{dr} = r_{WACC} = \left(\frac{E}{W}\right) R_{sl} + \left(\frac{D}{W}\right) R_d (1 - t_c)$

where E = market value of equity; D = market value of debt; and $W = E + D$

and R_s = cost of equity capital; R_{sl} = required yield on equity once levered

and R_d = market yield on debt

and t_c = corporate tax rate applied for tax deductions where interest has been paid on a debt loan

Equation 28 – Simplified equation showing the NPV of a mining operation as a function of the operational cash flows and discount rate.

In Equation 28 a modified discounted cash flow is used to calculate the sum of all the cash flows over the life of mine for *j*-number of periods (years in this case), discounted by the rate, r_{dr} per period to derive the NPV for each of the conditionally simulated realisations. Within each period, *j*, the sum of all the mine blocks (for i = 1 to n) contributing to the operational cash flow, *CF*, is calculated as a function of *V*, the volume in m^3 of the reserve block model, *SG*, the specific gravity (or density); *G*, the grade in carats per hundred tonnes (cpht); *R* is the processing recovery expressed as a percentage; *P* is the price (or revenue) per carat in USD.

The function $g(r_{fx})$ is the selected foreign exchange rate for each of the hedging scenarios considered where the currency differential between foreign (in this case USD) currency and domestic (in this case CAD) currency; and the total costs (determined in domestic FX rates) as the sum of all the mine blocks (1 to n) contributing to the operational cash flow, *CF* where C_f is the fixed portion of operating costs and C_v is the total variable portion of the operational costs, comprising the cost per tonne mined (*c/tm*) and cost per tonne processed (*c/tp*). For the purposes of this equation, no capital expenditure is included. The author uses the foreign exchange market convention and the FX rate in the direct sense, i.e. USD:CAD. It should also be noted that for simplicity, it was assumed that the FX rate had no effect on costs.

The discount rate, r_{dr} per period, was determined as a function of the weighted average cost of capital (WACC) assuming that the main source of funding is derived from debt funding from one or more banks, and that the equity portion is more likely to be derived from retained earnings generated from operational cash flows within a company's portfolio

6.2 'PHYSICAL' AND ECONOMIC PARAMETERS

It is acknowledged that diamonds are not a very liquid commodity in banking trading terms and hence, a tradable hedge market does not exist for diamonds. Consequently, the sale of diamonds is highly dependent on committed 'off take' agreements with selected sightholders, typically renewed on an annual basis. Negotiations around volume (or quantity), quality per size frequency distribution and diamond price per category are key issues for both suppliers and sightholders to agree on. Diamond suppliers use these volumes and prices, which are contingent on sightholder negotiations to produce their annual budgets and to predict future cash flows.

Diamond reserves are uncertain with respect to several stochastic variables such as grade, thickness and density, which results in uncertain production estimates. In addition to uncertain production estimates, management of producing mines also need to consider economic uncertainties such as diamond pricing, diesel/gas prices, cost of labour, foreign exchange rates. Internal corporate studies (unpublished) reveal that, of these economic uncertainties, diamond prices and FX rates often have the biggest impact on a mine's profitability in NPV terms.

Diamond prices are largely affected by macroeconomics (in relation to supply-demand trends) as there is a large degree of subjectivity and private negotiations between diamond producers and sightholders to secure tangible volumes and agree on pricing, which cannot easily be quantitatively modelled using stochastic models. For this reason, a constant nominal diamond price was used for modelling purposes to keep price constant in sensitivity analysis studies.

A case-study of an underground diamond mine is presented where diamonds are contained in an irregular, structurally-deformed dyke (similar to a narrow vein gold deposit) that intruded into a fractured granitic host rock. Geological variabilities considered in the IEM are in the form of unsystematic (specific) risks as a function of the uncertain thickness of the mineralized dyke, the grade and its undulating top surface (referred to as v1):

- thickness (in metres) related to the volume of the dyke;
- grade (in carats per hundred tonnes); and the
- geometrical variability (v1) of the top surface of the dyke.

A virtual ore body (VBod) was created as a version of reality using non-conditional geostatistical simulation based on data from a combination of actual drill hole data, bulk-samples and face mapping from an exposed part of the dyke. A non-conditional simulation was used to capture the full extent of variability as opposed to a conditional simulation where limited, available sampling data are used that may generate a reduced range of variability. The author recognises that a degree of subjectivity is introduced in terms of which spatial variability model to use but felt that this was acceptable based on a historic understanding of geological and mineralisation variability at different scales for similar styles of deposits.

The VBod was used to represent reality upon which three distinct virtual sampling campaigns were drilled to generate data (referred to as the 75m, 50m and 25m sampling campaigns). For each sampling campaign, the resource and reserves estimates were recalculated using the necessary thickness, grade and v1 information to estimate the cash flows and generate the NPV. Of the three sampling campaigns, only the 75m campaign was selected for this study to demonstrate the combined impact of physical stochasticity combined with economic stochasticity in order to save considerable computing time. Resource models for v1, grade and thickness were generated based on sampling data from the 75m sampling campaign.

Ordinary kriging was used to generate estimates for the grades and thicknesses of selective mining units (SMU) of 4m by 4m. The data were also used to generate 100 conditional simulations into each SMU. E-type estimates were generated by calculating the mean of all conditional simulated values in each SMU for each variable (v1 geometrical variability, grade and thickness).

Reserve considerations focused mainly on mining and treatment processes. Each SMU is analogous to a mine blast that was assessed to determine whether it met the necessary mining and plant criteria before either contributing to the call of 3,150 tonnes per day or being sent to the waste dump if it comprised more than 70% waste. A conventional room and pillar method was used with an option of slashing and drifting depending on whether the dyke thickness was less than a specified mining threshold. An average extraction rate of 75% was imposed.

Each large planned mining block of size 250m by 250m was depleted based on a combination of rim tunnels, stope tunnels and stope slashing. An average daily call of 3,150 treatment tonnes was imposed on the project by management. A simplified treatment model was assumed for this study based on a linear relationship between the proportion of ore and waste. Recovery efficiency improved as the proportion of mineralised dyke increased. A plant surge capacity constraint was included to assess the impact of varying dyke thickness on the feed rate variability using an ‘event-based’ simulation. A total stockpile capacity of 3,000 tonnes was created, which included capacity from an underground storage bin.

The IEM was designed so that the financial model was directly correlated with the mining and treatment database enabling all production estimates, revenues and costs to be accumulated from a blast by blast basis to a daily basis and collected quarterly and annually. These production outputs became inputs into the cash flow model. DCF valuation was used to calculate NPVs at an initial discount rate of 10%. NPV were calculated in real money terms, after royalties and tax deductions, allowing for inflation.

Figure 48 illustrates the modelled relationships of conditional simulations for the 75m sampling campaign in relation to the kriged and E-type estimates, expressed in cumulative discounted cash flow terms. Figure 48 demonstrates that resource variability has the potential to materially affect a project's production estimates, i.e. the non-linear impacts of ore body variabilities are modelled on the designed mining and processing constraints, see Nicholas et al. (2006, 2008) for further detail.

Note that in Figure 48, modelled on an underground diamond mine, the kriged and E-type estimates are relatively positioned in the middle of the conditional simulations as all estimates including the simulations were run through an IEM to capture non-linear reserve impacts on the financial model. This should be compared to case study two discussed in Chapter four (see Figure 16), modelled on an open-pit diamond project where the kriged estimate lies distinctly above the conditional simulations – only the conditional simulations were run through an IEM in that case study.

The NPV of a project is directly influenced by the variability of its reserves in relation to the mine plan and processing constraints, and management's ability to mitigate any negative-impacting issues affecting its business plan.

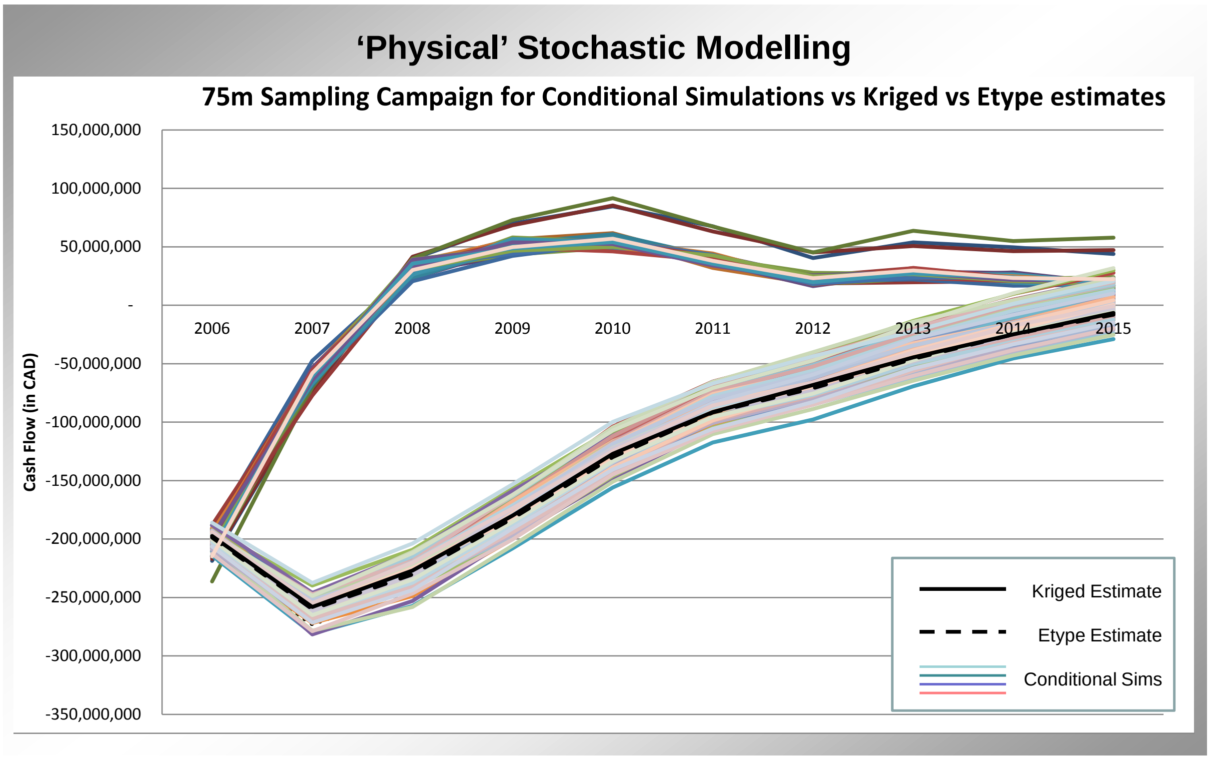


Figure 48 – The 75m sampling campaign depicts the cumulative discounted cash flows (DCF) of the conditional simulations (multi-coloured lines) relative to the kriged and E-type estimates – bottom of diagram. The discounted cash flows (non-cumulative) are shown towards the top of the diagram.

6.3 REAL OPTIONS VALUATION

A high-level overview of Real Options Valuation (ROV) was provided in chapter two of this thesis. This section elaborates upon the appropriate key points.

Copeland and Antikarov (2001) identified six main variables that influence the value of real options:

i. *The value of the underlying risky asset* – in the case of real options, this may be a project, investment or acquisition. As the stock goes up, the price of a call option increases and that of a put option decreases. The reverse is true when the stock goes down. The rate of change is denoted by the symbol δ (Delta).

ii. *The exercise or strike price* – the amount of money invested to exercise the option if you are buying the asset (call option); or the amount of money received if you are selling the option (put option). As the exercise price of an option increases, the value of the call option decreases and the value of the put option increases. This determines the intrinsic value of the option. If the option strike price is OTM (Out-of-the-money) where for a call option, the strike price is higher than the price of the underlying stock, it has no intrinsic value. An option has intrinsic value when it is ITM (In-the-money) where for a call option, the underlying stock price is higher than the strike price.

iii. *The time to expiration of the option* – the value of the option increases as the time to expiration increases; or alternatively, extra money is required to buy more time for the life of the option. The rate of change in time is denoted by the symbol θ (Theta).

iv. *The standard deviation of the value of the underlying risky asset*. The value of an option increases as the volatility of the underlying asset increases – i.e. there is more upside potential. If a stock is volatile, the option prices will be high; if the stock is not volatile, the option prices will be low. Volatility has two components, viz. historical volatility is denoted by the symbol *v* (Vega) and implied volatility ζ (Zeta).

v. *The risk-free rate of interest over the life of an option*. As the risk free rate goes up, the value of the option also increases.

vi. *The dividends that may be paid out by the underlying asset*. Dividend pay-outs will decrease the option value.

When it comes to the modelling of options, these Greek parameters are important to understand in terms of their impact on the option value:

- δ measures the rate at which an option price will change relative to the stock price (or price of the underlying asset), which is equivalent to speed.
- γ measures the rate at which the δ changes with respect to the price of the underlying asset, which is equivalent to acceleration.
- θ measures the rate of change of time value.
- *v* measures the rate of change of the option value versus the parameter sigma.
- ρ measures the sensitivity of the option price relative to changes in the risk free interest rate. This factor usually does not interest option traders.

These parameters measure the sensitivity to change of the option price under a slight change of a single parameter while holding the other parameters fixed. Formally, they are partial derivatives of the option price with respect to the independent variables. Traders will typically make a choice of which parameters to hedge to limit exposure. Financial institutions will usually set limits for the parameters that their trader cannot exceed. δ is the most important Greek and traders will zero their delta at the end of the day, while γ and *v* are also important but not as closely monitored.

The real investment opportunities (or real options) of a mineral project corresponds with the call options on stocks.

Call option on a stock	Real option on a project
Current value of stock	Gross present value of expected cash flows
Exercise or strike price	Investment cost
Time to expiration	Time until opportunity disappears
Stock value uncertainty	Project value uncertainty
Riskless interest rate	Riskless interest rate

Table 14 – Comparison between financial and real options in a mineral project

To preclude arbitrage opportunities, the prices of European call and European put options must satisfy a certain relationship. The put-call parity relationship is defined as the call price plus the present value of the exercise price, which must be equal to the put price plus the current stock price – see Equation 29:

$$w + Ke^{-r(T-t)} = u + S_t$$

w = call price

K = exercise price

u = put price

S_t = current stock price

Equation 29 – Put call parity relationship

Put-call parity is a convenient way to value a call option, given the price of a European put option with the same exercise price and expiration (or vice versa).

Black and Scholes (1973) used stochastic partial differential equations (PDE) to calculate the option price as a function of the change in the share price less the present value of a bank loan based on a fictive portfolio (see Equation 30).

$$w(S,t) = S.N(d_1) - K.e^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\left[\ln\frac{S}{K} + \left(r + 0.5\sigma^2\right)T - t\right]}{\sigma\left(T-t\right)^{0.5}}$$

$$d_2 = d_1 - \sigma\left(T-t\right)^{0.5}$$

Equation 30 – Black and Scholes (1973) equation

Option pricing was originally developed for valuing stock options and other derivatives. Black and Scholes (1973) believed that if options were correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks.

They made the following assumptions to derive their formulae and to simplify the mathematics. The short-term interest rate is known and is constant through time. The Black and Scholes model assumes that the price of the underlying stock follows a geometric Brownian motion with a variance rate proportional to the square of the stock price. Stock prices are assumed to be lognormally distributed, as opposed to normally distributed which would introduce the unrealistic prospect that stock prices could drop by more than 100%. While it is acknowledged that stock prices can rise more than 100%, they cannot drop by more than 100% to create a negative price. Hence, the assumption of lognormality is an important consideration.

The Brownian motion, random walk process is one in which the change in value over any time interval is independent of any changes that have occurred in preceding time intervals, and the size and direction of the changes in value are in some sense random. For stock prices, the applicability of a random walk is based on the assumption that the stock market is efficient, i.e., that the stock price at a given moment reflects all the information available at that moment. Stock prices change for reasons, but changes that are about to happen will be due to new information, which cannot be predicted ahead of time.

Black and Scholes (1973) also assumed that the stock pays no dividends or other distributions. The option can only be exercised at maturity (European option). There are no transaction costs in buying or selling the stock option and there are no penalties to short selling. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate. It is also assumed that there are no liquidity restrictions with respect to trading shares in that it is possible to sell or buy any quantity at any time.

A lognormal random walk (also known as Geometric Brownian motion with a drift) was used to describe the behaviour of the spot price; a fictive portfolio was set-up to mimic reality and the 'no-arbitrage' argument was invoked to equate the return on the portfolio to the riskless rate of return. They found that the trick was to set up an option equivalent by combining stock investment and borrowing. The net cost of buying the option equivalent must equal the value of the option, Brealey and Myers (2003).

Merton (1973) showed that the value of an option is always greater than the value it would have if it were exercised immediately ($S_0 - X$). This general property of the relations between option value and stock price is illustrated in Figure 49 for a call option.

The maximum and minimum option values are shown by the dark blue lines which represent the maximum and minimum value bounds of the option. The three curved lines represent possible option values with different maturity dates at a fixed exercise price of USD50.00. *T1* has a shorter maturity date than *T2*, which in turn expires sooner than *T3*. For a call option the value of the option increases as the stock price increases, at a constant exercise price. When the stock price becomes large, the option value approaches the stock price less the present value (PV) of the exercise price. The value of the option increases with both the rate of interest and time to maturity. The probability of large stock price changes, during the remaining life of an option, depends on the volatility of the stock price per period and the number of periods until the option expires.

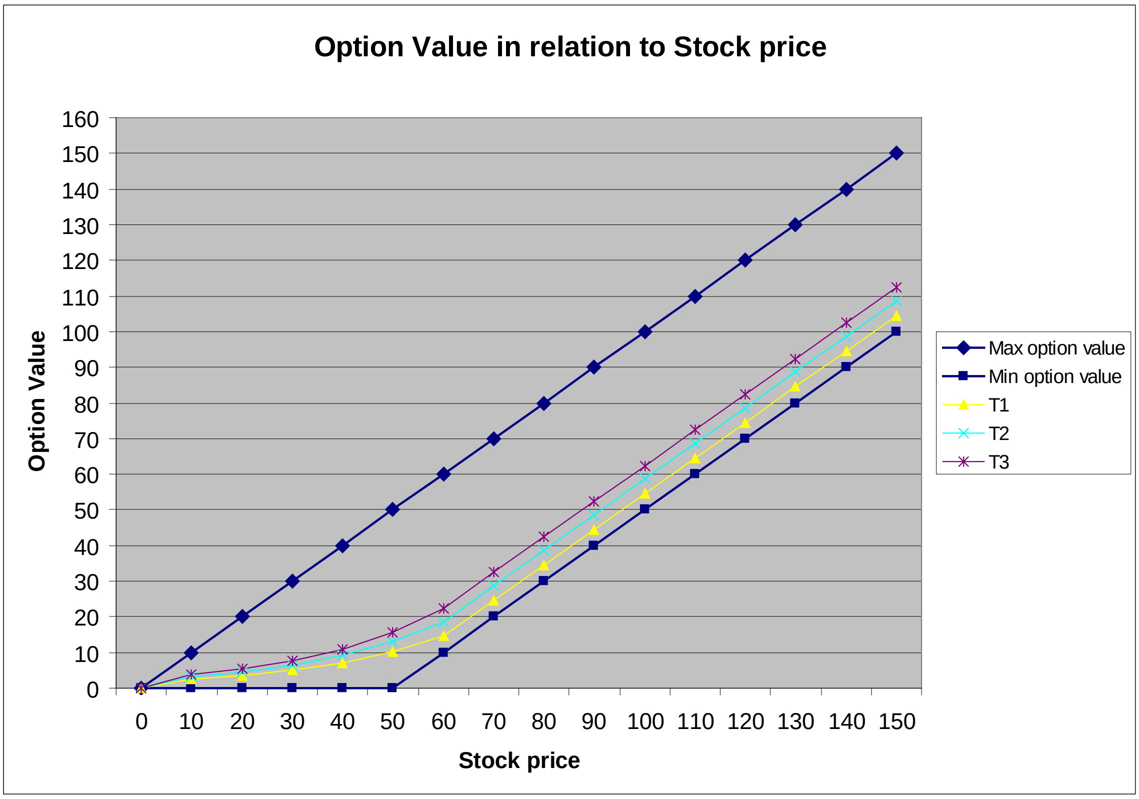


Figure 49 - A call option with a strike price of $50, adapted from Black and Scholes (1973). Note how the value of the option increases as the stock price increases. The opposite is true for a put option.

Using the assumption of 'no arbitrage', the difference between the value of a call and the value of a put option for the same stock (if both could be exercised at maturity) must obey the following boundary condition:

$$w(S_0,t) - u(S_0,t) = S_0 - X$$

which can be rewritten as:

$$w(S_0,t) - u(S_0,t) = S_0 - Xe^{r(t-T)}$$

where $w(S_0,t)$ is the value of a call option

$u(S_0,t)$ is the value of a put option

S_0 is the price of the underlying stock

t is the current time

T is the time to maturity

r is the risk free rate

X is the exercise or strike price

e is the base of natural logarithms with constant = 2.7182818

From the above, the value of the European call and put options, respectively, are:

$$w(S_0, t) = S_0 N(d_1) - Xe^{-r(T-t)} N(d_2)$$

$$u(S_0, t) = Xe^{-r(T-t)} N(-d_2) - S_0 N(-d_1)$$

where $N(d_1)$ and $N(d_2)$ are cumulative normal probability density functions; and

σ^2 is the volatility or variance

σ is the standard deviation

$$d_1 = \frac{\ln(S_t / X) + (r + \sigma^2 / 2)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

Equation 31. European call and put options.

A calculated example of the Black and Scholes call option value is shown in Table 15 for two scenarios, comparing 0% and 8% risk free rates (purely for illustrative purposes), and contrasting these two scenarios in Figure 50 for a range of different volatilities.

Black & Scholes European Call Option Value

X	Starting value of stock	53.3	53.3
r	Risk free rate	0.00%	8.00%
T	total time period	5.00	5.00
σ	volatility (sigma)	0.01	0.01
c	Strike price or Exercise price	50	50

Call = $S_0N(d_1)$ - Xe^{-rT} $N(d_2)$
Put = Xe^{-rT} $N(-d_2)$ - $S_0N(-d_1)$

d1 = ln(S/X) + (r+σ^2/2)T / σ(sqrt T)
d2 = d1 - σ(sqrt T)

d1	unit normal variable d1	2.869	20.758
d2	unit normal variable d2	2.847	20.736
N(d1)	cumulative normal probability of d1	0.998	1.000
N(d2)	cumulative normal probability of d2	0.998	1.000
Call	value of the Call option	**3.301**	**19.784**

Table 15. A calculated example of the Black & Scholes European option value.

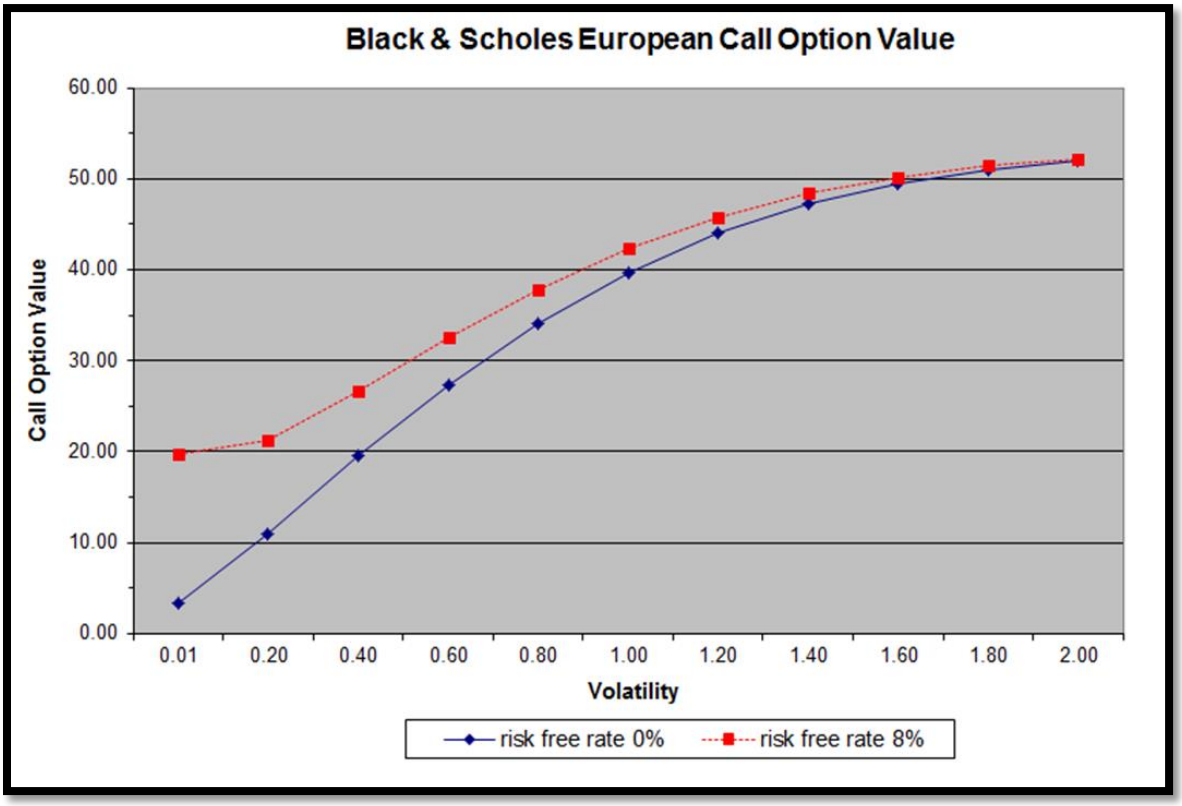


Figure 50. Graph showing the calculated Black & Scholes European call option values (y-axis) for a range of volatilities (x-axis) comparing two scenarios whereby the risk free rates are 0% and 8%, respectively.

6.4 HEDGING STRATEGIES

6.4.1 Overview

Hedging is widely practiced, from huge corporations to small individual FX investors. Armstrong et al (2009) observed that attitudes to hedging practices in the mining industry vary from one sector to another and that, in general, gold miners have been active in hedging whereas diversified groups have shied away from it. Large gold miners and their shareholders typically criticize the practice of forward sales because locking in prices ahead of production closes off opportunities to benefit from a rise in the metal's value. However, it is prudent practice for mid-tier, emerging gold producers to hedge against falling gold prices to secure operating margins, especially where these companies secure debt loans from banks who may insist on mandatory hedging in these circumstances.

Hedging involves using market instruments to offset the risk of any negative movement in price. The easiest way to do this is to hedge an investment with another investment, i.e. by simultaneous buying and selling of the same trading instrument. Hedging is generally defined as holding two or more positions at the same time, where the purpose is to offset the losses in the first position by the gains received from the other position. “Most businesses insure or hedge to reduce risk, not to make money” (Myers, 2003). In some cases large corporates attempt to insure or hedge away operating divisions’ risk exposures by setting-up internal, synthetic (or fictitious) internal markets between each division and the treasurer’s office with the objective of mitigating operating managers’ risks outside their control.

There are several arguments for and against hedging and the rationale behind implementing hedging in practice, which have been raised by Stulz (1984), Smith and Stulz (1985), DeMarzo and Duffie (1995), Graham and Rogers (2002) and Hull (2003). One of the more convincing arguments in favour of corporate hedging is that a hedge can be put in place to protect cash flow margins for systematic variables that are beyond the control of management, and allow a company to focus on those operational issues that are within its control. Often, banks will insist on mandatory hedging (commodity and/or foreign exchange rate) as a condition precedent to lending money, to protect the company’s cash flow margins throughout the loan tenure period (Mineweb, 2013).

The benefits of hedging do not come without costs (Leland, 1998; and Allayannis and Ofek, 2001). These include fixed costs, direct transaction costs and indirect costs for ensuring that managers transact appropriately. Direct costs are related to the costs of trading and include the costs of setting up and maintaining information systems sophisticated enough to track positions. Indirect costs include the need to monitor positions on trading desks to ensure that hedging and speculative positions are within predetermined limits.

Hedging strategies in this research are contemplated from a company/corporation’s perspective rather than from a banker/trader or an individual investor’s perspective. While it could be argued that a company may not need to hedge, as a private investor can mitigate their risks by hedging themselves, in most cases this is not the case. Depending upon the type of hedge instrument used, the quantity of hedging and the tenure, large corporations require fairly substantial banking limits to be put in place before the hedge can be dealt, which are typically beyond the financial reach of private investors.

A financial institution that sells an option to a client in the over-the-counter (OTM) markets is faced with the problem of managing its risk. If the option is the same as one that it is traded on an exchange, the financial institution can neutralise its exposure by buying on the exchange the same option it has sold. However, when the option is more bespoke in terms of meeting the client's needs and does not correspond to the standard products traded by exchanges, hedging the exposure is more difficult (Hull, 2003). One strategy available to the financial institution is to do nothing, referred to as adopting a 'naked' position. An alternative to a 'naked' position is that the financial institution can adopt a 'covered' position, which attempts to mitigate the risk from the perspective of a balancing portfolio theory.

A company's simplistic perspective on FX rates may involve fixing, hedging or keeping the FX rate floating over a specified time period, typically ranging from a few months up to three years. Fixed exchange rates are treated as a permanent (or "nominal flat") over the specified period while the floating exchange rate may drift, up and down, according to certain market trends. Floating FX rates are usually more volatile as they are free to fluctuate over time. The volatility in FX rates results in an increase of exchange rate risk and may adversely affect the economic viability of a company/project.

Hedging strategies presented in this section refer to one of five different foreign exchange scenarios considered by management over the period January 2006 to December 2009. Based on the cash flow model of a diamond mine (presented in case study one of chapter four), this period was considered to be the most sensitive to foreign exchange rate fluctuations when considering the time value of money. Five scenarios are considered:

- A flat nominal foreign exchange rate of 1.21 (reflecting management's over-simplified assumption of an average FX rate over a three year period);
- Actual (historic) foreign exchange rates (from www.FXblog.org/category/canadian-dollar);
- No hedging but stochastic spot foreign exchange rates following the GK model;
- Hedging with zero-cost foreign exchange rate collars; and
- Hedging with calls evaluated using the Garman-Kohlhagen call option models (with an additional consideration for volatility uncertainty in the input parameters using a range of FX strike rates).

For each scenario, the same methodology was employed to consider simultaneously the impact of physical reserve stochasticity with FX rate uncertainty in cash flow and NPV terms. One hundred FX rate scenarios were generated for each month of the mine schedule (for the period 2006 to 2009). A NPV was generated for each of the 100 conditionally simulated reserve realisations (one for each FX rate considered) and the full NPV distribution captured. Similarly, 100 NPV were generated for the E-type estimate, Kriged estimate and VBod to consider each of the five FX rate scenarios.

6.4.2 FX Rate Models

Many models have been developed for interest rate and foreign exchange rates, ranging from simple extensions of Black and Scholes (1973) to Vasicek (1997) and to the latest models with stochastic volatility. The author chose the Garman and Kohlhagen (GK) model (1983) which is a simple extension of Black and Scholes, as the risk-neutral process for the stock price is:

$$dS = (r - q)Sdt + \sigma SdW$$

Equation 32 – Risk-neutral valuation from Black and Scholes with dividend returns, q.

In a risk-neutral world the total return from the stock would be r; and the dividends would provide a return of q. Hence, the expected growth rate, or drift, in the stock price is $r - q$. S, the spot exchange rate (which is the value of one unit of the foreign currency measured in the domestic currency) is, by virtue of satisfying Equation 32, a geometric Brownian motion process similar to that assumed for stocks.

In this model the drift term is replaced by the difference between the domestic and foreign interest rates. The stochastic process for an exchange rate is the same as setting $q = r_f$. If S_t denotes the spot exchange rate at time t and r_d and r_f are the domestic and foreign interest rates, then (see Equation 33):

$$dS_t = \left(r_d - r_f\right) S_t dt + \sigma_s S_t dW_t$$

where S_t = spot exchange rate at time t; and r_d = domestic interest rate;

r_f = foreign interest rate; dW_t = Brownian motion element; and

σ_s = volatility of the exchange rate;

defined as $\sigma_s = \frac{\sigma_{stdev}}{\sqrt{P}}$ representing the annualized volatility;

σ_{stdev} = standard deviation of the returns; and

$\sqrt{P}$ = square root of the time period, P, of the returns

Equation 33. Garman Kohlhagen (1983) equation.

The approach of not imposing a hedging programme and exposing a mining project to the variable spot FX rate is a deliberate management strategy and in the context of this chapter, should be perceived as one of the scenarios that are available to management when considering the economic viability of a project (or company). FX stochasticity was modelled using a Garman and Kohlhagen (1983) model. A total of 100 simulations were run over a 10-year period emulating the FX uncertainty to generate monthly FX rates (cf. Figure 51).

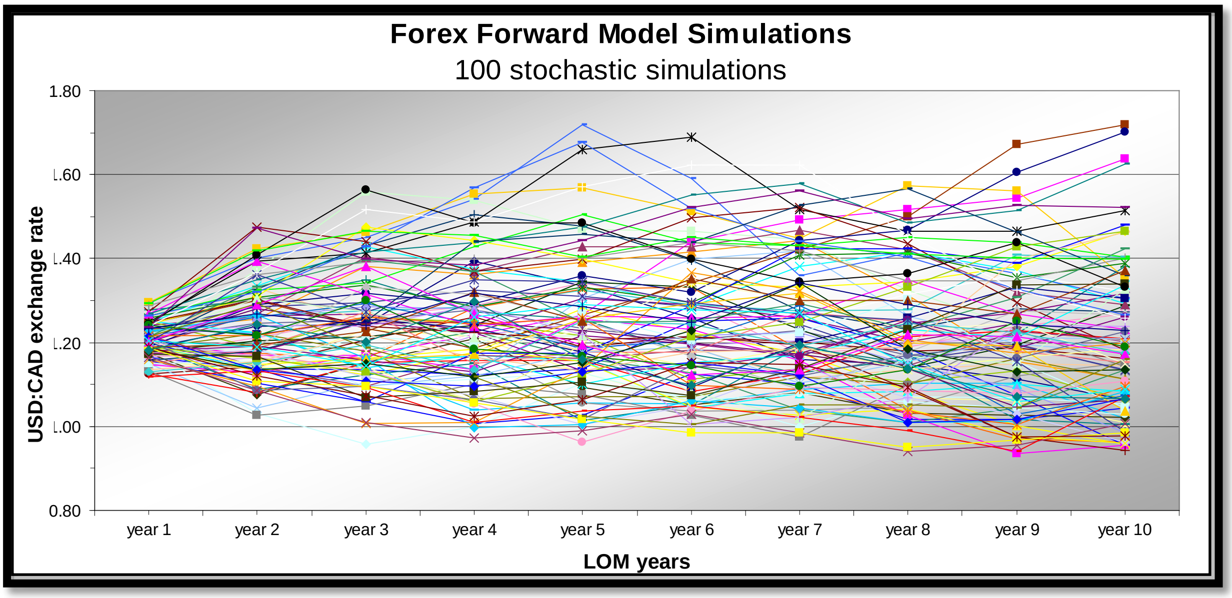


Figure 51 – Spot FX rate generated from the Garman-Kohlhagen options model, year 1 (2006) to year 10 (2015).

Figure 52 depicts the impact of running variable spot FX rates through the mining DCF algorithm to derive NPVs for each of the 100 FX rate scenarios. Note that because there is no hedging strategy imposed in this strategy, the operational cash flows are fully exposed to both the downside and upside scenarios (shown by the 'unconcatenated' NPV probability distribution).

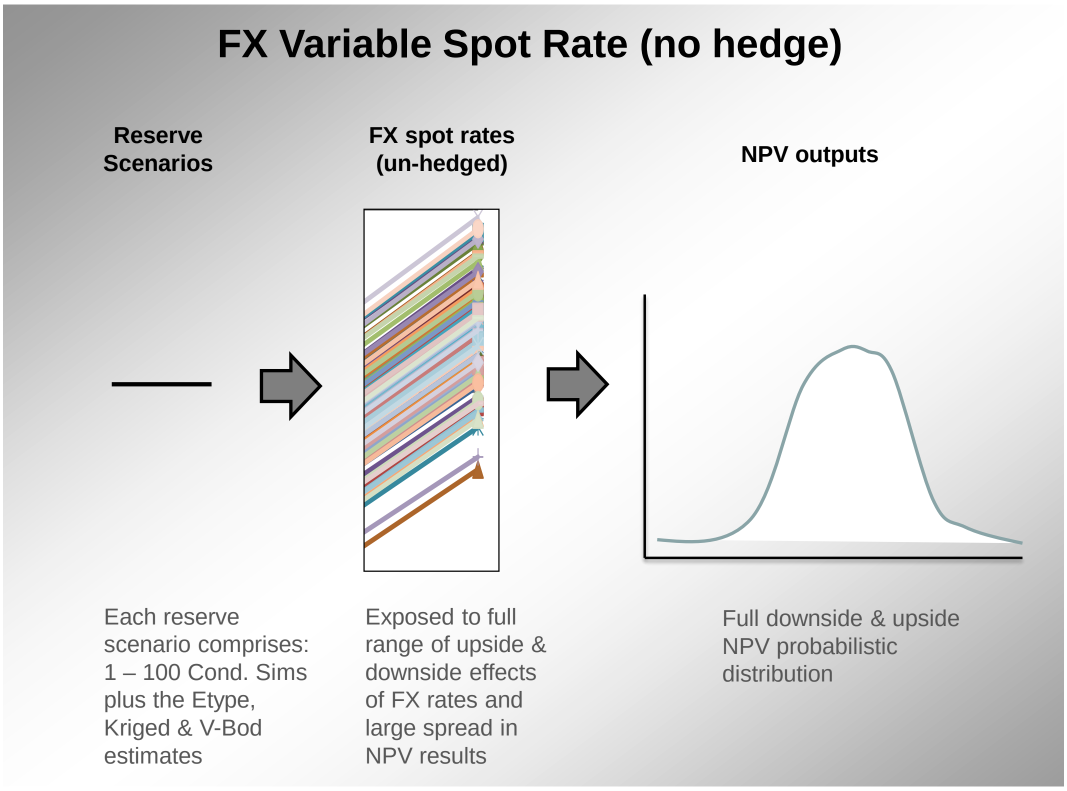


Figure 52 - Graphical representation of the impact of exposing a project to the variable spot FX rate

Mark Garman and Steven Kohlhagen founded the Garman Kohlhagen model in 1976. This is an analytic valuation model for European options on currencies using an approach similar to that used by Merton for European options on dividend-paying stocks. Two advantages of this model are that the exchange rates generated are lognormally distributed and hence positive, and that the parameters are easy to estimate.

The call option for the G-K model is defined in Equation 34.

Call G-K model

$$c = \left(S_t * e^{-r_f *(T-t)} \right).N(d_1) - \left(K * e^{-r_d *(T-t)} \right).N(d_2)$$

where

$$d_1 = \frac{\left(\ln \frac{S_t}{K} + (r_d - r_f) * (T-t) \right)}{\left(\sigma * \sqrt{(T-t)} \right)} + \frac{\left(\sigma * \sqrt{(T-t)} \right)}{2}$$

or

$$d_1 = \frac{\left(\ln \frac{S_t}{K} + (r_d - r_f) * (T-t) \right)}{\left(\sigma * \sqrt{(T-t)} \right)} + \frac{1}{2\left(\sigma * \sqrt{(T-t)} \right)}$$

$$d_2 = d_1 - \left(\sigma * \sqrt{(T-t)} \right)$$

S_t is the current spot exchange rate (domestic currency per unit of foreign currency)
K is the strike exchange rate
N is the cumulative normal distribution function
r_d is the domestic risk free interest rate
r_f is the foreign risk free interest rate
T is the time to maturity/expiration of the option (in years)
σ is the implied volatility for the underlying exchange rate

Equation 34 – Call option for the Garman Kohlhagen (1983) equation

The Garman Kohlhagen model is based on a number of assumptions:

- The distribution of terminal currency exchange rate (returns) is lognormal.
- There are no arbitrage possibilities.
- Transactions cost and taxes are zero.
- The risk-free interest rates, the foreign interest rates, and the exchange rate volatility are known functions of time over the life of the option.
- There are no penalties for short sales of currencies.
- The market operates continuously and the exchange rates follows a *continuous Ito process*.

An example of deriving Garman Kohlhagen call option values from Equation 34 is shown in Table 16 and graphically depicted in Figure 53.

Garman Kohlhagen Call Option Value		
S	current spot exchange rate	1.20
rd	domestic risk free interest rate (p.a)	3.87%
rf	foreign risk free interest rate (p.a)	4.35%
T	total hedge time period	3.00
t	evaluation time period (months)	1.00
σ	volatility of exchange rate	11.77%
K	strike exchange rate	1.00
d1	unit normal variable d1	0.93
d2	unit normal variable d2	0.72
N(d1)	cumulative normal probability of d1	0.82
N(d2)	cumulative normal probability of d2	0.76
Call	Value of one Call option	$0.185
	Total dollars (millions): Call option	18.549

Table 16. An example of a calculated Garman-Kohlhagen call option value, referring to the Canadian domestic interest rate and the foreign interest rate referring to the USA. The call option value is shown in millions of Canadian dollars based on $100m of production cash flows.

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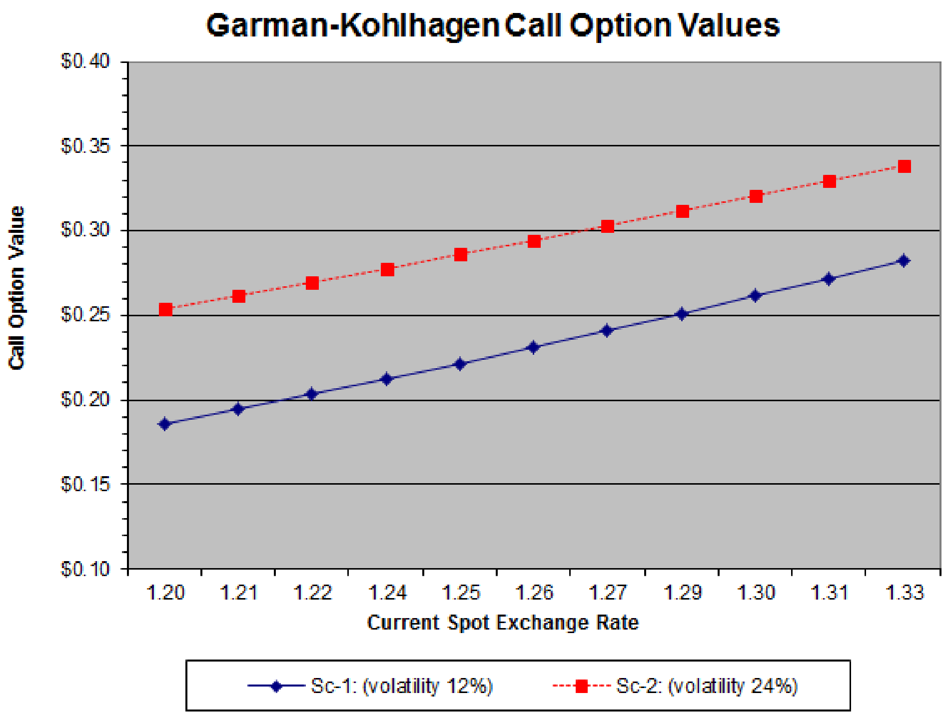

Figure 53. An example of Garman-Kohlhagen call option values for two scenarios, Sc-1 at a volatility of 12% and Sc-2 at 24%. In both scenarios the values of the call options increase with the current spot exchange rate.

Figure 54 depicts the scenario in which for each reserve scenario, a G-K call option model is put in place that limits the downside risk exposure to the cash flows. The net result is that the downside exposure in the NPV probability distribution is capped according to the agreed strike rate, whereby the call option is exercised when the agreed strike rate is reached. The G-K option comes at a cost (based on the option call price premium) which needs to be deducted from the overall NPV of the project. It is worthwhile noting that this type of hedging will need to be declared to shareholders and will need to appear on the balance sheets of a company.

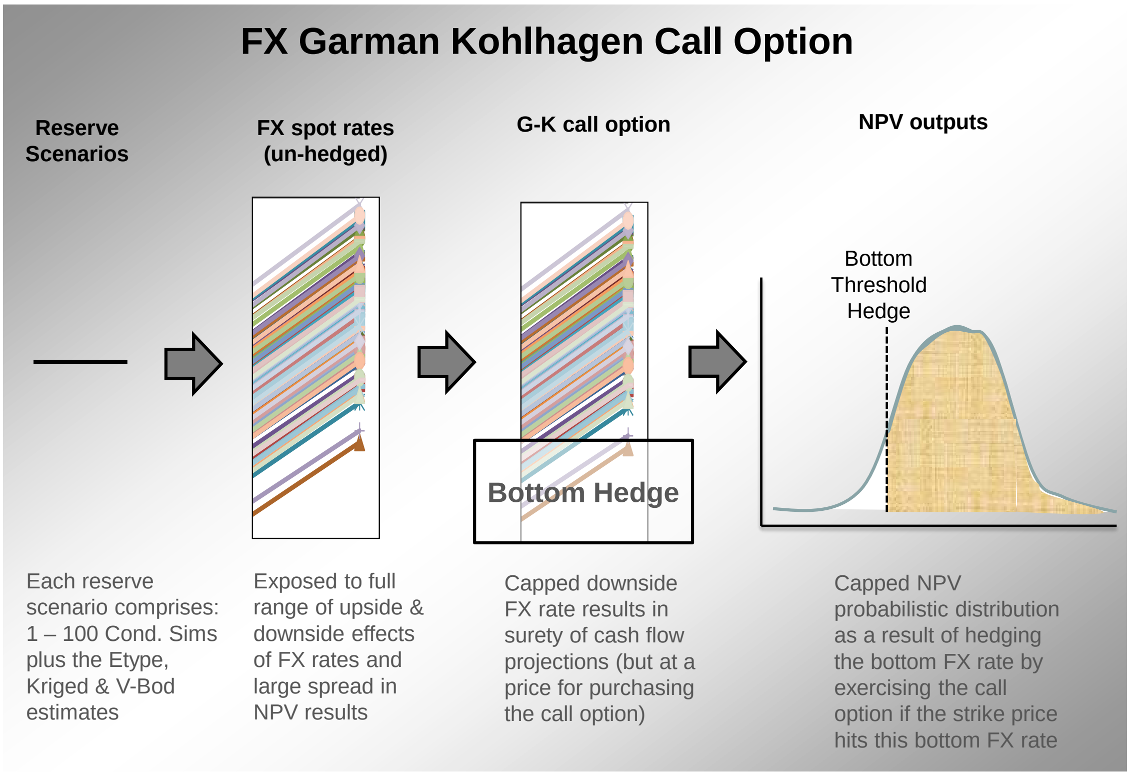

Figure 54 – Graphical representation of the impact of zero-cost collars on the FX rate

The zero-cost collar, or costless collar, is established by buying a protective put (called 'the floor') while writing or selling an 'out-of-the-money' covered call (called 'the cap') with a strike price at which the premium received is equal to the premium of the protective put purchased, The Options Guide (2012) and Financial Review (2012). Zero-cost collars can be established to fully protect existing long stock positions with little or no cost since the premium paid for the protective puts is offset by the premiums received for writing the covered calls, or stated in another way, the premium income from selling the call reduces the cost of purchasing the put. The amount saved depends on the strike price of the two options. If the premium of the long call is exactly equal to the cost of the put, the strategy is known as a "zero cost collar".

By establishing a zero-cost collar hedge strategy, a long-term stockholder will sacrifice any profit if the stock price appreciates beyond the striking price of the call written. However, this hedge strategy provides maximum downside protection. In a FX rate zero-cost collar, the investor seeks to limit exposure to changing foreign exchange rates and at the same time lower its net premium obligations.

For the purposes of this study, a company management strategy implements a zero-cost collar for the FX rate over a three year period by 'giving away' the upside advantage where the USD FX rate appreciates above a specified threshold ($x + S1$) but simultaneously provides downside protection where the USD FX rate drops below a specified threshold ($x - S2$). In this instance the value of x is deemed to be equal to parity (1.00 USD:CAD). Here $S1$ is the maximum benefit of a favorable move in FX rates and $S2$ is the maximum tolerable unfavorable change in payable FX rate.

The reader is reminded that the example referred to in this section is that of a diamond mining company earning its revenue in USD and converting the majority of it back into its domestic CAD currency to service its operational costs. This hedging strategy assumes that a mining company implements its FX hedge as a 'once-off' strategy locked-in for a three-year period rather than 'rolling' its hedge each month or quarter. The 'once-off' hedging provides greater certainty and 'smoothing' of cash flows protecting the company from FX rate volatility for the duration of the three-year period, but on the other hand, rolling its FX hedges every quarter could provide greater flexibility in establishing the FX rate collar thresholds at that time. There is no clear solution and depends on management's overall strategy and economics for the project.

One source of risk not mentioned thus far is counterparty risk. If the stock price expires below the 'floor' of the downside threshold then the counterparty may default on the put contract, thus creating the potential for losses up to the full value of the stock (plus fees). If not properly managed, options can pose significant risk to a banking institution because options held by the bank are usually exercised at the advantage of the holder which could be to the disadvantage of the bank (see Houpt and Embersit, 1991; Wright and Houpt, 1996; and Angbazo, 1996 for more information on risks to banks).

Figure 55 depicts the impact of implementing a zero cost FX rate collar on the mining DCF algorithm to derive NPVs for each of the 100 FX rate scenarios. Unlike the variable spot FX rate, where there was no hedging strategy imposed, the operational cash flows are not fully exposed to downside and upside risks (shown by the ‘capped’ NPV probability distribution). This provides management with a degree of certainty that the project cash flows will be protected from FX rate volatility for the specified period of three years.

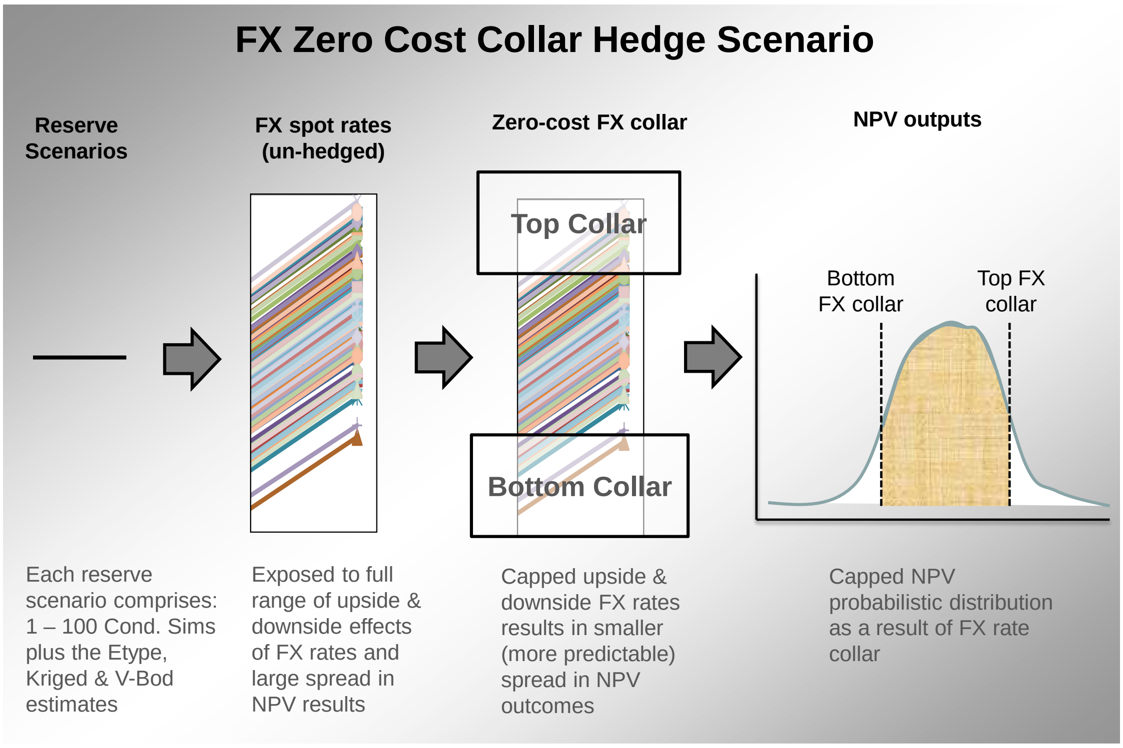

Figure 55 - Graphical representation of the impact of implementing a zero-cost collar on the FX rate

Finally, one of the key advantages of using a collar strategy is that it takes the return from the probable to the definite (Financial Review, 2012). It is presumed that when an investor owns a stock (or another underlying asset) and has an expected return, that expected return is only the mean of the distribution of possible returns, weighted by their probability. The investor may get a higher or lower return.

When an investor who owns a stock (or other underlying asset) uses a collar strategy, the investor knows that the return can be no higher than the return defined by strike price on the call, and no lower than the return that results from the strike price of the put. This theory is based on the presumption that there are no other stochastic input variables that materially affect returns. In the application of a zero-cost collar hedge strategy to this study, it should be noted that this hedge strategy merely reduces the number of probabilistic outcomes (rather than providing ‘definite’ returns) by ‘capping’ the downside and upside FX rate scenarios as there are several other stochastic variables considered to estimate the NPV returns.

There are several other forms of protective hedging strategies similar to the zero-cost collar option strategy, such as bull spreads, bear spreads, butterfly spreads, calendar spreads, strangles and straddles which are beyond the scope of this study (the reader is referred to McMillan, 2002 and Hull, 2003 for further information).

6.5 OPTION MODELLING

This section discusses the input parameters, modelling assumptions and model outputs used to evaluate management hedging strategies, considering both physical reserve and economic uncertainties.

Volatility is an important factor involved in the decision making of investors and policy makers with high volatility of the underlying risk asset increasing the value of the option, see Davis (1998), Copeland and Antikarov (2001) and Hull (2003) for more. Volatility is typically defined as the standard deviation of the change in value of the log return of a financial instrument with a specific time horizon, used to quantify risk of the financial instrument over that period. Davis (1998) and Trigeorgis (1990) note that miss-estimation of the input volatility parameter can have a material impact on the cost of the option.

Historical volatility is the volatility of a financial instrument based on historical returns, typically annualized and can be represented by the formulas noted in Equation 35.

$$\text{volatility} = std\left(\ln\left(\frac{Q_t}{Q_{t-1}}\right)\right)$$

true if returns are conditionally homoskedastic

$$\text{volatility} = std_{t-1}\left(\ln\left(\frac{Q_t}{Q_{t-1}}\right)\right)$$

true if returns are conditionally heteroskedastic

(volatility at time t - 1 represents the stdev of time t log return, conditional on information at time t - 1)

Equation 35 – volatility equations for homoskedastic and heteroskedastic models

A univariate stochastic process Q is said to be homoskedastic if standard deviations of terms Q_t are constant for all times t, otherwise it is said to be heteroskedastic (Holton, 2004b). Heteroskedasticity can take two forms. A process is unconditionally heteroskedastic if unconditional standard deviations σ^t are not constant. It is conditionally heteroskedastic if conditional standard deviations $\sigma^{t|t-1}$ are not constant. Heteroskedasticity is important in finance because asset returns in capital, commodity and energy markets usually exhibit heteroskedasticity. Given that FX rates exhibit non-constant volatility, but periods of low or high volatility are not known in advance, FX rates appear to be conditionally heteroskedastic.

Using an econometrics approach, an autoregressive conditional heteroskedastic (ARCH) model considers the variance of the current error term to be a function of the variance of the previous time period's error terms. ARCH relates the error variance to the square of a previous period's error. This methodology may be used to model FX rates. If an autoregressive moving average model is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedastic (GARCH) model.

In the available literature of modelling time series volatility, the authenticity and the popularity of ARCH family (GARCH, EGARCH and stochastic volatility models) is recognized universally. According to Engle (1982) and Bollerslev (1986), time series' models are more reliable for capturing the volatility in financial time series as these models are specifically designed for volatility modelling. Hsieh (1989) used 10 years (1974 – 1983) of daily closing-bid prices, consisting of 2,510 observations, for five countries in comparison with the US dollar to estimate the autoregressive conditionally heteroskedastic (ARCH) and generalized autoregressive conditionally heteroskedastic (GARCH) models along with the other modified/altered types of ARCH and GARCH.

Methods such as the exponentially weighted moving average (EWMA) use weighting schemes to give more weight to recent data. In this instance, the weights decrease exponentially as a function of time. In practice, variance rates tend to be more mean reverting, Hull (2003). The GARCH model incorporates more mean reversion (recognizing that over time the variance tends to get pulled back to a long-run average level), whereas the EWMA model does not, and hence, may be more appealing in theory.

For this research, a simplified method of estimating volatility was used to calculate the instantaneous standard deviation of historic changes Davis (1998) and Hull (2003) based on unweighted 30-day average data (Equation 36).

$$\sigma_s = \sqrt{\frac{\tau}{n-1} * \sum_{i=1}^{n} \left(u_i - \bar{u}\right)^2}$$

where $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ and i=1 .. n

τ = the number of observation periods per year

i = the observation number

$\bar{u}$ = the mean log relative rates $\bar{u}$

$$where,\ \bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

Equation 36 – The instantaneous standard deviation model to calculate historic volatility

An important consideration in Equation 36 is that volatility increases as the time interval increases (usually not in proportion), which is a key aspect of the random walk theory. The standard deviation scales (increases) in proportion to the square root of time as defined in Equation 37.

$$\sigma = \frac{\sigma_{stdev}}{\sqrt{P}}$$

$$\sigma_T = \sigma\sqrt{T}$$

where σ = annualized volatility

σ_{stdev} = standard deviation of the returns

$\sqrt{P}$ = square root of the time period, P, of the returns

σ_T = generalized volatility for time horizon, T

For Example:

$$\sigma_{\text{year}} = \sigma_{day}\sqrt{252}$$

$$\sigma_{\text{day}} = \frac{\sigma_{year}}{\sqrt{252}}$$

assuming 252 trading days per year to calculate volatilies

Equation 37 – Calculation of daily volatilities as a function of the annualized volatility.

Equation 36 and Equation 37 were used to derive the input volatility parameter into the G-K options model based on historical monthly FX rates from January 2003 to December 2005 over a three year period in order to predict the input parameter for the G-K options model. A volatility parameter of 11.77% was derived and used as an input to generate call option prices on a monthly basis (over a three year period commencing from January, 2006).

The alternative to using a historical volatility rate is using an implied volatility rate, which involves 'back-calculating' the price of an option (call or put). In the case of a call option actively traded on the underlying stock, the option's price is back-calculated to derive the implied volatility rate. However, there is only a very small likelihood that a similarly priced option exists on the market if the option has been customized to suit the client, i.e. not a vanilla styled put or call option but a hybrid of several options to make the option as appealing as possible to the client. Hence, the use of an implied volatility is impractical in this instance.

Due consideration was given to the utilisation of a 'volatility smile' which compares the implied distribution and lognormal distribution for valuing foreign currency options (Hull, 2003). It is well known that the implied distribution has heavier tails than the lognormal distribution, which results in the implied distribution giving a relatively higher price for the option. Hull examined the daily movements in 12 different exchange rates over a 10-year period, and concluded that his evidence supported the existence of heavy tails and the volatility smile used by traders. The lognormal model generally under-estimates the standard deviation of the foreign exchange rates, relative to the real world data, beyond one standard deviation away from the mean.

Davis (1998) notes that miss-estimation of the input volatility parameter can have a material impact on the cost of the option. Trigeorgis (2002) showed that a 50% increase in the input volatility parameter resulted in a 40% increase in the option value. Many authors on this topic recognized the importance of the volatility parameter in their options modelling and produced option pricing as a function of a range in volatility parameters. To demonstrate the impact of volatility on the option price, this author used the Garman and Kohlhagen model to generate a G-K call option as a function of increasing volatilities (ranging from 12% - 116%, increasing by increments of 10% each time) and for a range of strike prices on the FX rate (from 1.00 to 1.10) **–** see Figure 56.

The colours in the 3-D surface grade from reddish-brown to pink as the call option price increases relative to the strike rate and volatilities. If volatility is kept constant at 12% with a FX strike rate of 1.00, the option price is CAD 0.19 and decreases to CAD 0.12 at a strike rate of 1.10 (the differential between the strike and spot rate is reduced). If the FX strike rate is held constant at 1.00, the option price increases from CAD 0.19 to CAD 0.75, for volatilities of 10% and 116%, respectively. This option pricing 3-D surface model clearly demonstrates the impact of both volatilities and management strategy (in selecting the appropriate strike rate relative to the spot rate) on the option price.

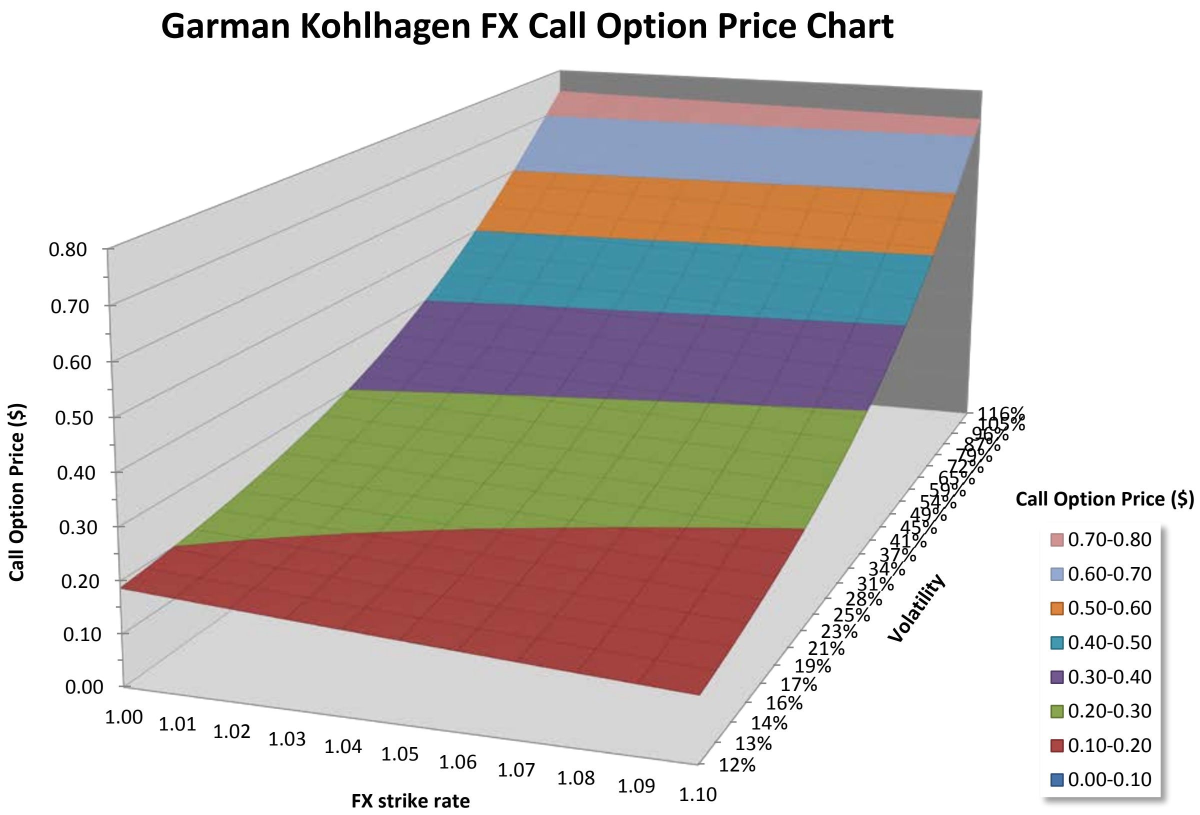

Figure 56 – Garman Kohlahagen **c**all option pricing in 3-D surface view as a function of the two input parameters, FX strike rate and volatility.

Risk free rates are important considerations for input into the G-K option pricing model, and were sourced for the periods 2006 to 2009. The author examined both the three-year and 10-year yield curves to evaluate the impact on the G-K model outputs. The three-year monthly rate as at January 2006 was 3.87% and 4.35% for Canada and the USA, respectively. The 10-year rate was 4.11% and 4.42 for Canada and the USA, respectively. Both scenarios were run through the G-K model and only a negligible difference of 1% was noted in the calculated option price. Rates were derived from the Treasury Board of Canada, (Trading Economics, 2012) in Figure 57 and Figure 58 below (the author notes that figures depicting three year bonds was unavailable from this website at the time).

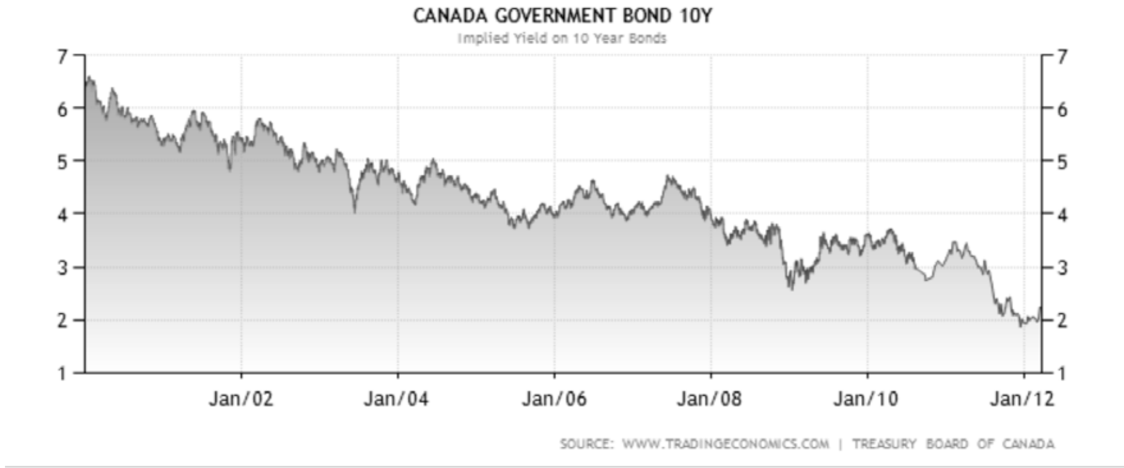

Figure 57 – Risk free interest rate for Canadian 10 year government bonds (Trading Economics, 2012).

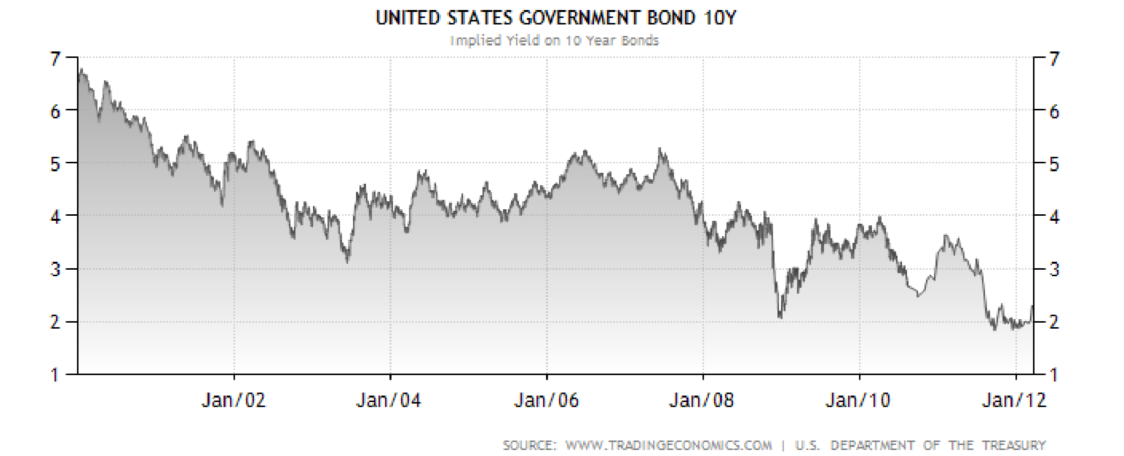

Figure 58 - Risk free interest rate for USA 10 year government bonds, (Trading Economics, 2012).

The current spot rate (as at January, 2006) was 1.20. The three-year bond yields as at January 2006 were used as inputs for the interest rates (3.87% for Canada and 4.35% for the USA).

6.6 ANALYSIS

A total of 100 FX simulations were generated using the G-K model (see Figure 51). The financial impact of running the variable spot FX rate through the IEM to derive the NPVs for each of the 100 conditional simulations is depicted in Figure 59, with the NPV (in Canadian dollars) shown on the y-axis and each of the 100 FX scenarios on the x-axis. It can be observed that the kriged estimate is marginally above (greater than) the VBod for each of the 100 FX scenarios (shown on the x-axis). Note that the FX rates were allowed to vary unconstrained when calculating the NPVs for each scenario.

The multi-coloured lines are spread evenly on either side of the kriged and VBod scenarios for all 100 FX scenarios. Over the first three years, FX scenario 4 has the lowest average FX rate of 1.06 USD:CAD generating the lowest NPV of -CAD86.7 million (with an average NPV of -CAD55.4 million for the minimum values of the 100 conditional simulations), which explains why almost all the conditionally simulated scenarios lie below the horizontal (x-axis). On the other side of the spectrum, FX scenario 89 has the highest average FX rate of 1.40 generating the highest NPV of CAD121.5 million (with an average NPV of CAD89.3 million for the maximum of the 100 conditional simulations).

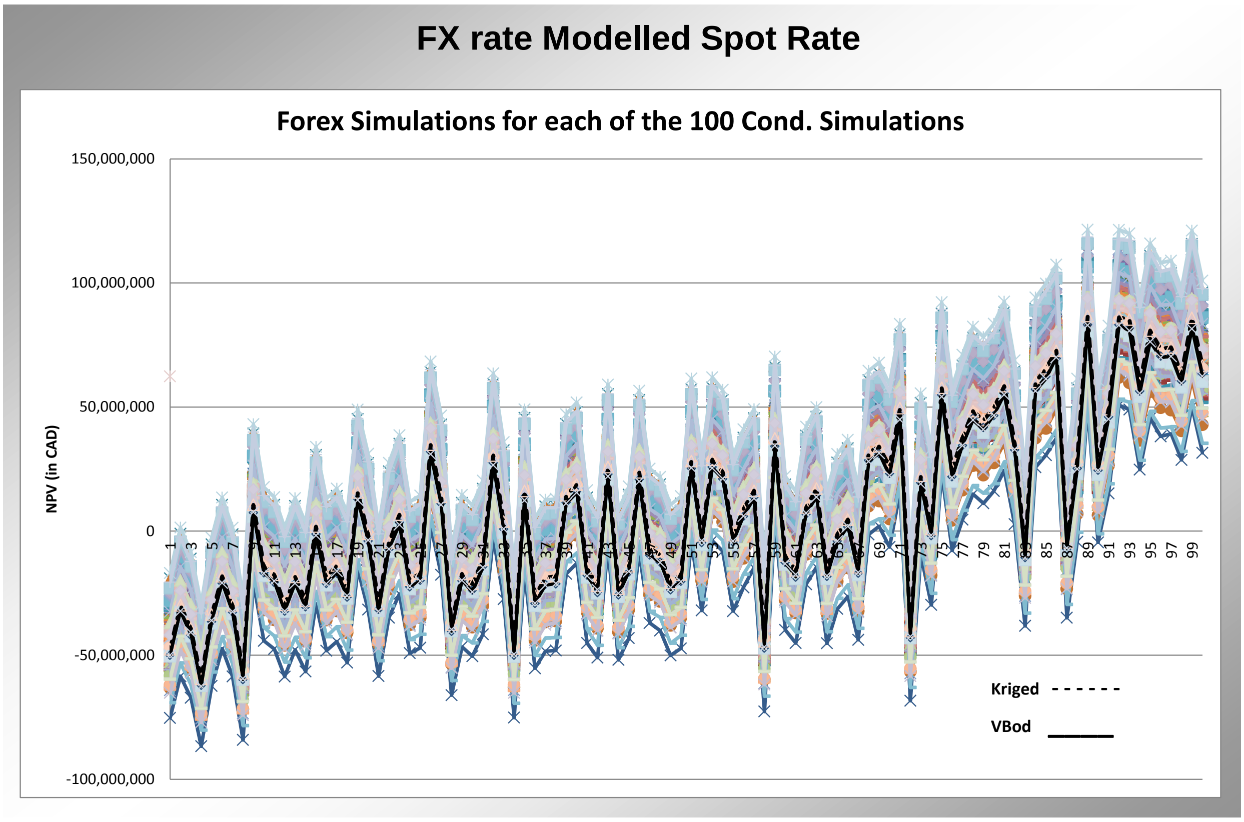

Figure 59 – FX modelled spot rate for each of the 100 conditional simulations. The conditional simulations are shown by the multi-coloured lines while the solid black line represents the VBod scenario, and the kriged estimate is presented by the black dashed line.

It is important to note that the average NPV estimates were derived by running each spatially conditionally simulated realisation of the ore body through the IEM, and mapping it to the stochastically generated FX rates to generate 100 NPV. The average of these 100 NPV was then compared relative to each other. The derivation of these average NPV should be compared with the NPV of the E-type estimate, where the latter was derived from calculating the P50 (fiftieth percentile) of the 100 resource values in the block model.

This E-type estimate was used to derive the NPV as a function of the stochastic FX rates. The NPV for the E-type estimate is -CAD62 million (on average 12% less); similarly for FX scenario 89, the E-type NPV estimate is CAD83 million. It was observed in each of the 100 FX rate scenarios that the E-type estimate 'on average' under-estimated the average NPV derived from the full conditionally simulated distributions by circa 9% (with a significant CoV of 1,410% indicating its high variability).

Figure 60 shows the results of classical statistics performed on conditionally simulated NPV outputs after considering resource uncertainty and FX stochasticity for variable FX rates.

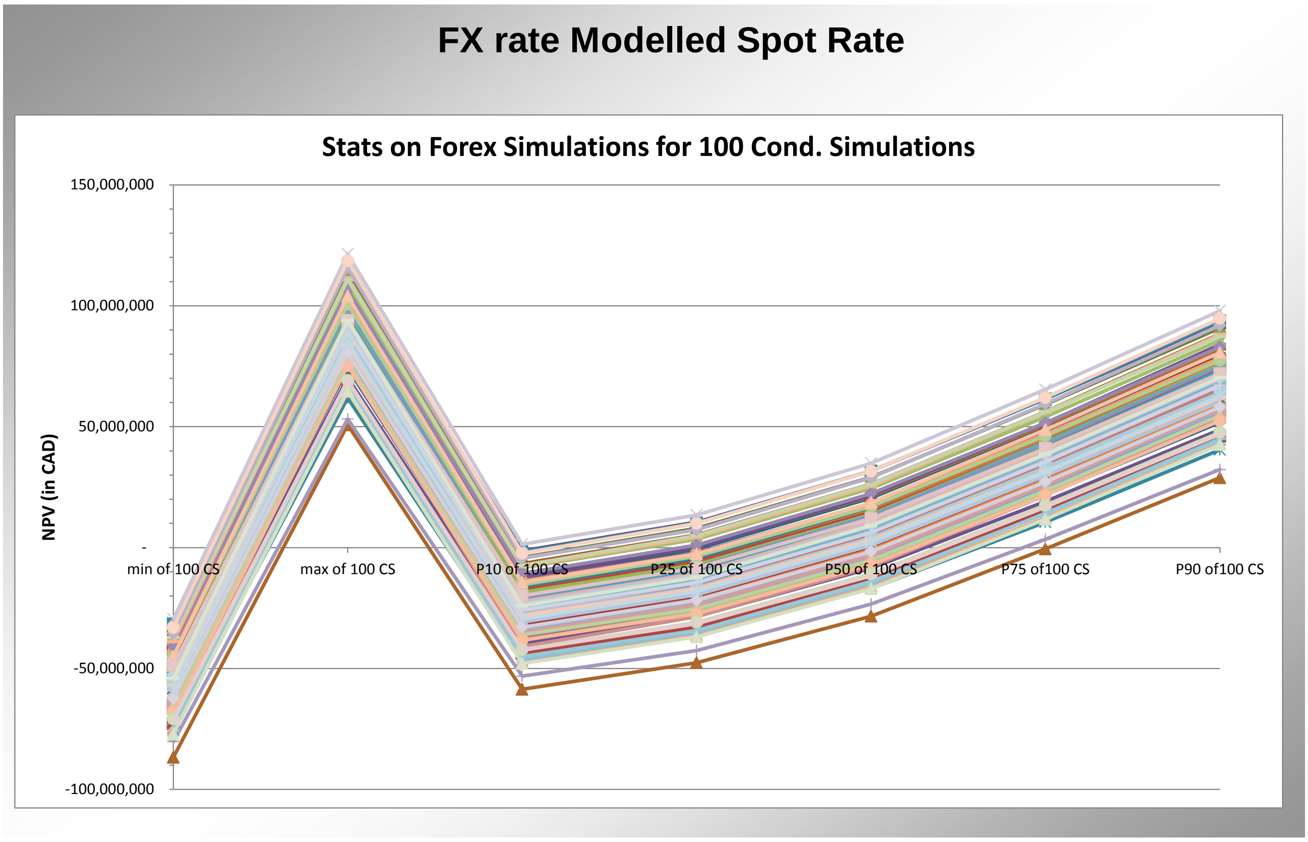

Figure 60 – Basic statistics for the FX spot modelled rate.

From Figure 60, it can be observed that the minimum NPV ranges between -CAD86.7 million to -CAD29.5 million with a mean of -CAD55.4 million. The maximum NPV ranges from CAD50.8 million to CAD121.5 million with a mean of CAD89.3 million. Hence, the full spread of possible NPV outcomes ranges from -CAD86.7 million to CAD121.5 million with the average of the 50th percentile (P50) NPVs equal to CAD13.4 million.

Figure 61 depicts the financial impact of running a three year FX rate collar hedge through the IEM to derive the NPVs for each of the 100 conditional simulations, with the NPV (in Canadian dollars) shown on the y-axis and each of the 100 FX scenarios on the x-axis. A bottom FX rate collar of 1.00 and top collar of 1.25 were used to constrain the range in which the FX rates could vary when calculating the NPVs.

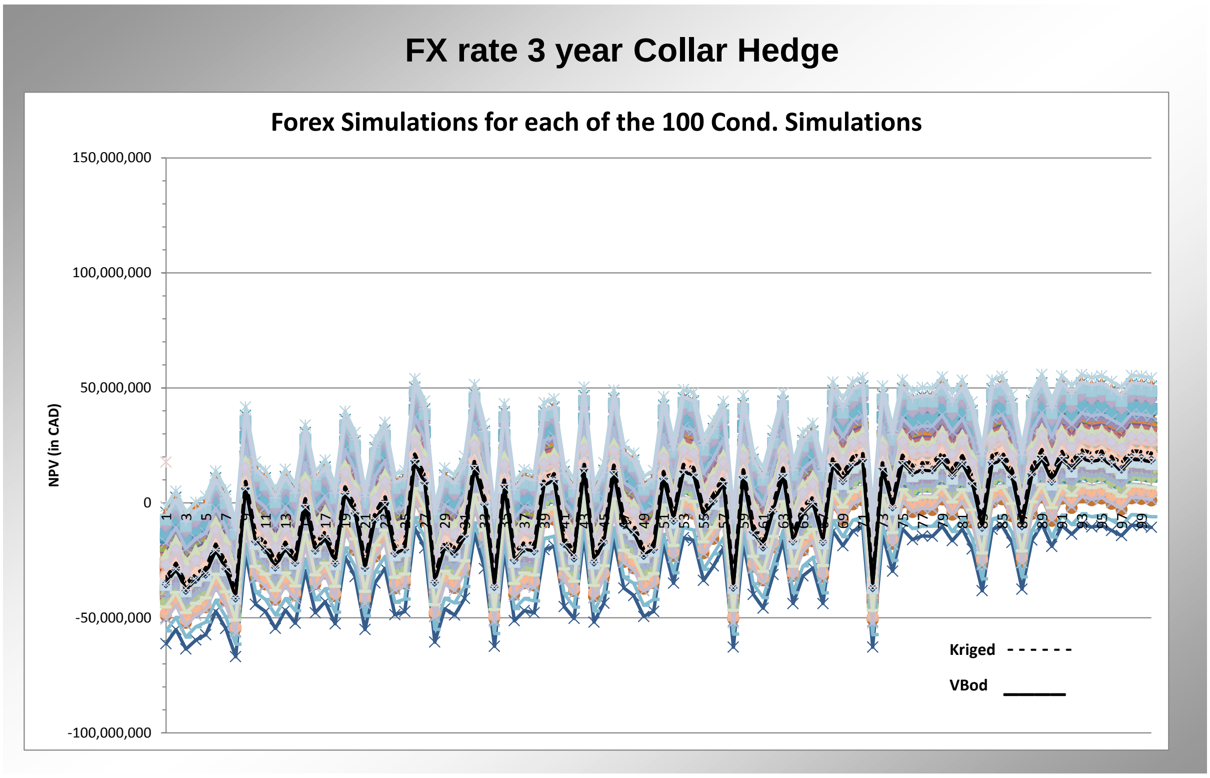

Figure 61 – FX three year hedge option output for each of the 100 conditional simulations. The conditional simulations are shown by the multi-coloured lines while the solid black line represents the VBod scenario, and the kriged estimate is presented by the black dashed line.

Unlike the wider spread of NPV shown in Figure 59 (with a range in NPV of -CAD30 million to CAD121 million), the majority of the NPV in Figure 61 lie approximately between -CAD50 million and CAD50 million. FX scenario 8 has the lowest average FX rate of 1.07 USD:CAD generating the lowest NPV of -CAD66.9 million (with an average NPV of -CAD34.8 million for the minimum values of 100 conditional simulations). FX scenario 89 has the highest average FX rate of 1.25 generating the highest NPV of CAD55.9 million (at an average NPV of CAD26.3 million for the maximum of 100 conditional simulations).

Figure 62 highlights the basic statistical data carried out on the conditionally simulated NPV outputs after considering both resource uncertainty and FX stochasticity for variable FX rates.

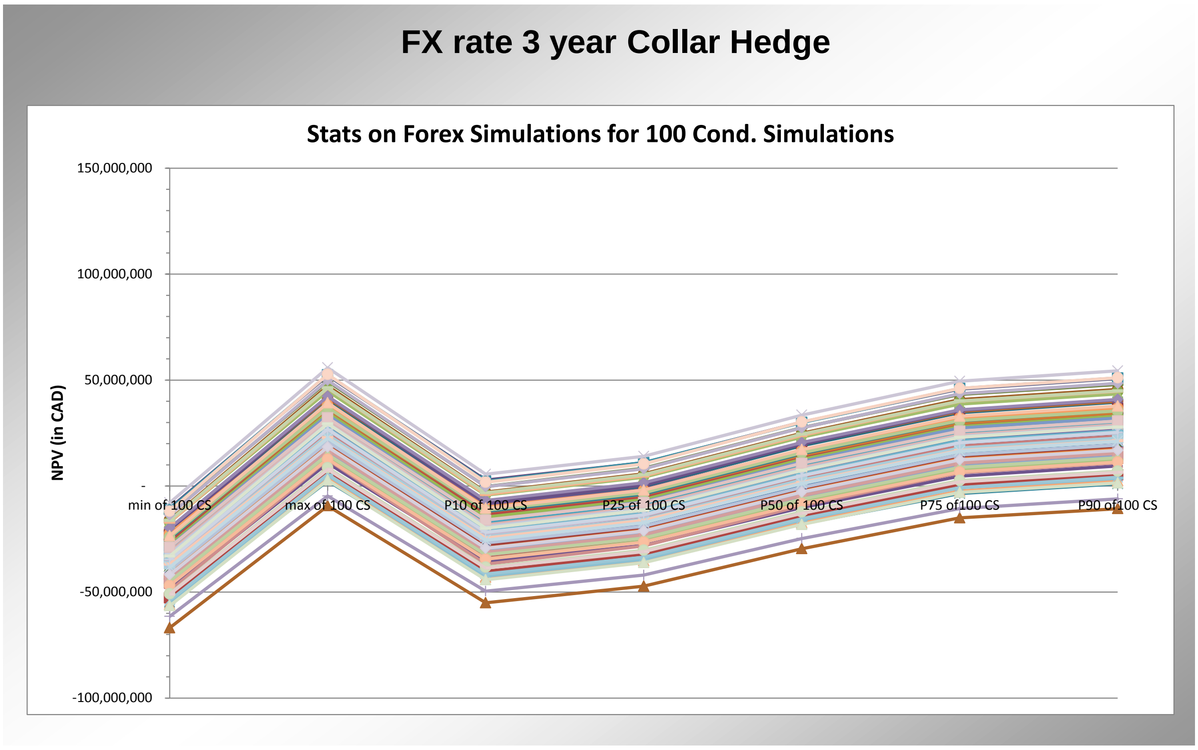

Figure 62 – Basic statistics for the FX rate three year collar hedge.

Due to the bottom and top capping implemented through the FX rate three year hedge, the minimum, maximum and percentile ranges (P50 – P90) depicted in Figure 62 are much closer together (tighter spread) compared to the variable spot FX rate scenario in Figure 60.

From Figure 62, it can be seen that the minimum NPV ranges from -CAD66.9 million to -CAD7.9 million with a mean of -CAD34.8 million. The maximum NPV ranges from -CAD9.3 million to CAD55.9 million with a mean of CAD26.3 million. The full spread of possible NPV outcomes extends from -CAD66.9 million to CAD55.9 million with the average of the 50th percentile (P50) NPV equal to CAD2.9 million.

Figure 63 highlights the financial impact of implementing a hedge using the Garman Kohlhagen (G-K) FX rate model through the IEM to derive the NPVs for each of the 100 conditional simulations, with the NPV (in Canadian dollars) shown on the y-axis and each of the 100 FX scenarios on the x-axis. To generate the NPVs for each scenario, an FX strike rate of 1.00 was used with no cap imposed on the higher FX rates.

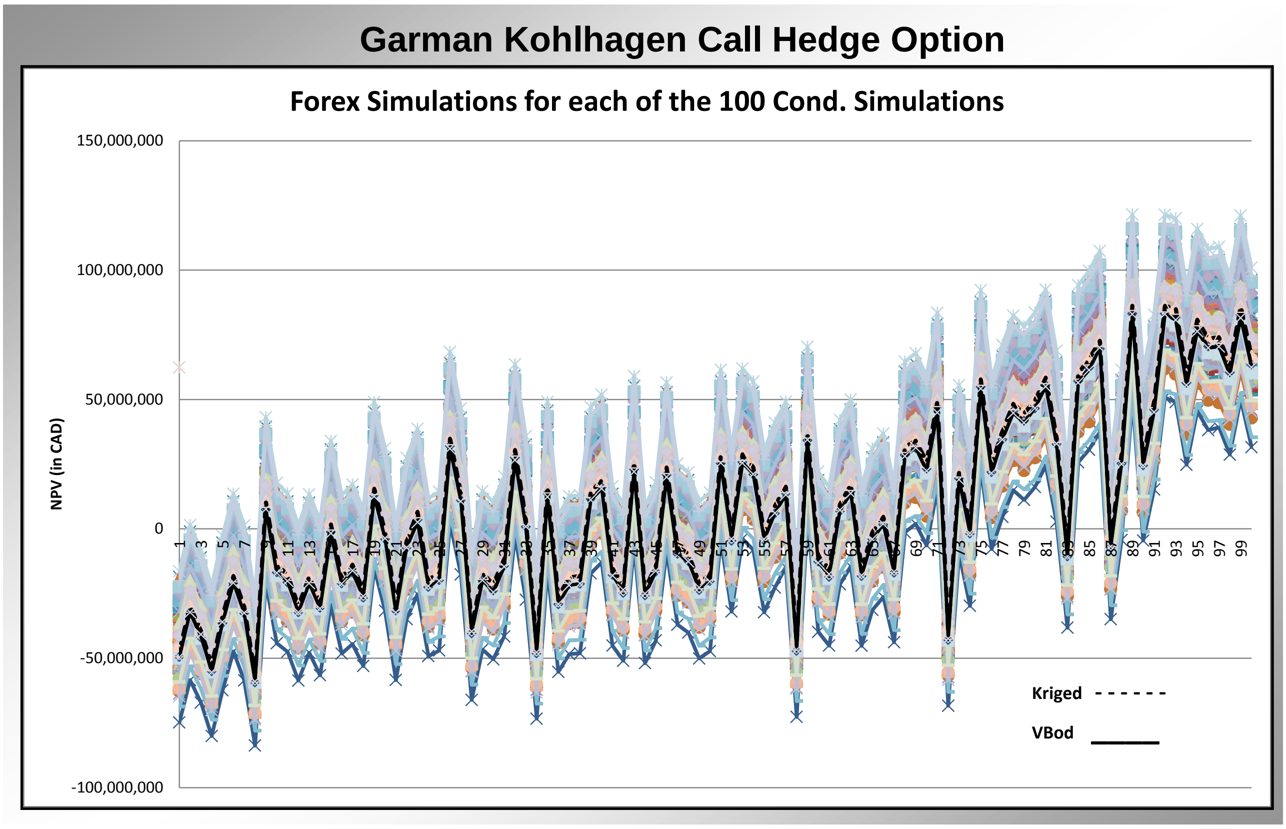

Figure 63 – Garman Kohlhagen call hedge option output for each of the 100 conditional simulations. The conditional simulations are shown by the multi-coloured lines while the solid black line represents the VBod scenario, and the kriged estimate is presented by the black dashed line.

While the spread and trend in NPV in this chart may appear similar to that of Figure 59, the majority of the NPV in Figure 63 derived from the conditional simulations (multi-coloured lines), lie approximately between -CAD83.7 million and CAD121.5 million. FX scenario 8 had the lowest average FX rate of 1.07 USD:CAD generating the lowest NPV of -CAD83.7 million (with an average NPV of -CAD52.6 million for the minimum values of the 100 conditional simulations). FX scenario 92 had the highest average FX rate of 1.41 generating the highest NPV of CAD121.5 million (with an average NPV of CAD88.9 million for the maximum values of the 100 conditional simulations).

Figure 64 highlights the results of basic statistical data carried out on the conditionally simulated NPV outputs after considering both resource uncertainty and FX stochasticity for variable FX rates.

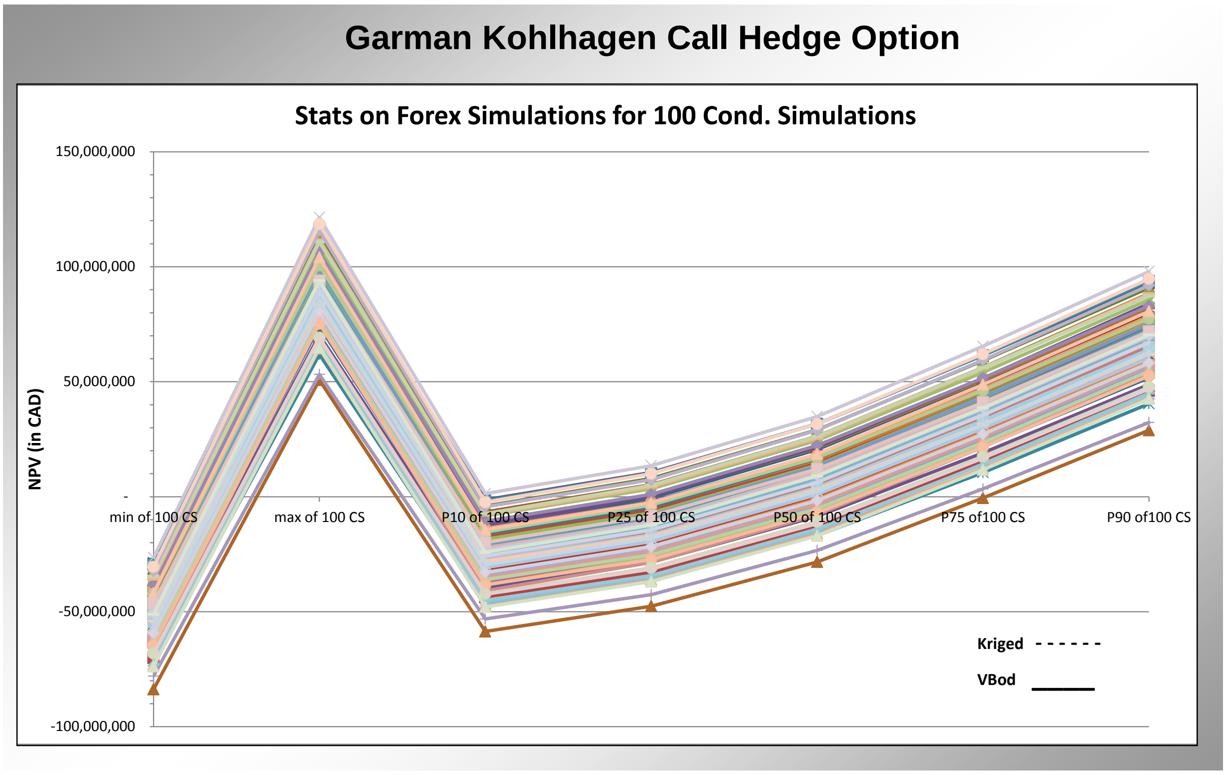

Figure 64 – Basic statistics for the Garman Kohlhagen call hedge option

It can be observed from Figure 64 that the minimum NPV ranges between -CAD83.7 million to -CAD26.4 million with a mean of -CAD52.6 million. The maximum NPV ranges from CAD50.8 million to CAD121.5 million with a mean of CAD89.3 million. Hence, the full spread of possible NPV outcomes ranges from -CAD83.7 million to CAD121.5 million with the average of the 50th percentile (P50) NPV equal to CAD13.5 million.

To facilitate comparisons, Table 17 lists key differences in NPV terms for the conditionally simulated outcomes between the three FX rate hedging structures, viz. the variable spot rate, the three year collar and the Garman Kohlahagen (GK) option model.

FX rate Hedged vs Un-hedged Scenarios
(for 100 Conditional Simulations)

	Variable Spot Rate	3yr Collar Hedge	GK Option
Minimum NPV	-86.7	-83.7	-83.7
Average Min NPV	-55.4	-52.6	-52.6
Maximum NPV	121.5	55.9	121.5
Average Max NPV	89.3	26.3	89.3
Average P50 NPV	13.4	1.5	13.5

Table 17 – Comparison between three hedging structures for the conditionally simulated NPV outputs (in millions of Canadian dollars), viz. the variable spot rate, the three year FX collar hedge and the Garman Kohlagen (GK) Option model.

From Table 17, there is a clear distinction between the NPV outputs of the variable spot rate and the three-year collar hedge; and similarly between the three-year collar hedge and the GK option model. The highlighted cells in blue in the table emphasise the same NPV outputs between scenarios. This is to be expected, and further confirms the calculation consistencies in the model, as the "3yr Collar Hedge" had the same bottom threshold as the "GK Option" and shows the same minimum NPV outputs. However, unlike the "3yr Collar Hedge" that had an upside threshold, the "GK Option" was uncapped and hence produced the same maximum NPV outputs as the un-hedged "Variable Spot Rate".

The 'average' referred to in the table above is derived as a function of calculating the average of all the average NPV across the 100 conditional simulations, which in turn has been derived from the matrix of NPV for each of the 100 conditional simulations run through the 100 FX rates. Thus, the 'average' in the table refers to a much smoothed 'average of an average' NPV output. The Average P50 NPV for the "GK Option" is marginally higher than the "Variable Spot Rate" but notably higher than the "3yr Collar Hedge".

The average P50 NPV is higher for the un-hedged variable spot rate scenario than the three-year hedged collar scenario, which may imply that it would be preferable to remain un-hedged for the first three years. However, it is recognised that the rationale supporting the implementation of a FX rate hedge is not to maximize the upside NPV but to minimize the downside scenario. There is, however, a financial cost to a mineral company for minimizing this downside scenario.

The cost of implementing a FX three-year collar hedge may be interpreted in two ways. Firstly, the mineral company will require a line of credit (or "credit limit") with a bank or financial institution. This means that the provider of the hedging strategy will need to review a mineral company's financial statements and assess the health of the company to determine whether a credit line should be extended.

If the mineral company is deemed to be a 'wholesale client' of the bank and already has an existing banking credit limit; or the mineral company wanting the hedge is prepared to place a cash deposit with the bank for an equivalent amount to the required credit limit, there is a good likelihood that the company would have credit limits extended to it to allow the hedge to be placed. This would allow the mineral company to deal the three-year FX collar hedge without incurring any material additional hedging costs.

Secondly, however, if a mineral company does not have an existing line of credit with a bank, then the cost of implementing a credit limit to deal a FX rate hedge must be calculated. A general 'rule-of-thumb' for banks to calculate the credit limit is provided below:

$$C_l = \sigma * \sqrt{t} * FV$$

where

C_l = credit limit requirement

σ = volatility

t = time period

FV = face value of transaction

Equation 38 – 'Rule-of-thumb' calculation of the credit limit requirement for banks

If a volatility input of 11.77% is used with time equal to three years and a face value of circa CAD230.0 million (which is the P50 of the total revenue generated over the first three years from the 100 conditional simulations), then a credit limit requirement of CAD47.8 million is derived from the equation above. This represents approximately 21% of the cost of capital for the face value of the transaction. It should be noted that this limit of CAD47.8 million may be interpreted as the maximum credit amount required as usually the credit limit will decrease over time according to an amortised profile for the three year tenure of the hedge (for simplicity in this section, the full CAD47.8 million will be considered in hedging scenario comparisons).

Typically, this credit limit is an internal risk requirement and does not necessarily imply that that a mining company has to place the full equivalent amount on deposit as a cash-backing to allow the FX hedge to be placed. It is a bank's credit risk regulatory function that requires the credit limit allocation but it is also the bank's commercial discretion as to whether this is waived or not to determine whether hedging is allowed to be placed as an unsecured (no asset security or 'cash backing' serving as collateral) or a secured transaction. These considerations will influence the overall cost and NPV (shown previously in Table 17) for determining the economic viability of implementing the zero-cost hedging strategy.

At face value, the NPV outputs between the un-hedged variable spot rate and the GK option model appear relatively similar (less than 4% difference between the minimum NPV and 5% between the average minimum NPV). However, it should be noted that the NPV displayed in Table 17 excludes the actual cost of hedging the FX rate using the GK option model, which was calculated according to Equation 34. The actual cost of hedging the FX rate to convert revenue generated in USD to CAD is displayed in Table 18.

Cost of a Garman Kohlhagen Call Hedge Option

	Volatility = 12% and FX strike rate = 1.00			
	Total	**2006**	**2007**	**2008**
Min	208,555,419	70,672,549	65,720,020	72,162,849
P50	230,027,757	77,310,202	74,754,655	77,962,901
Max	247,507,951	82,783,797	81,543,701	83,180,453
	Volatility = 116% and FX strike rate = 1.00			
	Total	**2006**	**2007**	**2008**
Min	834,481,869	282,778,367	262,962,073	288,741,428
P50	920,398,011	309,337,260	299,111,882	311,948,869
Max	990,340,593	331,238,468	326,276,539	332,825,586

Table 18 – Calculated cost (in dollars) of hedging the Garman Kohlhagen call option for the three-year period (2006 – 2008) at minimum and maximum modelled volatilities of 12% (top) and 116% (bottom), respectively.

The minimum, maximum and fiftieth percentile costs of the GK hedging options shown in Table 18 were generated from multiplying the GK costs (shown previously as a 3-D surface view in Figure 56) by the range of revenues derived from running all 100 conditionally simulated outputs through the IEM. It is apparent from Table 18 that the total cost for the GK hedge option ranges from a minimum of CAD208.6 million (at a volatility of 12%) to a maximum of CAD990.3 million (at a volatility of 116%). These GK option costs represent approximately 18% of the minimum revenue (CAD1,152 million), to 77% of the maximum revenue (CAD1,294 million) for the three year period 2006 to 2008.

It can be deduced that the NPV for the GK option model scenario, after deducting the GK hedge costs are substantially less than the un-hedged variable spot rate scenario and less viable than the three-year zero-cost collar hedge. The GK NPV for the E-type estimate is CAD6.7 million less the CAD208.6 million costs (at a minimum volatility of 12%) for dealing the hedge, which results in a net loss of -CAD201.8 million. Similar sized net losses would be applicable to the conditional simulations outputs, the kriged estimate and the VBod model. The GK option model is not the preferred solution in this case.

Table 19 summarises the main differences between various hedging strategies. All NPV shown in the table for the GK call FX option exclude costs for hedging the NPV option. The values in Table 19 for the Avg. FX rate column depend on the scenario modelled. For each of the five hedging scenarios, a matrix of FX rates was generated for each month of the year (over a three-year period) and for each of the 100 conditional simulations; then the average across every row was calculated for all of the 100 conditional simulations. Finally, the average of these 100 values was derived, referred to as the Avg. FX rate.

	NPV Comparison for 100 Cond. Sims & 100 FX sims							
	NPV's in millions				FX rate statistics			
	Cond. Sims	Etype	Kriged	Vbod	Avg. FX rate	CoV %	min	max
Actual FX rate	-$46.31	-$53.13	-$50.18	-$51.42	1.09	6.72%	0.97	1.23
3yr Collar 1.10	$2.40	-$3.98	-$1.19	-$4.12	1.19	2.54%	1.10	1.25
3yr Collar 1.00	$1.04	-$5.35	-$2.57	-$5.48	1.19	2.89%	1.00	1.25
Fixed FX rate	$8.79	$2.46	$5.27	$2.10	1.21	-	-	-
Modelled spot FX	$12.88	$6.58	$9.42	$6.41	1.22	4.56%	0.93	1.61
GK call FX option	$12.93	$6.68	$9.51	$6.50	1.22	4.53%	1.00	1.61

Table 19 – Comparison of hedging strategies in NPV terms (in millions of Canadian dollars) relative to the E-type, Kriged estimate and VBod. Note that in each case, the mean NPV is shown (while in the case of the conditional simulations, the mean is calculated from the mean NPV over 100 conditional simulations).

Table 19 compares two subtly different FX collar hedging scenarios with each other where the bottom collar threshold in the first scenario is fixed at a FX rate of 1.10 compared to 1.00 for the second scenario (the latter rate corresponds with the bottom threshold of the GK option). The higher bottom collar of 1.10 provides an improved NPV of -CAD4.1 million for the VBod scenario as opposed to -CAD5.5 million for the 1.00 FX rate. However, the higher bottom threshold of the collar (at a rate of 1.10) is likely be more expensive to protect the minimum downside cash flow scenario, and financial institutions implementing this hedge may require more of the upside potential in the FX rate. For pragmatic reasons, the collar using the FX rate of 1.00 was compared with the GK call option hedging scenario as they both used the same bottom threshold.

The Modelled spot FX rate has an average rate of 1.22, resulting in a lower NPV than the GK call option output, also at an average FX rate of 1.22. The reason for this only becomes apparent by expressing the coefficient of variation (CV) as a percentage (4.56%) which is marginally higher than that of the GK option CV (4.53%). This is not the highest CV of the five scenarios even though the actual spread for the Modelled Spot FX rate is greater than that of the Actual FX rate although the latter has a greater CV of 6.72%. In the case of the Actual FX rate, the mean is materially lower than the Modelled Spot FX rate resulting in a higher CV.

Figure 65 supports the results discussed in Table 19 and shows why running the actual FX rates through the IEM resulted in the lowest NPV compared to the modelled FX outputs. The first ten simulated FX rate outputs for the Garman Kohlhagen (G-K) model and FX Collar (with a bottom floor of 1.00 and ceiling of 1.25 USD:CAD FX rate) are compared with the actual FX rates for this period. It can be observed that the actual FX rates (black dashed series) are visibly lower than the modelled rates and also portray higher variance compared to the smoother modelled FX outputs, bearing in mind that the FX models were generated from raw FX data prior to 2006. In general the FX Collar rates (blue series) are located relatively in the middle of the graphed data (due to its capped floor and ceiling rates) while the G-K rates (pink-orange series) have a wider spread. Only the first ten (of 100 each) simulated outputs of the G-K and FX Collar models are shown due to the sheer size of showing all data.

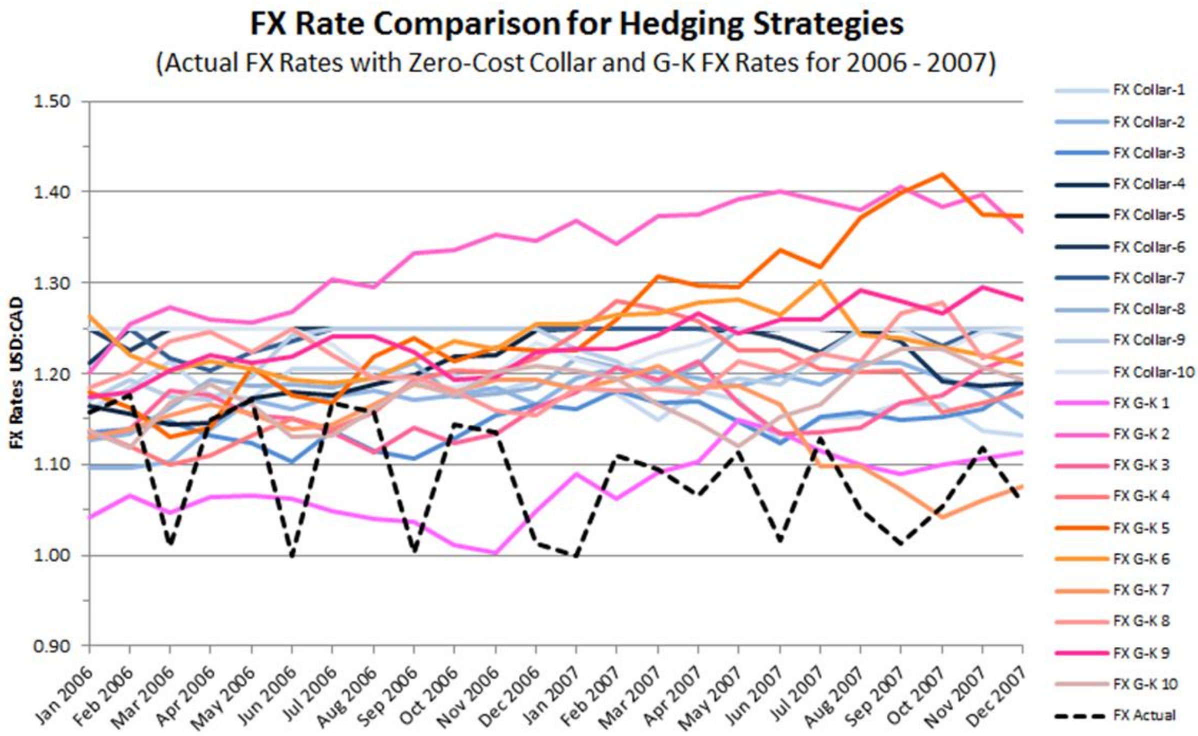

Figure 65. Comparison of actual FX rates for the period 2006 – 2007 (black dashed series) with the first ten simulated outputs for the Garman Kohlhagen (G-K) model (pink-orange series) and the Zero-Cost Collar Rates (blue series). In general the FX Collar rates are located mostly in the middle (due to its capped floor and ceiling rates) while the G-K rates have a wider spread. Actual FX rates appear more variable and lower than the modelled FX outputs.

6.7 CONCLUSIONS

As the DCF NPV calculation includes the time value of money, the FX rate in earlier years for the Modelled spot FX rate has a lower rate on average than the GK call option output even though in later years the FX rate increases. The statistical mean of the FX rate over the entire cash flows in this case is less significant than the variability of the rate during the earlier periods of the project. The focus of a hedging strategy should not simply be on breaking even over a specified time period by balancing wins and losses in each financial year from hedging the foreign exchange rate in dollar terms. It should focus on cash flows over time, and in particular, on the higher risk periods in the life of mine schedule to protect the downside scenario.

When the costs of hedging are considered in the calculated NPV estimate, it is clear that the GK option is too expensive to make financial sense in this instance as the total loss to the mine ranges from -CAD202 million to -CAD240 million (at a modelled volatility of 12%).

One possibility is hedging only 50% of the FX rate (as opposed to 100% in this model) and exposing the project to the variable spot rate. However, it could be deduced in this instance that the GK option would still be the most uneconomically viable scenario due to the excessive costs associated with this type of option hedge.

If the NPV for the GK option (CAD9.51 million) is ignored because of the higher costs associated with implementing a GK hedge, the kriged NPV estimate provides the second-highest return of CAD9.42 million if the project is exposed solely to the variable spot FX rate for the first three years. However, the E-type estimate derived from the conditional simulations, has a NPV of CAD6.58 million which is closer to the VBod NPV of CAD6.41 million; while the mean NPV for the conditional simulations is CAD12.88 million. This represents a total variation of 3% - 101% from the VBod NPV based on the variable spot FX rate, which would represent a significant challenge to management to implement an appropriate hedging strategy.

The impact of the Actual FX rate on calculating the NPV is apparent in that the mine runs at a distinct loss, ranging from -CAD46 million to -CAD53 million. The real NPV based on running the Actual FX rate through the VBod revealed a loss of -CAD51 million. The VBod represents reality so it can be argued that the true value of this project ranges from -CAD51 million to CAD6.50 million depending on whether FX hedges were implemented or not. In the absence of perfect information, i.e. assuming that comparisons with the VBod and actual FX rates were not possible in the real world, an interpretation of the NPV from Table 19 may provide an interesting, yet incorrect, conclusion with respect to whether a mineral company should hedge or not. After the GK option, the kriged NPV estimate provided the second-highest return of CAD9.42 million if no hedge was put in place, and instead the project was exposed to the variable spot FX rate for the first three years.

When the cost of the GK call option is deducted from the NPV estimate, it may appear as if the kriged modelled (variable) FX spot rate is the preferred hedging strategy. The impact of the actual un-hedged FX rate volatility for the kriged estimate was circa 5.3 times lower than the modelled volatility and resulted in a worst-case NPV compared to the other scenarios. The preferred scenario is to hedge using the zero-cost collar, which would limit losses to a mean NPV of negative CAD5.5 million for the VBod model (as opposed to a loss of negative CAD51.4 million if the project was exposed to the actual variable FX spot rate). Once unsystematic risks are taken into consideration, the NPV from the mean of the conditional simulations for the modelled FX spot rate is the highest (post consideration of the GK option costs) at CAD12.88 million, which may further influence management's decision not to hedge.

Cost and revenue estimates within a mineral projects' cash flow are derived from a reserve estimate, which may result in the project (or company) having material exposure to commodity prices and/or FX rates if reserve variability is greater than anticipated, negatively impacting production targets. This affects cash flow projections and could result in significant financial losses if an inappropriate hedging strategy is implemented. A key advantage of hedging is that it provides a smoothing effect of cash flows that delivers certainty and less volatility to management when evaluating the overall economic viability of a mineral project.

Finally, the economic threshold should be evaluated in DCF NPV terms, rather than only cash flow or revenue terms to correctly evaluate the impact of 'risk in time' as a function of the time value of money. Once the economic threshold has been determined to identify the trigger point between a project incurring a financial loss versus a profit, numerous hedging strategies at various strike rates and volatilities should be modelled.

Chapter 7 : Conclusions and Recommendations

7.1 SUMMARY AND CONCLUSIONS

The fundamental arguments in each of the previous chapters of this thesis are summarised below.

7.1.1 Chapter 1

Chapter one set out the problem statement from the perspective of a project economist or project manager 'within' a company that has to evaluate whether a mineral project is economically viable or not, considering both technical and economical project risks in the face of uncertain and limited information.

The main objective of this research was to compare quantitatively conventional evaluation methods with an innovative, 'spatially-aware' IEM evaluation technique that captures the non-linear effects of the response variables (such as recovery) related to production constraints taking into consideration the short (block-by-block) spatial and temporal scales.

The author also strived to understand how the financial impact of economic uncertainties can be incorporated within an integrated evaluation framework to generate a range of NPV outputs based on stochastic inputs, given the main resource (or 'physical') uncertainties in a mineral project. Lastly, this research explored some of the hedging strategies that management can consider to mitigate economic (or systematic) risks while simultaneously considering resource (or unsystematic) risks.

7.1.2 Chapter 2

Chapter two discussed the literature review and seminal papers in the areas of risk analysis, project evaluation, finance and real options valuation. The author recognised that an evaluation framework should be designed to encapsulate and integrate complexity across the entire evaluation cycle, i.e. resource estimation, mine planning and processing, and financial and economic modelling, with specific emphasis on understanding the impact of management's decisions to not incorporate appropriate flexibility in the mine and/or processing plant design to sufficiently cater for resource variabilities. The evaluation model has to strike a balance between simplified estimation techniques and sufficient incorporation of aspects of the project that would make a material difference to the investment decision.

A gap identification process revealed that the standard practice for evaluating mineral deposits often involves using a single resource and reserve model whereupon sensitivity analyses are conducted; this approach does not adequately capture the range of variability associated with the compounding effects of resource uncertainties. The combined impacts of non-linear resource variables on mining and treatment constraints within an integrated evaluation model, and their cumulative impact on the cash flow model has not previously been adequately documented, especially in the case of diamond projects. The evaluation challenge is further complicated when attempting to reproduce both geospatial and temporal scales in an integrated evaluation model.

The author postulated that failure to account correctly for spatial and temporal risks, by estimating the average annual production totals instead of accumulating the effects of short-scale interactions of resource variables on the mining and processing constraints into annual production totals, could result in material errors in estimating a mineral project's NPV. There was no previously documented robust methodology that quantifiably evaluates the financial costs/benefits of operational and management flexibilities in any specified period as a risk mitigation strategy, given the combined effects of spatial resource uncertainties, mining and treatment constraints and economic uncertainties.

The author recognised that addressing these gaps required research to focus on designing and developing an integrated evaluation modelling (IEM) framework that would allow unsystematic, technical risks related to the resources and reserves of a diamond mine, to be evaluated at the correct spatial and temporal scales. These spatial uncertainties would be combined with stochastic foreign exchange rate models in an IEM framework to quantify their collective financial impacts on a project's NPV, reproducing the spatial integrity of the data throughout the entire evaluation pipeline. Lastly, conventional evaluation methods would be compared to this new IEM technique and compared to a virtual ore body (VBod) 'reality' to determine the impact of various risk mitigation decisions (hedging strategies) that management could implement.

7.1.3 Chapter 3

In chapter three the author described the experimental design and techniques used to develop an integrated evaluation model (IEM) framework. The IEM is based on a unique ‘bottom-up’ methodology that follows every block through the mining and processing value chain, i.e., it accurately captures the spatial variability of resource variables in the ground (grade, density, processing characteristics, etc.). This variability is then propagated through the processing value chain at a mining block (or selective mining unit, “SMU”) scale. The ‘bottom-up’ (as opposed to ‘top down’) evaluation approach is necessary to capture correctly resource variabilities and their non-linear impacts on the reserve model, with specific regard to key mining and processing constraints.

The two main advantages of the IEM approach are that firstly, it reproduces accurately the spatial resource characteristics of block models at the appropriate temporal scale; and secondly, direct linkages are created between the resource, reserve and financial models within a single software environment. This allows the accurate and rapid assessment of multiple scenarios for a mineral project and the easy evaluation of the cost/benefits of implementing risk mitigation strategies.

The author also introduced the concepts of an ‘evaluation bias’ which demonstrates the impact of short-scale variability within each SMU on the planned production constraints, and ‘scheduling errors’ that highlight potential errors that can take place when selecting blocks for processing based on the well-known ‘time value of money’ approach applied within a conventional DCF financial framework. Failure to correctly account for spatial and temporal risks, by estimating the ‘average’ annual production totals instead of accumulating the effects of short-scale interactions of resource variables on the mining and processing constraints into annual production totals, may result in material errors in estimating a mineral project’s value.

7.1.4 Chapter 4

Chapter four introduced the concept of a virtual ore body (VBod) that was created using a non-conditional geostatistical simulation from a combination of actual drilling information, bulk-samples and face mapping from an exposed part of the dyke. An unconditional simulation was used to try and model the full range of possible variances and for simplicity, one simulation was assumed to be reality (the VBod) rather than as a single realisation of a particular orebody. The VBod was sampled on selected grid spacings to generate three sampling scenarios (or campaigns). These samples are assumed to be representative of drill hole data extracted from a deposit. Samples from each campaign were used to generate kriged estimates and a set of conditional simulations for each of the three sampling scenarios. These kriged estimates and conditional simulations formed the basis of the inputs into an IEM approach to capture correctly the correlations and system linkages of resource variables on the mining and processing constraints of diamond projects.

Two examples of diamond mines, one open-pit and one underground, were used to expound on the significance of using an IEM approach to evaluate spatial and temporal resource variability impacts upon production forecasts and cash flow models. A similar exercise was conducted on a gold mine to prove that this IEM approach could also be extended to other commodities.

For the first case study, whilst kriging produced the best unbiased linear estimates for key resource variables, the smoothing effect of kriging made the kriged estimates less sensitive to production constraints, thereby over-estimating the NPV. The second case study demonstrated that conditional simulations can be used alongside kriged estimates to quantify the financial impact of resource uncertainties without adjusting the discount rate to compensate for technical risks.

The financial impact of grade, density, yield and revenue per carat uncertainties were quantified in terms of production, cash flows, discounted cash flows and in NPV terms. The author advocated the use of an IEM as an alternative to the approach of applying mining and treatment modifying factors (derived from annual averages) to production figures, which are likely to provide ‘smoothed’ estimates of the actual variability that will be encountered on a daily basis.

The author discovered that mining operations that operate under strict reserve constraints or are characterised by resource complexity/heterogeneity do not have the luxury of unlimited mining and treatment flexibilities, and hence, cannot easily respond to changes in tonnages or grades as a function of resource variability. In the case of marginal operations with limited capital expenditure, the impact of this limited responsiveness is further exacerbated by the presumption of ‘smoothed’ ore horizons due to kriging with limited sampling data.

Depletion of simulated blocks in space and in time allowed accurate quantification of the financial impact of variability during each year. While volume, grade and density estimates showed little variation in simulations over the life of mine on an annual scale, it was the variability of these simulations within each year and the selection and sequencing of blocks over time that directly influenced the contribution to cash flows. The use of an IEM approach showed that the highest variability in cash flows occurred early on in the life of mine (“risk window”) had the biggest impact on the time value of money. This highlighted the need for an efficient operational plan to ensure that the right tonnes from the right areas are mined and treated during the right time.

In addition to the discoveries made in the first two case studies, the third case study applied to a gold mining operation, demonstrated that the financial benefit of grade control systems and stockpile management could be quantified in financial terms by running different scenarios through the IEM. Given the non-linear relationships between the resource, mining, processing and financial constraints, this particular problem could not have been solved through any form of closed-form mathematical model – a simulation approach was necessary.

7.1.5 Chapter 5

Chapter five compared different risk analysis methodologies, in particular conventional sensitivity analyses and Monte Carlo simulations with an IEM approach, to evaluate their relative advantages and limitations in mineral project evaluation. The author demonstrated that sensitivity analysis and Monte Carlo simulations could provide an improved understanding of project risks but there were limitations of using these techniques when compared to an IEM approach, which more accurately reproduced the spatial and temporal risks in a mineral project. Sensitivity analysis was not recommended for assessing spatial (physical) resource parameters in an evaluation risk model as it could not correctly capture the correlations or variance between variables, whereas conditional (spatial) resource simulations can do so.

The author also showed that Monte Carlo simulations could mislead decision-makers in thinking that they accurately captured the range of possible outcomes with the expected outcome safely lying within the modelled variability range. This is not always true, as demonstrated in the case study showing that material NPV errors (in the range of 160% - 180%) could be realised.

The author also investigated the most appropriate method of incorporating technical risks in mineral projects. Relationships between the technical component of the discount rate, capital expenditure and techno-economic factors were quantified through heuristic experiments. These outcomes together with the VBod were used to provide a quantitative breakdown of the technical component of the discount rate, using the concept of variance reduction curves, in an attempt to find an alternative technique to the IEM approach. While this method produced interesting results that could be extended to other resource variables in this specific case study, it was deemed to be too time consuming and conclusions derived were exclusive to that case study, limiting the extrapolation of the findings from this technique to other problems.

Finally, it was demonstrated that as additional sampling information was acquired, the overall project variance reduced as a function of gaining more information and reducing uncertainty. The fewer the number of spatially representative sample data, the more the likelihood increases of under-estimating the true variance and assuming a smoother profile with less variability if linear estimation techniques were used. The objective of implementing an IEM approach for mineral project evaluation was not to recommend closer spaced sampling grids but to find a balance between the required sampling drilling density and the derivation of quantitative estimation errors in NPV.

7.1.6 Chapter 6

Chapter six quantified the financial impact of managerial flexibilities by evaluating different hedging strategies that simultaneously considered production and economic uncertainties within an integrated evaluation modelling framework. All modelled outputs were calculated in NPV terms for a diamond mine using a modified DCF approach. The importance of linkages within an IEM framework were validated for unsystematic (project specific) risks related to resource/reserve parameters and systematic (economic) risks to evaluate the most appropriate management hedging strategy for a diamond mining project.

In this chapter instead of conventionally evaluating multiple hedging strategies for foreign exchange (FX) rate uncertainty using a ‘single’ production scenario as a basis (typically generated from a kriged resource estimate), each hedging strategy was run against ‘multiple’ realisations of the ore body generated from conditional simulations (discussed previously in chapter four) on a one-to-one mapping with the FX rate. This uniquely quantified the financial differences of each hedging strategy between the stochastic output from geostatistical conditional simulations of the mineral deposit with the kriged and VBod models.

Based on the VBod and actual FX rates for the period 2006 - 2009, it was proven that the true value of this project ranges from negative CAD51 million to CAD6.50 million depending on whether FX hedges were implemented or not to protect the project revenues from the volatile exchange rate. However, in the absence of perfect information, i.e. assuming that comparisons with the VBod and actual FX rates were not possible in the real world, the author revealed that management may be inclined to make the incorrect decision (in this case study) not to hedge.

Management’s decision would have been strongly influenced by the NPV generated from the kriged results and mean of the conditional simulations, modelled on the variable FX spot rate, which produced NPV greater than any of the hedged positions. However, it was clear from the case study that the impact of the actual ‘un-hedged’ FX rate volatility for the kriged scenario, which was circa 5.3 times lower than the modelled volatility, resulted in a worst-case NPV compared to the other scenarios than management could have realised at that time. The preferred scenario was to hedge using the zero-cost collar, which would have limited losses to a mean NPV of negative CAD5.5 million for the VBod model (as opposed to a loss of negative CAD51.4 million if no hedging was in place and the project was exposed to the actual variable FX spot rate).

The author deduced that the focus of the hedging strategy should not simply be on ‘breaking even’ over a specified time period by balancing wins and losses in each financial year from hedging the FX rate in dollar terms. It should focus on cash flows of the operation over time, and in particular, on the higher risk periods in the life of mine schedule (especially in the earlier years from a time value of money perspective) to ensure that profit margins are sustainable, by protecting project revenues negatively affected by FX rate volatility. The author advocates that a mineral resource company should determine the economic threshold (in DCF NPV terms) that a project or portfolio of projects can withstand, based firstly on the key physical properties of the resources/reserves, then secondly on a combined range of economic criteria within an integrated evaluation framework.

7.1.7 Final Conclusions

This thesis covered a range of topics from geostatistics to real options valuation to evaluate a mineral project, with a consistent theme throughout each of the chapters. Project risks, pertaining to both systematic and unsystematic risks, need to be assessed in an integrated approach to ascertain whether their financial impact is material or not. The greater the perceived variability of key systematic and unsystematic variables, the more the mine has to consider flexibility in its mining and processing schedules and management hedging strategies; but the real costs of attaining that flexibility needs to be evaluated using an IEM framework.

Each deposit may have several resource variables, such as grades, density and rock type characteristics that differ in variability. These variabilities affect reserves with respect to the planned mining and processing constraints used to calculate the production figures for input into the financial model. The NPV of a project depends on the estimated values and variabilities of the variables in question from a resource and reserve perspective; the business plan including the mining, processing, refining, marketing and sales plans; the project's exposure to key economic risks; and management's ability to mitigate risks as a function of various hedging strategies.

Once resource variabilities have been modelled using an IEM to quantifiably measure their impact on reserve constraints and consequent influence on the financial model, it is the author's belief that the next stage should involve management modelling various scenarios via the IEM to mitigate these risks. For example, case study one in Chapter four highlighted the challenge of having a limited stockpile capacity of only 3,000 tonnes per day which created a bottle-neck to feed the processing plant at the planned rate of 3,150 tonnes per day. Management should now incorporate additional capital costs in their financial model for a larger underground stockpile capacity to remove this constraint and then re-run the IEM to ascertain the cost versus benefit thereof, which is where the IEM becomes an important tool to appropriately measure management's risk mitigation scenarios.

The case studies covered in this thesis demonstrated that each project has its own unique challenges, whether in terms of geological, mining or processing complexities or with respect to uncertainties associated with systematic and/or unsystematic variables. The author designed and developed an IEM framework for each case study but the principles were consistent throughout. The IEM framework is essentially a simulation model that strives to capture, replicate and model the key spatial and temporal linkages between resources, reserves and the financial model.

7.2 ORIGINAL CONTRIBUTIONS OF THIS RESEARCH

In the last few decades, innovative and faster processing risk analysis methods have been developed to model risks related to resource uncertainty. Research on these methods focused on commodities other than diamonds, such as gold, iron ore, coal, base metals, and oil and petroleum. Where diamond risk modelling took place, it concentrated mainly on alluvial deposits rather than kimberlite pipes, which are the main sources of diamonds.

Complex resource estimation problems are typically expressed through 'simplified' mathematical equations to solve global or local geostatistical problems. However, the production and financial impacts of non-linear resource-to-reserve relationships cannot be approximated (with any degree of reasonable accuracy) using a closed-form mathematical solution as each project has its own set of resource and reserve variables, which interacts with mining and processing constraints in a sequential, non-linear and unique way.

The author recognised that failure to account correctly for spatial and temporal risks in a mineral evaluation model, by estimating the 'average' annual production totals instead of accumulating the effects of the short-scale (e.g. daily) interactions of resource variables on the mining and processing constraints into annual production totals, may result in material errors in estimating a mineral project's value.

This research is deemed to be unique in that this study is applied to kimberlite diamond projects which have not been previously researched to this extent and in this context. It transcends conventional discipline boundaries covering the areas of geostatistics; mine planning; financial theory; and real options valuation. This thesis uniquely quantifies the impact of key resource variabilities and their non-linear interaction with reserve constraints in financial terms, using a modified DCF NPV approach, founded on a single, integrated evaluation framework.

The actual IEM toolkit is not an 'off the shelf' software solution but is rather a bespoke solution of MS Excel spreadsheets with Visual Basic Application (VBA) programming coding, developed by the author, that links each component of the evaluation pipeline together. This coding was later enhanced into a web-enabled, dot.net programming environment linked to a SQL database for larger-scale commercial risk evaluation projects, sponsored by Quantitative Group, a Perth-based mining consultancy, but further discussion is beyond the scope of this thesis.

7.3 RECOMMENDATIONS FOR FUTURE WORK

The areas below are recommended to supplement this research in terms of increased depth of study and across a broader range of applications.

7.3.1 Application of multiple VBod's in an IEM Framework

While the limitation of using only one virtual ore body (VBod) is acknowledged, this research focused on developing the methodology of an IEM framework. The use of a VBod provides a way of quantitatively comparing the relative financial outputs. Without the use of a VBod, the evaluator would not be in a position to accurately quantify the benefits of an IEM approach compared to conventional evaluation techniques. Furthermore, as computer technology develops in terms of processing speed and memory storage capabilities, the opportunity exists to generate multiple VBod repeating this exercise for several 'realities' based on each VBod, creating a multi-probabilistic and more objective IEM framework.

7.3.2 Evaluating Uncertainty of the Variogram for Estimations and Simulations

The author recognizes that the variogram is vital to any geostatistical estimate or simulation but typically, the precision of a variogram estimated from sample data by the method of moments is unknown. No uncertainty on the experimental or fitted variogram models was included in this study when generating kriged estimates and conditional simulations. Various authors have considered variogram uncertainty in a number of different contexts. Journel and Huijbregts (1978) distinguished between theoretical and local variograms, also referred to as ergodic and non-ergodic variograms, respectively. They noted that the key difference is that ergodic variograms are defined as the average over all realisations of the underlying stochastic model, and non-ergodic variograms only over the realisation actually sampled (not related to a stochastic model). Isaaks and Srivastava (1988) argue that ergodic variograms provide more reliable confidence intervals of predictions than non-ergodic variograms as they capture the character of spatial variability unique to the domain over which it is defined.

Cressie (1985) used theoretical expressions to fit variogram models in such a way that it accounts for the difference in accuracy of the experimental semi-variance at each lag distance. Webster and Oliver (1992) measured the uncertainty of variograms estimated from different sampling schemes to determine whether the sampling schemes were adequate for variogram estimation. Davis and Borgman (1979) conducted considerable research into sampling distributions of variograms. Srivastava and Parker (1989) evaluated several methods of calculating variograms and concluded that correlogram and pairwise relative variogram methods were the most robust of the methods studied. It is generally impractical to design a sampling programme that is tailored to calculating variograms.

Rossi et al. (1992) compared the use of ergodic versus non-ergodic variograms on ecological studies to show that the use of non-ergodic covariances and correlograms provide a more effective description of lag-to-lag spatial dependence by accounting for the changing local means and variances. They suggest that the non-ergodic approach may be better than the traditional variogram estimator for reproducing the true underlying spatial structure. Müller and Zimmerman (1999) and Bogaert and Russo (1999) suggested techniques for designing sample schemes where the sample points are positioned to minimize the value of a theoretical expression of variogram uncertainty. They use theoretical expressions to quantify the expected error in the experimental variogram as an approximation to the variogram of the random process which generated the field.

Further research is recommended on the impact of variogram uncertainty on the kriged estimates and conditional simulations for selected case studies in this thesis based on sampling campaigns derived from a VBod. In previous studies theoretical expressions have been derived to approximate uncertainty in estimates of the experimental variogram and fitted variogram models. These expressions typically rely upon various statistical assumptions about the data and express variogram uncertainty as functions of the sampling positions and the underlying variogram. These expressions can be used to design efficient sampling campaigns for estimating a particular variogram.

One research path of interest would be to focus on the role of sample data on reducing variogram uncertainty, as a function of acquiring more data by increasing samples derived from an even sampling grid, and also by selectively positioning sample points to minimize the value of a theoretical expression of variogram uncertainty. A second research path would be to compare the ergodic variogram approach with the use of non-ergodic covariances and correlograms to quantify potential differences in terms of kriged estimates and simulation outcomes and their associated impact on the financial model.

7.3.3 Deeper Analysis into Calculating Input Volatilities

In chapter six the author recognised the importance of calculating historic volatilities for the foreign exchange rate as an input into the Garman-Kohlhagen FX rate predictive model. A simplified method was selected to calculate the instantaneous standard deviation of historic changes based on an approach from Davis (1998) and Hull (2003) using unweighted 30-day average data (see Equation 39).

$$\sigma_s = \sqrt{\frac{\tau}{n-1} * \sum_{i=1}^{n} \left(u_i - \bar{u}\right)^2}$$

where $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ and $i = 1 \,..\, n$

τ = the number of observation periods per year

i = the observation number

$\bar{u}$ = the mean log relative rates

$$where,\ \bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

Equation 39 – The instantaneous standard deviation model to calculate historic volatility.

The author acknowledges that more sophisticated variations to this approach exist such as the autoregressive conditional heteroskedastic (ARCH) and generalized autoregressive conditional heteroskedastic (GARCH) models. These may be used to model the historic FX rates to better detect trends between linear and non-linear dependencies of the means and variances of the data. Bera and Higgins (1993) remarked that "a major contribution of the ARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables."

When dealing with nonlinearities, Campbell, Lo and MacKinlay (1997) distinguished between 'Linear Time Series' where shocks are assumed to be uncorrelated but not necessarily identically and independently distributed (iid); and 'Nonlinear Time Series' where shocks are assumed to be iid, but there is a nonlinear function relating the observed time series and the underlying shocks. Taking this into consideration, the author recommends that further work on selected FX rate modelling techniques be investigated to evaluate the impact of the following approaches on the predictive Garman-Kohlhagen FX model:

- *GARCH* uses a general autoregressive moving average model, which is a general auto-correlation of data. If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity.

- *NGARCH* is a nonlinear asymmetric generalized autoregressive conditional heteroskedastic model, which is a way of applying adaptive fuzzy logic to infer future prices or returns.
- *IGARCH* is an integrated generalized autoregressive conditional heteroskedastic methodology, which is a restricted form of the GARCH model where the persistent parameters sum up to one, and therefore there is a unit root in the GARCH process. If the processing is linear, the unit root will be one in the calculations.
- *EGARCH* is an exponential general autoregressive conditional heteroskedastic that uses a conditional variance and a general error distribution (a standard normal variable can also be used).
- *QGARCH* is a quadratic generalized autoregressive conditional heteroskedastic technique, which is used to model the symmetric effects of positive and negative shocks, but not applicable to modelling extreme market conditions like the recent global financial crisis (GFC) period in 2008 - 2009.

7.3.4 Alternative hedging strategies

The author used hedging strategies such as zero-cost collars and an adaptation of Black and Scholes called the Garman-Kohlhagen call option, but there are other forms of protective hedging strategies similar to the zero-cost collar option strategy, such as bull spreads, bear spreads, butterfly spreads, calendar spreads, strangles and straddles that were beyond the scope of this study – see McMillan (2002) and Hull (2003) for further information.

Further investigations into these strategies are recommended to ascertain their impact on developing a more robust hedging strategy. The alternative hedging strategies to be investigated include the following:

- *Bull spreads*: A bull spread consists of two or more options of the same type, and results in a profit when the underlying asset increases.
- *Bear spreads*: A bear spread consists of two or more options of the same type, and results in profit when the underlying asset decreases (the opposite to bull spreads).
- *Butterfly spreads*: A butterfly spread requires three options of the same type, with three strike prices. It’s formed by taking equal positions in the high- and low-priced options and twice the opposite position in the middle-priced option.

- *Calendar spreads*: A calendar spread consists of a short expiry option and a long expiry option of the same type and same exercise price.
- *Strangles*: A strangle is made up of a long call and a long put with the same expiry but different strike prices, and profits from large price movements.
- *Straddles*: A straddle is made up of a long call and a long put with the same strike price and expiry, and profits from large price movements.

7.3.5 Application of Bayes Law for Modelling of Uncertainty

Discrete Failure Events (DFE) may have a low probability of occurrence for many mining operations but if they do occur, their consequential impact can be financially material, negatively affecting management's flexibility options and may require additional expenditure to address these issues once they have occurred. Typical DFE for mining operations include excessive rainfall that could flood the deepest (and often the most high-grade) parts of open-pits or stopes of an underground operation; or open-pit wall failures that may be caused by geotechnical failures and/or excessive rainfall patterns; or in the case of the underground diamond mine (case study one, Chapter four) flooding due to collapse of the mine stope roof (which lies beneath a frozen lake), may result in 'writing off' material parts of the mine plan affecting the NPV.

Some operations that are located in regions of high rainfall, e.g. Batu Hijau (a copper-gold mine in Indonesia), will have a higher probability of DFE associated with flood events. It is considered good operational management practice to have an understanding of the likelihood of occurrence of a DFE to evaluate the production and financial impacts, if any. The author recognised the importance of this and investigated modelling DFE using an application of Bayes theorem (Bayes and Price, 1763) applied qualitatively (Medow and Lucey, 2011) to a set of mining data to evaluate the probability of a DFE involving flooding of the underground diamond mine discussed in chapter four of this thesis (using Equation 40).

$$\boxed{P(A|B)=\frac{P(A).P(B|A)}{P(B)}}$$

Equation 40. Bayes Theorem

A current probability matrix, P(A) was defined based on the mine flood probability and the number of production days affected by flooding. A probability decision tree was used to calculate the expected value of the tonnages in the affected stopes of the flooded mine. The reliability of this predictive information was evaluated by comparing historical trends, i.e. actual flooding events in the past were compared with the model's predicted outcomes to determine the posterior distribution, P(B|A). This was used to weight the predictive model by the reliability of information and lastly, a full probabilistic decision tree was generated to estimate the impact of the various flooding scenarios on the number of mine stopes (tonnages) affected and the expected value determined.

The author recommends that this independent study could be incorporated into the IEM framework to include DFE into the uncertainty framework to provide a more comprehensive probabilistic risk assessment of a mine/project's risks. Monte Carlo simulations could also be included within the current probability matrix.

7.3.6 Further Work on Economic FX Modelling

In this research the author elected to use a combination of a resource VBod and actual FX rates (retrospectively evaluating a time period between 2006 – 2009) in order to compare various hedging scenarios to each other and to 'reality'. It is recommended that various time periods be selected to retrospectively evaluate the FX rate volatility (other than the 2006 – 2009 period) on nominated hedging strategies combined with resource uncertainties. It would be interesting to assess if there are any trends in the preferred hedging strategy based on multiple FX rate evaluation periods for this diamond study.

There is a broader application of this research into management flexibilities that attempt to produce robust hedging strategies for FX rates based on predictive models. The USD:CAD FX rate correlation was used in this research as it seemed logical, particularly as the leading case study involved a diamond operation in Canada where revenue was earned in USD, then swapped into the local CAD currency to service operational costs. An opportunity exists to quantify a relationship between commodity prices and the FX rate to assess the influence of commodities on the FX rate to develop better predictive models of FX rates to assist with developing robust hedging strategies.

Initial research into a comparison between the AUD:USD and USD:CAD revealed a good correlation between these two currencies and the commodity index (see Figure 66 and Figure 67).

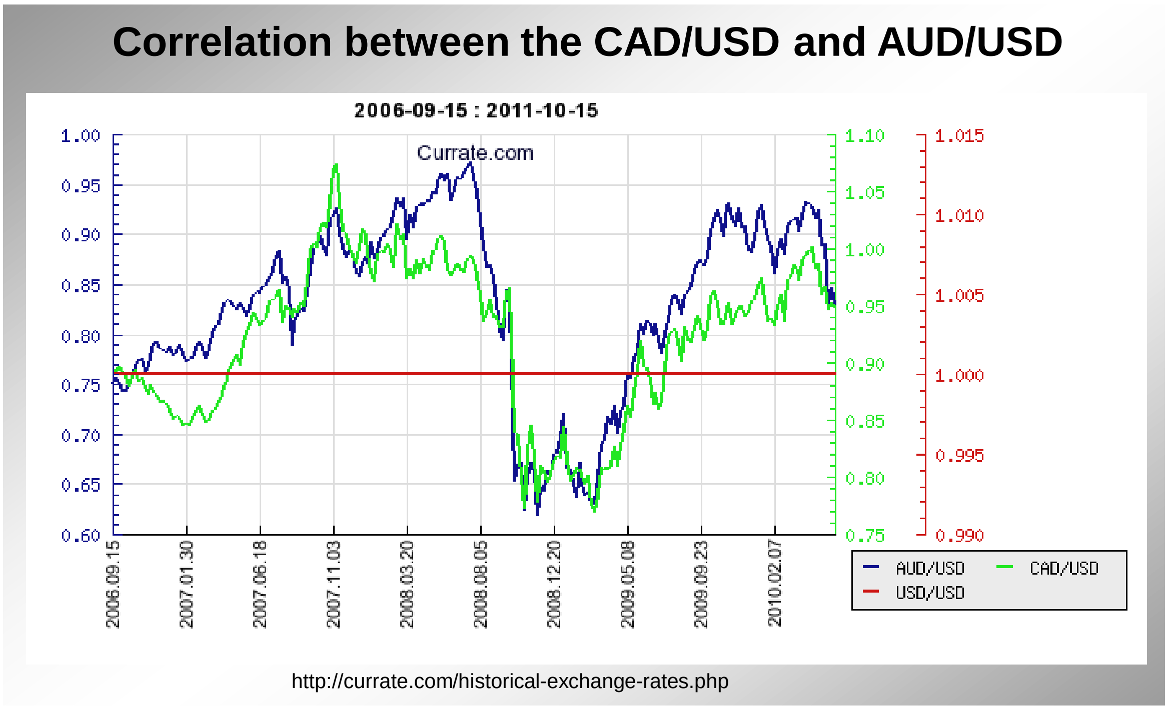

Figure 66 – Correlation between the Canadian-United States Dollar and the Australian-United States Dollar.

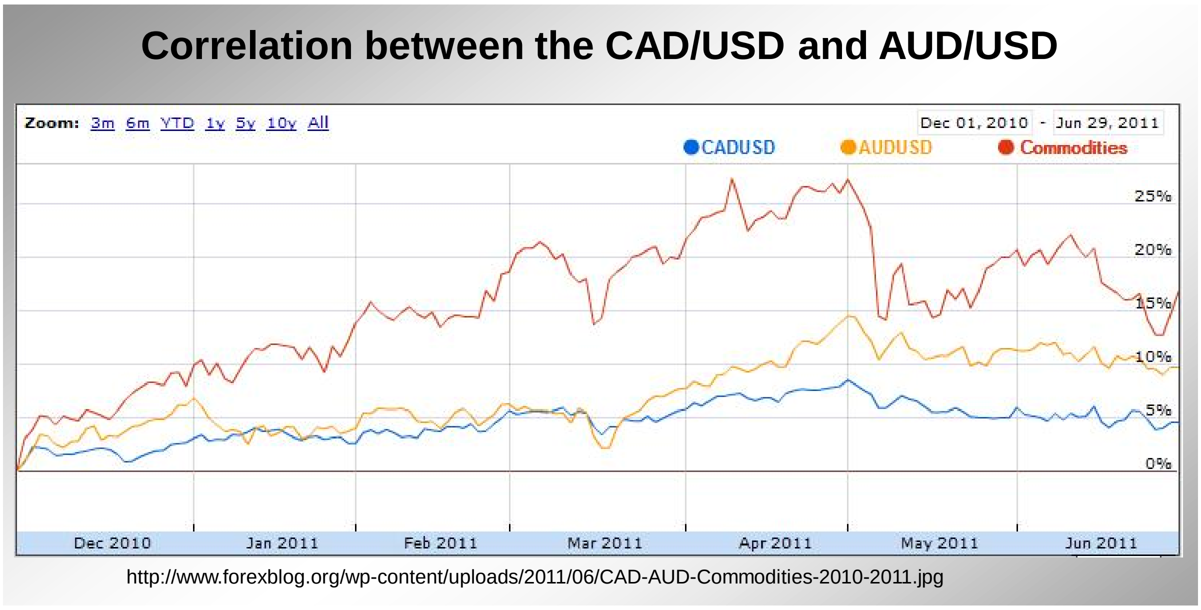

Figure 67 – Correlation between the Canadian-United States Dollar and the Australian-United States Dollar in relation to the commodities index.

In Figures 59 and 60, both currencies are characterized as ‘commodity currencies’, which implies that a rise in commodity prices is matched often by a proportionate appreciation in the AUD and CAD relative to the US dollar. It can be observed from the chart that the year-long commodities boom and sudden drop corresponds to similar movement in commodity currencies. Both currencies are seen as attractive proxies for risk and are associated with rising commodity prices translating into stronger currencies. It is interesting to note that both Australia and Canada are natural resource economies but it appears that the Australian FX rate has strengthened more than the Canadian dollar, which could be principally due to Australia’s proximity to China and influenced by its large exports of iron ore and coal to China.

While the focus of this thesis has been on diamond projects (with exception of the gold application of case study three discussed in chapter four), future extension of this research using an IEM approach to evaluate unsystematic risks combined with FX rate uncertainty for other commodities is recommended. The relationship of commodity prices (especially iron ore and coal) to the AUD:USD FX rate could be a stimulating extension of this research to evaluate its impact on hedging strategies, considering combined economic and resource uncertainties. This would expand the concept of an IEM framework into broader evaluation mining applications especially iron ore, which has similar complex unsystematic stochastic variables to diamonds that need to be incorporated in the evaluation model other than grade uncertainty.

This concludes chapter seven and this thesis.

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