

AN INTEGRATED RISK EVALUATION  
MODEL FOR MINERAL DEPOSITS

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# ABSTRACT

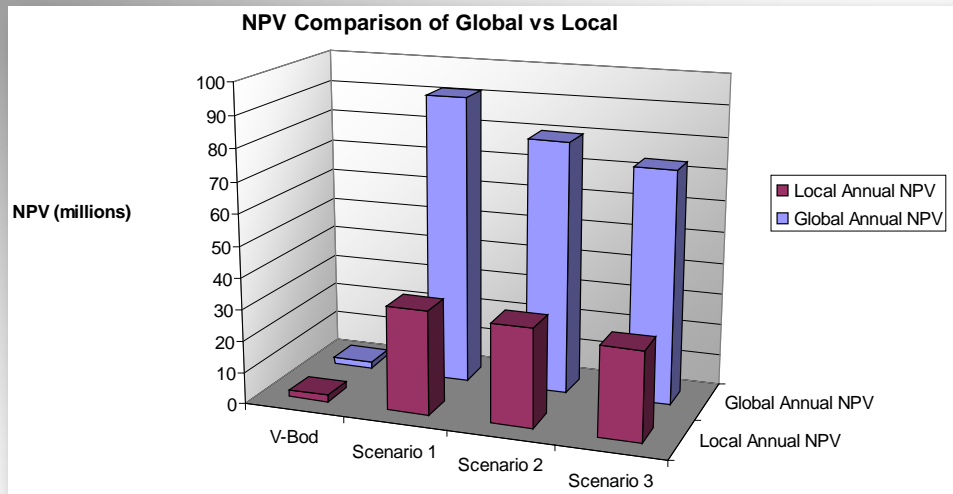
The core asset of most mining companies is its mineral resources and reserves. The company produces ore from its reserves, which is a subset of its mineral resources associated with varying levels of geoscientific confidence and uncertainty. One of the key evaluation challenges is to distil technical complexity into a financial model that is usually designed to focus only on one or two key valuation indicators, such as net present value (NPV) or internal rate of return (IRR).

The driver behind this research was whether conventional evaluation techniques for mineral projects can evaluate accurately both the spatial and temporal characteristics of project risks in financial terms, due to their inherent nature to understate the true variance, and under-value or over-value the actual NPV. How can conventional evaluation methods be compared to an innovative, integrated evaluation technique that quantifies the non-linear impacts of spatial resource variables on production constraints in financial terms, measured at the appropriate temporal scale?

To answer these questions, this research focused on developing an innovative risk evaluation methodology for two different diamond deposits and one gold deposit to incorporate spatial, non-spatial and financial data across the evaluation pipeline. The author developed an integrated evaluation modelling (IEM) framework based on a unique bottom-up methodology that follows every estimation block through the mining and processing value chain, i.e., it accurately captures the spatial variability of all relevant value chain variables in the ground and their correlated impacts on production constraints such as grade, density and processing characteristics. This variability is propagated through the processing value chain at a mining block (or selective mining unit, “SMU”) scale.

The IEM approach revealed differences in NPV between a ‘bottom-up’ (or Local) evaluation method and a ‘top-down’ (or Global) evaluation method – see Figure 1. While the actual NPV for the virtual ore body (VBod) was CAD 2.1 million, the figure shows that the local evaluation method (bottom-up) more closely approximated the actual NPV of the project than the global (top-down) evaluation method, which materially over-estimated the NPV.

### Global 'top-down' evaluation method compared to a Local 'bottom-up' method



	V-Bod	Scenario 1 (75m)	Scenario 2 (50m)	Scenario 3 (25m)
Global NPV (CAD million)	-	91.6	80.1	73.9
Local NPV (CAD million)	2.1	32.9	31.4	28.3

**Figure 1** compares the Local (bottom-up) and Global (top-down) evaluation methods over three different sampling campaigns (75m, 50m and 25m spaced drill holes).

The author demonstrated that cash flow constituents derived from annual estimates in a top-down approach will not correctly reflect the asymmetries due to operational variability on a local, daily basis. The 'bottom-up' evaluation method represented a more accurate way of deriving annual cash flow estimates needed to make decisions on projects by accumulating the appropriate values from a bottom-up approach, i.e. daily, monthly, quarterly then derive annual estimates for NPV forecasts.

The two main advantages of the IEM methodology are that firstly, it accurately reproduces the spatial resource characteristics of blocks at the appropriate temporal scale; and secondly, direct linkages are created between the resource-reserve-financial models within a single software environment. This allows multiple scenarios to be rapidly assessed for a mineral project and the cost/benefits of implementing risk mitigation strategies to be easily evaluated.



This research also quantifies the financial impact of managerial flexibilities by evaluating selected hedging strategies that simultaneously consider production and economic uncertainties within an IEM framework. All modelled outputs are calculated in NPV terms using a modified DCF approach. The importance of linkages within an IEM framework is validated between unsystematic risks, with respect to key resource-to-reserve stochastic variables, and systematic risks considering the impact of foreign exchange rates.

The author concludes that the greater the variability of key systematic and unsystematic variables, the more the mine has to consider flexibility in its mining and processing schedules and management hedging strategies; but the real costs of attaining that flexibility needs to be evaluated using an IEM framework. The confidence in a NPV estimate for complex mineral projects cannot easily be quantified using any closed-form analytical or mathematical solution. Complex, non-linear relationships between resources, reserves, financial and economic parameters requires a simulation model based on an IEM framework to provide a robust solution.

# STATEMENT OF ORIGINALITY

Submitted by Grant Nicholas to the University of Adelaide as a thesis for the degree of Doctor of Philosophy to the faculty of Engineering, Computer and Mathematical Sciences, March, 2014.

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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GRANT NICHOLAS

30/03/2014

DATE

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# GLOSSARY

<b>AR</b>	Autoregressive time series model.
<b>ARIMA</b>	Integrated Autoregressive-Moving Average time series model.
<b>ARMA</b>	Autoregressive-Moving Average time series model.
<b>AUD</b>	Australian Dollar.
<b>CAD</b>	Canadian Dollar. Note that for market convention purposes, the foreign exchange rate (USD:CAD) is referred to in the ‘Direct’ sense, i.e. specifying the number of CAD required to buy or sell one United States dollar (USD).
<b>Call option</b>	Provides the holder of the option with the right but not the obligation to buy the underlying asset by paying the exercise price agreed upfront in the contract. A call option is referred to be ‘in the money’ when the price of the underlying asset is greater than the exercise price and a profit could be made by exercising the option. Conversely, the call option is ‘out of the money’ if the price of the underlying asset is less than the exercise price.
<b>CAPM</b>	Capital Asset Pricing Model.
<b>CPHT</b>	Carats Per Hundred Tonne (a measure of diamond grade).
<b>Correlation</b>	A measure of the dependency between two variables; or may be calculated as a measure of spatial dependency of a single variable at a distance interval.
<b>Covariance</b>	A measure of the dependency between two variables; or may be calculated as a measure of spatial dependency of a single variable at a distance interval.
<b>Conditional simulation</b>	A geostatistical tool which can be used to generate punctual or block ‘realisations’ of mineral grades. Each realisation is intended to honour the histogram and semivariogram of the true grade distribution, as well as honouring known data points.
<b>Conditional distribution</b>	The probability distribution for a variable, given the known value of that variable at other locations in space.
<b>DCF</b>	Discounted Cash Flow.

<b>DTA</b>	Decision Tree Analysis estimates the maximum and minimum project value by evaluating the probabilities associated with different options discounted back at the traditional discount rate.
<b>European options</b>	Those options that can be exercised only on their maturity date while options that can be exercised at any time are referred to as American or Real Options.
<b>Exercise price</b>	The amount of money invested to exercise the option if you are buying the asset (call option); or the amount of money received if you are selling the option (put option). As the exercise price of an option increases, the value of a call option decreases and the value of a put option increases. This determines the intrinsic value of the option.
<b>FX</b>	Foreign exchange rate.
<b>Geometallurgy</b>	It is a cross-discipline approach between geostatistics, geology, mining and metallurgy with the objective of addressing some of the complexities associated with determining the value of a resource and whether it is economic to exploit.
<b>Geostatistics</b>	Mathematical techniques used to estimate properties which are spatially dependent.
<b>Heteroskedasticity</b>	Non-constant variance.
<b>Homogeneity</b>	The property of a spatial series when its characteristics are independent of location. Homogeneity is equivalent to stationarity.
<b>Homoskedasticity</b>	Constant variance.
<b>IEM</b>	Integrated Evaluation Modelling approach, which models the linkages and dependencies between resources, mine planning and the financial model.
<b>IRR</b>	Internal Rate of Return.
<b>Kriging</b>	A collection of generalised linear regression techniques for minimising an estimation variance defined from a prior model. In contrast to classical linear regression, kriging takes into account stochastic dependence among the data.
<b>Kriging variance</b>	The minimised value of the estimation variance. It is calculated as a function of the semivariogram model and locations of the samples relative to each other and the point of block being estimated.

<b>Kurtosis</b>	The kurtosis is a measure of the ‘peakedness’ of a data distribution around the mode. A kurtosis: equal to 3 suggests a normal, or Gaussian, distribution; < 3 implies a lower concentration near the mean than a normal distribution; and > 3 suggests that the distribution has an excess of values near the mean.
<b>MA</b>	Moving Average time series model.
<b>Markov process</b>	A stochastic process in which a prediction is determined solely by the closest n observations, and is stochastically independent from all remaining, more distant observations.
<b>MCS</b>	Monte Carlo Simulations - any number of procedures that use simulated random samples to methods make inferences about actual populations.
<b>Multivariate conditional simulation</b>	An extension of conditional simulation which also aims to ensure that the correct dependencies between simulated variables are honoured in each realisation.
<b>NPV</b>	Net Present Value.
<b>Nugget effect</b>	When the semivariogram does not pass through the origin and arises from the regionalised variable being so erratic over a short distance that the semivariogram goes from zero to the level of the nugget in a distance less than the sampling interval.
<b>OLS</b>	The regression analysis method of Ordinary Least Squares.
<b>Ordinary Kriging</b>	The general geostatistical estimation process often simply known as kriging. Unlike simple kriging, the mean is unknown.
<b>PDE</b>	Partial Differential Equation.
<b>Put option</b>	The converse of a call option – provides the holder of the option with the right but not the obligation to sell the underlying asset to receive the exercise price. A put option is referred to be ‘in the money’ when the price of the underlying asset is less than the exercise price and a profit could be made by exercising the option. Conversely, the put option is ‘out of the money’ if the price of the underlying asset is greater than the exercise price.
<b>Random field</b>	The application of time series analysis to the spatial variability of theory geotechnical properties, and unlike time series analysis, random field theory is not confined to one dimension.

<b>Range</b>	The distance over which the semivariogram becomes a constant.
<b>Real Option</b>	The application of financial options, decision sciences, corporate finance and statistics to evaluating real or physical assets as opposed to financial assets ( <i>cf.</i> definition of European options). A real option is the right but not the obligation to take an action (such as deferring, contracting, expanding or abandoning) at a predetermined cost, called the exercise price or strike price over the life of the option.
<b>Regionalised</b>	A variable which has properties that are partly random and partly variable spatial, and has continuity from point to point, but the changes are so complex that it cannot be described by a tractable deterministic function.
<b>Resource</b>	A ‘Mineral Resource’ is a concentration or occurrence of material of intrinsic economic interest in or on the Earth’s crust in such form, quality and quantity that there are reasonable prospects for eventual economic extraction. The location, quantity, grade, geological characteristics and continuity of a Mineral Resource are known, estimated or interpreted from specific geological evidence and knowledge.
<b>Reserve</b>	An ‘Ore Reserve’ is the economically mineable part of a Measured and/or Indicated Mineral Resource. It includes diluting materials and allowances for losses, which may occur when the material is mined. Appropriate assessments and studies have been carried out, and include consideration of and modification by realistically assumed mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors. These assessments demonstrate at the time of reporting that extraction could reasonably be justified.
<b>ROV</b>	Real Options Valuation.
<b>Simple Kriging</b>	The same as ordinary kriging, except that the mean is assumed known and thus, there is no need to impose the unbiasedness condition, which eliminates the final row from all matrices, as is the final column of the square matrix.
<b>Skewness</b>	The Skewness is a measure of the symmetry of a data distribution. A skewness of zero suggests a symmetrical distribution, a positive value indicates a right-hand skew, and a negative value indicates a left-hand



	skew.
<b>Spatial series</b>	A sequence of discrete or continuous data measured at specific locations - the spatial equivalent of a time series.
<b>Stationarity</b>	A term used to denote different degrees of invariance in the characteristics of random fields. If the mean and autocovariance of the series change with the lag, and not location, the series is said to be weakly stationary. If all higher moments depend on the lag, and not position, the series is said to be stationary in the strict sense.
<b>Systematic risks</b>	Risks related to economics, such as price and foreign exchange rates etc. that can be diversified.
<b>Time series</b>	A mathematical technique used to estimate properties which are analysis temporally or spatially dependent. When applied to geotechnical engineering, time series analysis is usually referred to as random field theory.
<b>Trend</b>	An abstract expression of the low frequency, large-scale systematic variation of a regionalised variable. The trend may also include bias in the test method.
<b>Turning bands</b>	A simulation algorithm that can produce both non-conditional and conditional results. The method works by simulating one-dimensional processes on lines regularly spaced in 3D. The one-dimensional simulations are then projected onto the spatial coordinates and averaged to give the required 3D simulated value.
<b>Unsystematic risks</b>	Project specific risks related to resource/reserve parameters such as grade, geology, density etc.
<b>USD</b>	United States Dollar (see FX rate convention in definition of CAD).
<b>USD/Carat</b>	United States dollar per Carat is an expression of revenue for diamonds.
<b>Semivariogram</b>	A quantification spatial correlation of a variable, usually calculated from sample information.
<b>VBod</b>	Virtual ore Body, which is an analogue of reality created through conditional simulations based on actual drilling results.
<b>WACC</b>	Weighted Average Cost of Capital.

*“As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.”*

*— Albert Einstein*