

# Stochastic Modelling *of* Fractures *in* Rock Masses

Younes Fadakar Alghalandis

*A thesis submitted for the degree of*

*Doctor of Philosophy*

School of Civil, Environmental and Mining Engineering  
Faculty of Engineering, Computer and Mathematical Sciences

The University of Adelaide



THE UNIVERSITY  
*of* ADELAIDE

March 2014

# Dedication

*to my mother and my father*

# Contents

1	Introduction .....	23
1.1	Stochastic approaches .....	24
1.2	Modelling .....	25
1.3	Fractures and fracture networks .....	26
1.4	Rock mass .....	26
1.5	Summary of literature review .....	27
1.5.1	Geothermal Energy System .....	28
1.5.2	Geometric modelling of fractures and fracture networks .....	30
1.6	Summary .....	39
1.7	Research objectives .....	41
1.8	Fracture network modelling .....	42
2	Fracture Network Modelling .....	43
2.1	Definitions and terminology .....	44
2.1.1	Point .....	44
2.1.2	The Centre of Geometry .....	45
2.1.3	Point processes .....	46
2.1.4	Line .....	47
2.1.5	Polygon .....	47
2.1.6	Convex-hull .....	48
2.1.7	Smallest enclosing circle, ellipse, sphere and ellipsoid .....	48
2.1.8	Largest Empty Circle, Ellipse, Sphere and Ellipsoid .....	48
2.1.9	Triangulation .....	49
2.1.10	Delaunay and Voronoi tessellations .....	49
2.1.11	Mathematical Morphology .....	51
2.1.12	Discrete Fracture Networks .....	54
2.1.13	Clipping and Cleaning .....	60
2.1.14	Marked Point Processes .....	62
2.1.15	Fracture growth concept .....	63
2.1.16	Voronoi tessellation .....	66
2.1.17	Block theory .....	67
2.2	Additional processing stages for fracture network modelling .....	67
2.3	Mathematical Morphology operations applied blocks .....	68
2.4	Extension and trimming .....	70
2.5	Three-dimensional modelling of fracture networks .....	71
2.5.1	Fracture network as pipe model .....	73
2.6	Conditional fracture network modelling .....	74
2.6.1	Conditioning fracture locations using Simulated Annealing .....	74
2.6.2	Conditioning to existing fractures .....	75
2.6.3	Conditional simulation to a point cloud .....	77
3	A General Framework for Fracture Intersection Analysis: Algorithms and Practical Applications .....	81
3.1	Introduction .....	85
3.2	Fracture Network Modelling .....	86
3.2.1	Locations from Poisson point processes .....	88
3.2.2	Orientations from Fisher distribution .....	88
3.2.3	Sizes from exponential distributions .....	89
3.2.4	Rotation matrices .....	90
3.2.5	Translating the resulting polygons .....	91
3.3	Intersection Analysis (in 3D) .....	91
3.3.1	Intersection Density .....	93
3.3.2	Lengths of Intersection Lines .....	94
3.3.3	Effects of Fracture Length on Percolation State .....	95

3.4 Case Study - Leeds Fracture Data Set.....	96
3.5 Conclusions.....	99
4 The RANSAC Method for Generating Fracture Networks from Micro-Seismic Event Data.....	101
4.1 Introduction.....	105
4.2 The Problem of Fitting Lines/Planes to Point Cloud: Conditional Simulation.....	106
4.2.1 Enhanced Brute-Force Search.....	107
4.2.2 RANSAC method.....	111
4.2.3 Fracture Extents.....	115
4.3 Results and Discussions.....	115
4.3.1 EBFS and RANSAC applied on two-dimensional Point Cloud.....	115
4.3.2 EBFS and RANSAC applied to a simulated three-dimensional Point Cloud.....	121
4.3.3 The Habanero Seismic Events Point Cloud.....	123
4.4 Conclusions.....	128
5 A Spatial Clustering Approach for Stochastic Fracture Network Modelling.....	129
5.1 Introduction.....	133
5.2 The Objective Function.....	135
5.3 The Simulated Annealing Method.....	137
5.4 A Spatial Clustering Technique.....	138
5.5 Proposed Stochastic Fracture Network Modelling.....	140
5.5.1 Phase 1: Initialization.....	140
5.5.2 Phase 2: Updating Parameters of Individual Fractures.....	141
5.5.3 Phase 3: Updating the Size of the Fracture Network.....	141
5.6 Optimal Number of Fractures.....	142
5.7 Growing the Network (“Split” and “Special-Split”).....	143
5.8 Pruning the Network (“Joint” and “Special-Joint”).....	144
5.9 Experiments.....	146
5.10 Extension to 3D Applications.....	148
5.10.1 A Real Case Study: Habanero Reservoir Dataset.....	151
5.11 Conclusions and Future Work.....	153
6 Characterisation of Fracture Networks.....	155
6.1 Fracture centroid density.....	156
6.2 $X_f$ .....	158
6.3 Intensity group.....	158
6.4 Fracture density.....	161
6.5 Largest empty circle.....	162
6.6 Largest empty convex-hull (inner-convex-hull).....	163
6.7 Distance map.....	164
6.8 Buffer effect.....	164
6.9 Convex-hull.....	165
6.10 Block area.....	166
6.11 Backbone density.....	166
6.12 Block centroid density.....	167
6.13 Intersection analysis.....	168
6.13.1 Fracture clusters.....	169
6.13.2 Intersection density.....	171
6.13.3 Extended intersection density.....	172
6.13.4 Inter-connectivity.....	172
6.13.5 Fracture normal intersection density.....	174
6.14 Effects of stress field.....	175
6.15 Comparison of density measures.....	177
6.16 Popularity index.....	177
6.17 Connectivity of fracture networks.....	179
7 Connectivity Field: A Measure for Characterising Fracture Networks.....	181

7.1 Introduction .....	185
7.2 The Connectivity Field.....	190
7.2.1 The Generalised Connectivity Field.....	196
7.2.2 The Probabilistic Connectivity Field.....	198
7.3 The relationships between the CF, GCF, DFC, ID, $X_f$ and P21 .....	200
7.4 Applications of the Connectivity Field group.....	205
7.4.1 CF and Flow Pathways .....	205
7.4.2 CF and Percolation State .....	206
7.4.3 Using CF to determine well locations and to design underground repositories .....	208
7.5 CF applied to a real three-dimensional fracture network .....	208
7.6 Concluding remarks.....	210
8 Connectivity Index and Connectivity Field towards Fluid Flow in Fracture-Based Geothermal Reservoirs .....	211
8.1 Introduction .....	214
8.2 Characterising Fluid Flow through a Fracture .....	216
8.2.1 Darcy's Law .....	216
8.2.2 Modelling Flow Pressure Heads.....	216
8.3 CI and CF for Flow Modelling.....	222
8.3.1 Preferential Flow Directions using CI .....	222
8.3.2 Incorporating Length and Aperture in CI: WCI .....	223
8.4 Comparison of the results .....	226
8.4.1 Preferential Flow Pathways using CF .....	227
8.5 Concluding Remarks.....	231
9 A Connectivity-Graph Approach to Optimising Well Locations in Geothermal Reservoirs .....	233
9.1 Introduction .....	236
9.2 Formulation and Methodology.....	238
9.2.1 Equivalent Length and Aperture for a Set of Fractures.....	238
9.2.2 Efficient Monte Carlo Sampling.....	241
9.3 Conclusion.....	244
10 Concluding remarks.....	245
10.1 Conclusions .....	247
10.2 Summary of contributions.....	256
10.2.1 Journal Papers .....	256
10.2.2 Conference Papers and Presentations.....	257
10.2.3 Talk .....	257
10.2.4 Developed and new terms, concepts, models, algorithms, frameworks and methods.....	257
10.3 Ideas for future research.....	260
1 Application of Connectivity Measures in Enhanced Geothermal Systems .....	265
1.1 Introduction .....	267
1.2 Connectivity in Fracture Networks .....	268
1.2.1 The Connectivity Measures.....	268
1.2.2 Relationships between CI and CF.....	270
1.3 Applications of CI and CF in characterising fracture networks .....	270
1.4 Concluding Remarks.....	272
2 Matlab Programs: Alghalandis Fracture Network Modelling (AFNM) Package .....	275
2.1 Introduction .....	276
2.2 License .....	277
2.3 List of functions .....	277
2.4 Functions for Two Dimensional Cases.....	279
2.4.1 Angles2D .....	279
2.4.2 Centers2D.....	279

2.4.3 Lengths2D.....	279
2.4.4 GenFNM2D.....	280
2.4.5 ClipLines2D.....	281
2.4.6 LinesXLines2D.....	281
2.4.7 LinesX2D.....	282
2.4.8 LinesToClusters2D.....	283
2.4.9 Density2D.....	283
2.4.10 Histogram2D.....	284
2.4.11 RandLinesInPoly2D.....	284
2.4.12 Sup2D.....	285
2.4.13 SupCSup2D.....	286
2.4.14 SupXLines2D.....	286
2.4.15 SupXNLines2D.....	287
2.4.16 P21G.....	287
2.4.17 ConnectivityIndex2D.....	288
2.4.18 ConnectivityField2D.....	289
2.4.19 BreakLinesX2D.....	289
2.4.20 Rotate2D.....	290
2.4.21 SortPoints2D.....	290
2.4.22 Isolated2D.....	291
2.4.23 Backbone2D.....	291
2.4.24 IsolatedLines2D.....	292
2.4.25 BackboneToNodesEdges2D.....	292
2.4.26 Expand2D.....	293
2.4.27 Resize2D.....	293
2.4.28 DrawLines2D.....	294
2.4.29 LinesToXYnan2D.....	294
2.4.30 ExpandAxes2D.....	295
2.4.31 Titles2D.....	295
2.5 Functions for Three Dimensional Fracture Networks.....	296
2.5.1 RandPoly3D.....	296
2.5.2 GenFNM3D.....	297
2.5.3 Sup3D.....	298
2.5.4 ClipPolys3D.....	298
2.5.5 PolysX3D.....	299
2.5.6 PolysXPolys3D.....	300
2.5.7 PolyXPoly3D.....	301
2.5.8 SupCSup3D.....	301
2.5.9 BBox3D.....	302
2.5.10 Expand3D.....	302
2.5.11 Resize3D.....	303
2.5.12 SaveToFile3D.....	303
2.5.13 SavePolysToVTK3D.....	304
2.5.14 SetAxes3D.....	305
2.5.15 DrawPolys3D.....	305
2.5.16 DrawSlices3D.....	306
2.5.17 VolRender3D.....	307
2.5.18 Vol3D.....	307
2.6 Generic Functions.....	308
2.6.1 Scale.....	308
2.6.2 ToStruct.....	308
2.6.3 KDE.....	308
2.6.4 Smooth.....	309
2.6.5 dict.....	309

2.6.6 Clusters .....	309
2.6.7 CheckClusters .....	310
2.6.8 Labels .....	310
2.6.9 Relabel .....	311
2.6.10 Stack .....	311
2.6.11 Group .....	312
2.6.12 FarthestPoints .....	312
2.6.13 PDistIndices .....	313
2.6.14 Occurrence .....	313
2.6.15 ConnectivityMatrix .....	313
2.6.16 FullCM .....	314
2.6.17 FNMTToGraph .....	315
2.6.18 LoadColormap .....	315
2.6.19 SaveColormap .....	316
2.6.20 SecondsToClock .....	316
2.6.21 Colorise .....	317
2.6.22 ShowFNM .....	317
2.6.23 Round .....	317
2.7 Example Full Programs .....	318
2.7.1 Example: Simulation of 2D Connectivity Index .....	318
2.7.2 Example: Two-dimensional Line Sampling .....	319
2.7.3 Example: Simulation of 3D Connectivity Index .....	321
2.7.4 Example: Intersection Analysis and Fracture Clusters .....	322
2.7.5 Example: Density Analysis .....	324
2.7.6 Example: Backbone Extraction .....	325

## Table of figures

- Figure 1.1:** Rock mass. Rock blocks on the left which are of interest of rock mechanics and geotechnics engineering, and fractures (fracture network) on the right which are of interest of water resources, petroleum, geothermal and mining engineering. ....27
- Figure 1.2:** Enhanced Geothermal energy System (EGS). Cold water is pumped into the injection well to reach the geothermal heat. The contact between the fluid and hot dry rock is made due to existing and or stimulated fractures building an appropriate and efficient connected network for heat exchange (chamber). Redrawn and painted from MIT (2010). .29
- Figure 1.3:** A real three-dimensional fracture network, the Leeds Rock Fracture Dataset (Dowd et al. 2009). On the right, fracture trace lines are generated by means of intersections between fractures and a horizontal plane (red rectangle). It follows that fracture trace lines can be simulated by means of appropriate distribution functions for their locations (e.g., Poisson), length (e.g., Power-law) and orientation (e.g., von-Mises). ...33
- Figure 1.4:** A historical visual review of fracture modelling proposals in the literature. (A) Orthogonal, (B) Baecher, (C) Enhanced Baecher (fractures can clip each other), (D) BART (random size), (E) Dershowitz (complex shapes on a plane), (F) Density model (inhomogeneous) and (G) Randomized polygonal shapes (images from Staub et al. 2002). .35
- Figure 1.5:** Realistic model of fractures and fracture network using polygonal shapes for fractures generated and distributed by means of marked point processes. Comparing this simulation with the real fracture data set shown in Figure 1.3(left) shows a high match with reality. ....36
- Figure 1.6:** Commonly used distribution functions in fracture network modelling. Those without a negative tail and having a long positive tail are commonly used for modelling of the length of fractures. Gaussian distribution in its polar form i.e., von-Mises distribution is used for orientations. Poisson distribution is used indirectly to model the location of fractures in space satisfying complete spatial randomness criterion. This figure is not meant to exclude the use of any other distribution for any of the attributes of fractures and fracture networks. ....38
- Figure 2.1:** CoG vs. CoM; CoG is resistant against density of points (vertices) and thus more suitable for representing fractures. ....46
- Figure 2.2:** Edge effect for a Voronoi diagram. (a) no correction, (b) periodic network correction, (c) buffer zone solution. ....50
- Figure 2.3:** The periodic boundary edge correction of a Voronoi diagram may fail to generate a satisfactory tessellation if the point pattern is not homogeneous. The red parts are Voronoi blocks (cells) that do not contain a point and these are, thus, redundant. Shortcomings such as this inhibit the application of the periodic method as the associated error is apparently greater than not applying any edge correction. ....51
- Figure 2.4:** Fundamental morphological operations. Opening is combination of erosion and dilation; while in closing first dilation then erosion applies. Numbers in the main map are areas of blocks. Note that opening operation is more like mechanical erosion i.e., narrower the block more chance to be completely dismissed. Also note that how closing fills opened areas. ....53



**Figure 2.5:** Point patterns: (a) regularly spaced, (b) randomly located and (c) clustered. Note that in all three patterns the number of points is 100; however, the resulting forms are significantly different. ....55

**Figure 2.6:** Point patterns can be homogeneous or inhomogeneous. Inhomogeneous point patterns expose intensity function rather than a single density value for the points. In (b) the intensity function is  $\lambda = \alpha x$  in which  $\alpha = 0.83$ . Both (a) and (b) have 100 points. ....56

**Figure 2.7:** Realisations of points in three-dimensions using (left) homogenous Poisson point process with the resulting intensity of 368 and (right) inhomogeneous Poisson point process (IPPP) with the resulting intensity of 682. The intensity function for IPPP was  $f(x, y, z) = 10x^2 + y^2 + z^2$ . ....56

**Figure 2.8:** Same locations and same lengths but different orientations. In (a) fractures are mainly oriented E-W; (b) partially oriented towards E-W and (c) are randomly oriented. For (a) von-Mises distribution with  $mean = 0$  and  $\kappa = 1000$ , for (b) with  $mean = 0$  and  $\kappa = 10$  and for (c) with  $mean = 0$  and  $\kappa = 0$ . ....58

**Figure 2.9:** Same locations and same orientations but different models of lengths. In (a) fractures are all the same size; in (b) they follow a power-law (exponential) distribution truncated between [0.01, 0.9] and (c) they are infinite in length. ....59

**Figure 2.10:** Various fracture network models can be generated by combining location, length and orientation models. The examples shown demonstrate the flexibility of the approach to modelling almost any form of fracture network. ....60

**Figure 2.11:** Fractures are clipped at the boundaries of the study region. Clipping is followed by a cleaning up stage in which clipped fractures that are too short are removed from the network. ....62

**Figure 2.12:** After clipping a FNM, the centres of clipped fractures are updated. For this example, updating the fracture centres has no significant effect on the density map of centre points. ....62

**Figure 2.13:** Fracture patterns generated by changing parameter values of a von-Mises distribution function for generating orientations of fractures. Pairs of values in captions refer to parameters for two different sets of fractures. ....63

**Figure 2.14:** Growing Fracture method; (a) 50 locations, (b) some fractures still growing, (c) the final result in which the growing process has stopped and a set of well-defined blocks is created. ....64

**Figure 2.15:** Rock blocks generated by means of GFNM for each setting listed in Figure 2.13. ....65

**Figure 2.16:** GFNM in which growing fractures are conditioned to the existing structures e.g., blue fractures etc. ....66

**Figure 2.17:** Voronoi tessellation fracture network modelling method; (a) 50 locations, (b) Voronoi network and locations (c) the final result in which a set of well-defined blocks is also created. ....66

**Figure 2.18:** Marking a subset of fractures as filled (closed due to precipitation, collapsing stresses etc.), so as to be removed from the fracture network. The criteria for

removal is for the acute angle of the fracture to the horizon to be less than $30^\circ$ and a probability of removal 70%; or 10% probability of removal for any fracture. ....	68
<b>Figure 2.19:</b> Blocks generated by GFNM and VFNM can be subjected to mathematical morphology operations such as erosion. These models simulate rock block erosion due to contact deformation or fluid flow, for example. It can be seen that GFNM is affected more than VFNM by the erosion procedure. The reason is the type of blocks. ....	69
<b>Figure 2.20:</b> Fully connected pathways (left), which can be used as input for evaluation of fluid flow (right) in fractured rock. ....	70
<b>Figure 2.21:</b> Blocks generated by extension and trimming. Fractures are extended or trimmed depending on the cost of extension or trim. This approach may result in convex or concave blocks. ....	71
<b>Figure 2.22:</b> A simple but robust method to generate polygonal fracture shapes for three-dimensional fracture network simulations. The method is based on generating convex polygons enclosed by a circle or an ellipse. As shown the latter provides a means of accounting for anisotropic shapes. The variety of the shapes in both examples demonstrates the ability to accommodate almost any polygon. Also note the size variation in the second method (ellipse). ....	72
<b>Figure 2.23:</b> Three-dimensional fracture network and associated pipe model. Pipes are generated by connecting the centres of each fracture to the centre of intersection line (point in vertex touching case). The radius of pipes can be defined locally according to the apertures of the associated fractures. ....	74
<b>Figure 2.24:</b> Conditioning fracture locations to given sample points. Simulated Annealing is used to honour given locations. ....	75
<b>Figure 2.25:</b> Conditioning fracture network to existing fractures observed in two boreholes. ....	76
<b>Figure 2.26:</b> Line detection using the Hough Transform. In (b) the generated lines in (a) are discretised into pixels added significant noise. The Hough transform for (b) is shown in (c) which results in linear objects shown as red in (d). The resulting lines (e) compared to the original lines (a) show a very good match despite the effects of noisy data added in (b). ....	78
<b>Figure 3.1:</b> 2D and 3D Fracture Network Simulations via Stochastic Marked Point Processes. ....	87
<b>Figure 3.2:</b> Framework to generate realistic fracture network by means of marked point process. ....	87
<b>Figure 3.3:</b> Demonstration of iteration number ( $n$ ) for generating 1000 random number with Poisson distribution of density 25. ....	88
<b>Figure 3.4:</b> Demonstration of the effect of the variation of $\kappa$ on Fisher function in the application for orientation angles of fractures. ....	89
<b>Figure 3.5:</b> A robust algorithm to generate polygonal shapes for fractures. ....	90

**Figure 3.6:** Possible intersection situations between two polygonal fractures in realistic fracture networks .....91

**Figure 3.7:** A full robust framework for fracture-fracture intersection analysis.....92

**Figure 3.8:** Pseudo-code for Segment-Plane intersection .....93

**Figure 3.9:** (left) Fracture Network HPPP; (right) Trace locations (green) and intersection points (red).....94

**Figure 3.10:** Density map of locations (left) and intersection points (right).....94

**Figure 3.11:** Distribution of the length of intersection lines in three-dimensional fracture network (class=length categories) .....95

**Figure 3.12:** Relationship between percolation sate reached and the variation in the range of length of fractures .....96

**Figure 3.13:** A 3D convex hull showing the block (left) and 387 fractures (right).....97

**Figure 3.14:** the result of intersection analysis which demonstrates the largest cluster of connected fractures (green) .....97

**Figure 3.15:** Distribution of the length of intersection lines in three-dimensional fracture network: Leeds Fracture Data Set (class=length categories).....98

**Figure 3.16:** Density map of locations (A) and intersection points (B) of 3D fracture network, Leeds Fracture Data Set .....99

**Figure 4.1:** A demonstration of BFS applied to a point cloud with a total of 367 points in two dimensions comprising 300 random points superimposed on a set of 5 lines discretised into a total of 67 points. Qn in the titles stands for quantiles computed from the ranks. ..109

**Figure 4.2:** LCF in action: Filtering lines and fitting the main orientations using a tolerance=0.1. (a) a small part of the original line data set produced by BFS; (b): clusters of line segments produced by our method and (c) the two fitted main lines (red) as proposed by LCF..... 110

**Figure 4.3:** Comparison of the performance of Least Squares (LS) and RANSAC for highly contaminated linear objects in a point cloud; RANSAC finds the best fit after 20 iterations which were completed in less than a second. The two lines around the fitted line in the RANSAC result correspond to the tolerance value  $\tau$  used to compute the cost function..... 111

**Figure 4.4:** Performance of RANSAC for line (a) and plane (b) fitting;  $q$  is the probability of outliers..... 113

**Figure 4.5:** Application of EBFS to a two-dimensional point cloud comprising 1,365 points; (a) point cloud with 30 embedded fracture lines represented using 365 points; (b) the same point cloud without the lines drawn; (c) lines detected by EBFS; (d) detected fracture lines in (c) are superimposed on the 30 initial hidden lines in (a). ..... 116

**Figure 4.6:** Lines resulting from RANSAC for different numbers ( $i \in [50,100,200,400]$ ) of iterations per stage. The embedded lines are shown in (d). As the number of iterations

increases the large numbers of linearly aligned points are correctly identified as embedded lines as shown in subplot (f). ..... 118

**Figure 4.7:** A demonstration of the effect on the RANSAC fitting process of varying the tolerance value. Figures (b) and (f) demonstrate clearly the importance of choosing the correct tolerance. Note that in reality, for example, field measurements, the ratio of overall inliers over outliers is much higher than that (0.192) used in these examples. .... 119

**Figure 4.8:** Performance of RANSAC as a function of varying the number,  $np$ , of trials per stage and a constant distance tolerance equal to 0.001 (a) and varying the distance tolerance  $tol$  with  $np$  fixed at 200 (b)..... 120

**Figure 4.9:** Plane detection using EBFS method. (a) three-dimensional point cloud comprising 109 oriented points (open circles) representing three fracture surfaces and 500 points (filled circles) with coordinates from a Poisson process; (b) resulting fractures (only the first three are shown). Crosses are random points associated with the detected fractures. .... 121

**Figure 4.10:** The greatest numbers of points are associated with the earliest selected planes..... 122

**Figure 4.11:** The fracture network generated by applying RANSAC to the simulated point cloud. Compared with BFS, RANSAC, with  $\{tol = 0.01; mcn = 500\}$  achieves an acceptably accurate result at significantly reduced computational cost,  $CRANSAC = CBFS/1208$ . The efficiency of RANSAC increases further as the amount of data increases. .... 123

**Figure 4.12:** Habanero point cloud data corresponding to seismic events recorded in the 2003 fracture stimulation. The number of points is 23,232 and the cloud is approximately horizontally oriented. .... 124

**Figure 4.13:** The construction of the first polygons requires assessment of the largest number of points and the greatest amount of time with the number and time declining exponentially with the number of stages. .... 125

**Figure 4.14:** Fractures fitted to the Habanero seismic point cloud data; (a) All 186 polygons where the first one involved 956 associated points, the second one 857, the third 786; (b) The 143 fitted polygons that have dip angle less than or equal to 15 degrees; (c) The first three polygons fitted by RANSAC. .... 126

**Figure 4.15:** (a) The number of points associated with the fitted fractures for the Habanero point cloud; (b) the areas of fractures; (c) the dip angles; and (d) the number of edges of the fitted fractures. .... 127

**Figure 4.16:** Histogram of dip angles for 186 fitted fractures for the Habanero seismic point cloud. .... 127

**Figure 5.1:** The lines  $[p1, p2]$  and  $[p3, p4]$  used in the product-similarity measure. .... 145

**Figure 5.2:** Results of the simulated dataset with 70 embedded lines; (a) simulated fractures; (b) point sampling with noise 0.01; (c) the final fitted fractures; (d) the number of fractures versus the iteration number; (e) the objective function values versus the iteration number; (f) the models of the length histogram for the actual and fitted fractures..... 146

**Figure 5.3:** Initial map before optimization.....149

**Figure 5.4:** Rose diagrams for the simulated dataset: (a) actual lines; (b) fitted lines...150

**Figure 5.5:** A summary of 30 simulations: (a) number of final fractures versus number of initial fractures; (b) function values after optimization; (c) actual (bold line) and fitted length histograms.....150

**Figure 5.6:** Results for Habanero dataset; (a) seismic point cloud; colours represent time domain of the seismic events; (b) initial fractures propagated from the borehole; (c) final fitted fractures; (d) distribution of point associations (i.e. number of points per fracture) .151

**Figure 5.7:** (a) Variation of number of fractures; (b) the total objective function value; (c) the amount (area) of fracturing during the optimization process; (d) distribution of the associated distances; (e) distribution of major axis; (f) distribution of minor axis. ....152

**Figure 6.1:** Fracture network density maps based on fracture centroid points (FCD map). A density map of this type can be seen as a quick and useful evaluation of concentration of fractures in the study region. It is, however, a biased estimation due to the simplistic representation of fracture lines as points. ....157

**Figure 6.2:** High fracture centroid density does not necessarily imply fracture intersections. In (a) despite its appearance (due to size of image resolution) there are no percolating sides. This example shows the serious shortcoming of using FCD for describing the connectivity and/or percolation state of the network. ....158

**Figure 6.3:** Summary of fracture intensity measures proposed by Dershowitz (1992). .159

**Figure 6.4:** P11 using two systems of scanline sampling: random (a) and regular (b). Histograms of P11(r) and P11 are shown in (c). ....160

**Figure 6.5:** P21 using Monte Carlo simulation. A number of 1000 square samples of size 0.1×0.1 were taken. In (c) in the title of histogram of P21 values, 31.757 is for entire study region while 34.513 is average of 1000 samples. ....160

**Figure 6.6:** It appears that the size of sample (w) influences the calculated P21 value. However, the sign of the influence varies among the different realisations, suggesting that the proposed relationship is strongly associated with each realisation. In the figure, each data point of each curve is produced by averaging 1000 samples. Ten realisations were used.....161

**Figure 6.7:** Fracture Density calculated for each cell in the grid by counting the number of fractures wholly contained within a cell and the number that intersect the boundary of the cell. GF<sub>n</sub> is the Generalised F<sub>n</sub> computed for grid sizes in the range [3, 25], i.e., cell size varying from 1/9 to 1/625 of the area of study region. ....162

**Figure 6.8:** LEC analysis. The largest one is highlighted. This measure is useful to determine isolated areas in the study region according to a distance of interest. Note that the edges of the study region have also been considered as constraints. The exponential distribution of the areas is apparent as shown in the histogram on the right.....163

**Figure 6.9:** Largest empty convex-hulls. Similar to LEC in its application but provides a much more realistic measure of empty space. Note that for any fitted convex-hull a maximum area circle can easily be found.....163

- Figure 6.10:** Distance map based on the distance from fracture trace lines. The darker the shade of blue the farther the location is from a fracture. This analysis may have applications in ranking a study region for safety issues. In (c) the contour values are the logarithms of the distance values..... 164
- Figure 6.11:** Convex-hull and buffer effect applied to a fracture network. Note that the buffer effect is applied only on clustered fractures assuming that the isolated fractures are not affected by expansion mechanisms. .... 165
- Figure 6.12:** Histograms of block areas for GFNM and VFNM. Smaller area blocks are more dominant in GFNM compared to VFNM..... 166
- Figure 6.13:** Backbone structure of an FNM. (a) FNM; (b) backbone; (c) density map of centres of backbone line segments. .... 167
- Figure 6.14:** Density map of block centres in an FNM, which is useful in identifying blocks of smaller area blocks in rock masses..... 168
- Figure 6.15:** All possible ways in which two lines can intersect. In the overlaying cases it is better to choose the centre of the overlapping segment although other choices are also valid. For example, when using intersection analysis for segmenting the overlaying lines both endpoints are reported. .... 169
- Figure 6.16:** Application of intersection analysis results in groups of fractures called fracture clusters. The size of a cluster can be defined as its number of member fractures (cardinality) or the area that the convex-hull of a fracture cluster covers (coverage)..... 170
- Figure 6.17:** Three-dimensional fracture network and fracture clusters. In (b) the first largest clusters are shown in decreasing order in red, green and blue. In (c) the next three largest clusters are shown. The cardinalities of the value clusters are shown next to them. .... 170
- Figure 6.18:** Pipe model constructed for the three-dimensional fracture network model shown in Figure 6.17. The first three largest fracture clusters are highlighted. The pipe mode clearly exhibits the possible domain for fluid transport for each fracture cluster..... 171
- Figure 6.19:** The intersection points between fractures are used for density mapping. 172
- Figure 6.20:** Intersection Density maps for an extended FNM. Every fracture trace has been extended to reach the boundary of the study area..... 172
- Figure 6.21:** Inter-connectivity ( $I_i$ ) between two sets of fractures in a fracture network.  $I_1$  in the title means  $i = 1$  i.e., fracture set 1 (fs1) and similarly for fs2. As shown, this measure is not transitive; depending on the choice of  $i$  two different values are calculated as 0.961 and 1.685..... 173
- Figure 6.22:** Inter-connectivity measured between three fracture sets (orientation classes)..... 174
- Figure 6.23:** Density maps of intersections between normals of fractures for GFNM and VFNM. The normal lines are the same size as the fractures. The VFNM model suggests that NID is associated with the BCD whereas GFNM is not..... 175

**Figure 6.24:** Mohr circles for extensional failures (e.g., due to hydraulic pressures) and associated fracture patterns. Labels “a” to “d” on Mohr circles correspond to patterns shown on sub-figures (a) to (d), respectively.....176

**Figure 6.25:** Various patterns of fracture networks in three dimensions due to extensional failure criteria. ....177

**Figure 6.26:** Comparison and correlation between various proposed measures. Correlation values are Pearson-r computed pixel-wise. ....178

**Figure 7.1:** Lattice-based connectivity (left) vs. fracture/support-based connectivity (right) measurement. In lattice-based methods the cell is the key element for defining connectivity in three forms: vertices, edges or faces. In support-based methods, however, connection is determined only via fractures. In the example shown (systematic sampling scheme),  $pv \leftrightarrow qv$  and  $qv \leftrightarrow rv$  but  $pv \nleftrightarrow rv$ . Also note that two disjoint supports can be connected via fracture(s). Two overlapping supports are not connected if there is no connecting fracture(s). ....187

**Figure 7.2:** Three scenarios for fully isolated fracture networks. As they are fully isolated the ID maps will be void while associated DFC maps are shown above. DFC contours are generated using the KDE method with automatic optimum bandwidth selection. ....189

**Figure 7.3:** The computational stages of the CF; (a): A realisation of a FNM; (b): The resulting CF block map. The darker the colour the higher the value of CF; (c): The smoothed contour map of super-sampled CF. (d): stages for central 9 cells.....194

**Figure 7.4:** A realisation of a FNM, the CF map computed on a  $25 \times 25$  grid and the interpolated contour map of the CF, from left to right respectively.....195

**Figure 7.5:** Lattice-based connectivity evaluation of a fracture network. For comparison, the example fracture network is the same as that used in Figure 7.4a for the CF map. Note the difference in the output of lattice-based connectivity and support-based connectivity evaluations on a synthesised ray form fracture network. ....195

**Figure 7.6:** An example of the evaluation of GCF; (a): First stage: evaluation of CF for different support sizes  $v$ ; (b): the fracture network; (c, d): The resulting GCF maps.....197

**Figure 7.7:** Box-plot of CF values for the various support sizes (1/3 to 1/20 of region  $\mathcal{R}$ ) shown in Figure 7.6a. An interesting observation is that the median CF value for different support sizes is less variable (note that here the CF values are not normalised). Increasing the resolution generates a longer positive tail for the CF distribution. ....198

**Figure 7.8:** Comparison of ID, CI and PCF for a set of 60 realisations of a FNM as input data. One realisation (#6) is shown as an example. ....200

**Figure 7.9:** Comparison of CF and ID: four realisations (cases) of a FNM and the corresponding ID and CF.....202

**Figure 7.10:** Comparison of various FNM measures: DFC, ID, CF, GCF and fracture clusters. ....203

**Figure 7.11:** Correlation coefficient between CF and GCF for 60 realisations of a FNM; While support size for CF was 0.16% of the size of study region  $\mathcal{R}$ , for GCF it varied from

0.16% to 100% of  $\mathcal{R}$ . CF and GCF are highly correlated with an average correlation coefficient of 0.81. ....204

**Figure 7.12:** Scatter plots for  $X_f$ , P21 and **CF** measures based on 700 realisations from FNM (locations: Homogeneous Poisson Point Process with  $\lambda = 100$ ; orientations: von-Mises with mean direction on X axis and  $\kappa = 0$ ; lengths: Truncated-Power-Law [0.03, 1]). Note that all measures are normalised to the range [0, 1]. Fitted curves are polynomials of order 4. Support size for **CF** was  $\mathcal{R}/625$ . ....205

**Figure 7.13:** Pathway analysis using the CF for the fracture network shown in Figure 7.6b. Two regions,  $\mathcal{R}1$  and  $\mathcal{R}2$ , are also identified as having the highest potential to change the percolation state of the region (see next section). The dashed line represents the major flow pathway while the dotted line shows a less developed pathway from the bottom to the top of the region. ....206

**Figure 7.14:** A system of fractures that is percolating for sides  $S_1$  and  $S_2$ ; the numbers shown are the CF values for each cell. The cells with higher CF values i.e., *cell*2,2 and *cell*(3,3) are used to assess the percolation state of the system. ....208

**Figure 7.15:** The CF for the Leeds Rock Fracture Data Set; (a to d): Stages for the preparation of the three-dimensional grid for CF evaluation; the evaluated CF maps are shown as: (e): the interpolated slice map, (f): the volumetric rendered map. ....209

**Figure 8.1:** Framework for studying directional flow in two-dimensional fracture networks: (a) four pressure head values set in the corners providing simple left-to-right pressure gradient, (b) four pressure head values set on main axial directions (c) 24 pressure head values set on a circle calculated via the Hamming filter, (d) marking isolated/partially isolated fractures after intersections analysis, (e) extracted and trimmed final pathways, and (f) pressure head solution for the boundary setting as of (c). ....219

**Figure 8.2:** The resulting orientations for flow are shown as rose diagrams: grey-filled for 15 degree bins and solid-black line for 10 degree bins. The boundary conditions were as in Figure 8.1c. For full directional coverage the boundary references were rotated by 15 degree steps; the resulting orientations are shown on the right. ....221

**Figure 8.3:** The framework for a finite difference method used to model the flow through fracture networks in two dimensions. The bottom section confined by dashed lines is repeated for any direction to obtain full coverage of orientations in the inlet pressure heads. ....221

**Figure 8.4:** (left) Directional SCI verifies homogeneity for the isotropic fracture network. (right) The anisotropy in the anisotropic fracture networks was clearly depicted by Directional SCI. ....222

**Figure 8.5:** The variation in the length of pathway and also in the aperture of the connected fractures in the pathway affects the heat exchange process in the EGS reservoir. Using CI the two pathways are reported as [1, 1] but using WCI they are reported as  $\{a, b: a, b \in [0..1] \subset \mathbf{R}\}$  depending on the lengths and apertures of fractures forming the pathways. ....222

**Figure 8.6:** CI and SPL vs.  $h$ ; larger  $h$  values result in lower CI values and higher SPL values. The fluctuation on the right of the SPL curve can be explained as an edge effect caused by the geometrical boundaries of the fracture networks. This computation was



conducted on 60 realizations on a grid of  $25 \times 25$  and for each h value 100 samples were taken.....224

**Figure 8.7:** Variations in the length and aperture of the pathways affect the flow direction depending on the configuration established by the interconnection between fractures and the support locations. (B) is the response of CI in the evaluation of flow direction of a system shown in (A) while (C) to (F) are assessments by means of WCI. (G) shows possible resulting flow directions.....224

**Figure 8.8:** Flow (velocity factor) through fractures using (top) Lattice Boltzmann method and (bottom) Finite Element method for (A) equal aperture, (B) variable aperture per pathway, and (C) variable aperture per fracture. FEM was done using the COMSOL software package. ....225

**Figure 8.9:** The framework and the code developed in this research have been checked extensively by applying it to various typical networks. As shown on the right the finite difference (FD), CI and WCI methods have correctly determined the flow direction for a particular case shown on the left.....225

**Figure 8.10:** The eight resulting flow directions from 70 realizations of a fracture network using three methods: CI as red, WCI as green and FD as blue rose diagrams. WCI matches the FD roses significantly better than CI. ....228

**Figure 8.11:** Histograms of orientation errors for CI and WCI based on 70 realizations of a fracture network model; WCI provides noticeably lower error values compared to CI; error statistics include the minimum ( $e$ ), mean ( $e$ ), mode ( $e$ ) and maximum ( $e$ ). This suggests WCI consistently provides more accurate results than CI for modelling flow directions. ..229

**Figure 8.12:** The procedure of determining ranked flow pathways using the CF measure. ....230

**Figure 9.1:** Equivalence between fracture and resistor networks. Worked examples of various pathway settings and equivalent weighted connectivity index. The same concept is used to determine the weight for different settings of pathways between the two wells....240

**Figure 9.2:** Synthetic fracture network ( $n_{fractures} = 200$ ,  $\theta \rightarrow vonMises \mu = 0, \kappa = 0$ ,  $\ell \rightarrow Powerlaw lmin = 0.1, lmax = 0.9$  its backbone structure.....242

**Figure 9.3:** Grid-based (SG) and random sampling (SR) schemes. Note that in the SG scheme the locations are not pairs, that is, the distance evaluation is applied during brute-force searching; while in the SR scheme the only pairs of locations generated are those that satisfy the distance distribution function. Notice that SR gives a much higher density of sampling points while still remaining practical. ....242

**Figure 9.4:** The shortest fluid flow pathway (red polyline; considering the effect of the length and aperture simultaneously) is determined between the two simulated wells (blue squares). The two cases are examples from a set of 10,000 trials. For each trial the total pathway weight is given at the top of the image.....243

**Figure 9.5:** Outputs from the proposed procedure for optimal well locations. The pathway with the lowest weight has been determined among 10,000 trials and is shown in the *left*. The associated weight is 0.009798 while the highest weight was 0.121129. In the *right*, a regional map of the suitability of well locations is shown on which the fracture network is superimposed. The darker the blue more suitable are the locations.....243

**Figure 10.1:** Rock mass: the matrix (blocks) and fractures. Modelling of fluid flow through fractures and the stability of rock blocks are two important applications of fracture network modelling.....248

**Figure 10.2:** RCF. The grid is centred on each point as shown. CF is calculated for each move. RCF is the sum of all resulting CF maps.....261

## Abstract

Fracture and fracture network modelling is a multi-disciplinary research area. Although the literature in general is significant, many research challenges remain. The complex geometry and topology of realistic fracture networks largely determine the static and dynamic mechanical properties of rock. In applications to hot dry rock geothermal reservoirs it is not possible to observe or measure fractures directly on any scale and the only data available are indirect measurements, such as seismic activity generated by hydraulic fracture stimulation. The lack of direct data and the complexities of the fracture characteristics make fracture network prediction and modelling in these applications very difficult. The ultimate purpose of the fracture and fracture network models is to evaluate the response of the fracture system to stress regimes and fluid flow. As understanding of the effective factors in the geometrical modelling of fractures and consequently topological properties of fracture networks increases, more accurate and hence more reliable results can be achieved from associated analyses. For flow modelling in geothermal reservoirs, the critical component of a fracture model is the connectivity of the fractures as this determines the technical feasibility of heat production and is the single most significant factor in converting a heat resource to a reserve. The ability to model this component effectively and to understand the associated system is severely constrained by the lack of direct data. In simulations, the connectivity of a fracture network can be controlled to a limited extent by adjusting the fracture and fracture network parameters (e.g., locations, orientations) of the defining distribution functions. In practical applications connectivity is a response of the system not a variable. It is essential to pursue modelling methods that maximise the extraction of information from the available data so as to achieve the highest possible accuracy in the modelling. Although the evaluation of fracture connectivity is an active research area, widely reported in the literature, almost all connectivity measures are based on degraded representations of the fracture network i.e., lattice-based. The loss of fracture connectivity information caused by using discrete representations is significant even when very high resolutions (assuming they are feasible) are used. This is basically due to the fact that the aperture dimensions of

fractures are several magnitudes smaller than their lengths. If discretisation is necessary, then a better approach would be to retain all connectivity information between fractures, i.e. for connectivity information to remain invariant to the resolution of the discretisation. Such a method would provide more reliable evaluation of connectivity. This thesis covers the modelling of fracture networks, the characterisation (particularly connectivity) of fracture networks and applications.

## Statement of Originality/Consent/Copyright

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint award of this degree.

I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines.

Younes Fadakar Alghalandis

10 March 2014

## Acknowledgments

I am thankful to my parents for their lifetime love and support; to my brother and sisters for their limitless kindness, support and encouragement; to my Iranian old friends; to my new friends worldwide; and to whom encouraged me to continue my study.

I would like to thank Professor Peter Dowd for his continuous support as the principal supervisor and Associate Professor Chaoshui Xu as the co-supervisor during my PhD study in the University of Adelaide. They were helpful in many aspects of my research. I am particularly in debt to Professor Dowd editing assistance especially on published papers. Without his help publication in high standard journals would not have happened.

I am also grateful for the allocation of a PhD scholarship funded by the Australian Research Council as part of ARC Discovery Project DP110104766 (Stochastic modelling of fractures in crystalline rock masses for hot dry rock enhanced geothermal systems) awarded to my supervisors.

Finally I am grateful to the University of Adelaide for admission to make this study possible.