Scanning Laser Doppler Vibrometry for Strain Measurement and Damage Detection

Stuart J. Wildy

School of Mechanical Engineering
The University of Adelaide
South Australia 5005
Australia

Supervised by

A/Prof Andrei Kotousov
A/Prof Ben Cazzolato
Dr John Codrington
Dr Sook-Ying Ho

Abstract

Numerous strain measurement and damage detection techniques have been developed over the last century. These techniques include strain gauges, digital image correlation, radiography and ultrasonic inspections. All have various advantages, as well as disadvantages, which make each suited to specific applications.

With the development of laser Doppler vibrometry, a number of techniques have been established for non-destructive evaluation, such as the measurement of bending strain, as well as damage detection using kinematic parameters, including displacement and curvature. With recent advancements in laser Doppler vibrometry technology (such as 3D scanning laser Doppler vibrometry for three-dimensional displacement measurements, improved velocity decoders and increased spatial resolution) the door has been opened to develop techniques for measuring surface strain from in-plane displacements, as well as the development of new damage detection techniques based on the fundamental principle of deformation:- the governing differential equation of displacement.

The extensive literature review contained in this thesis identified a number of gaps in the field, including the evaluation of the accuracy of quasi-static bending strain measurements using current 1D SLDV technology, the precision of full-field surface strain measurement techniques utilising 3D SLDV, and new detection techniques based on the violation of the governing differential equations of displacement. Thus, the research contained in this thesis focussed on these areas.

The first part of this thesis presents an investigation into the use of 1D and 3D scanning laser Doppler vibrometry for non-contact measurement of quasi-static bending strain in beams and surface strain in plates, respectively. The second part presents a new damage detection technique based on the governing differential equations of displacement in beam and plate structures. Two algorithms are developed to determine a violation in the governing differential equations created by either a delamination in a composite beam with out-of-plane displacements, or by a crack in a plate with in-plane displacements.

Declarations

Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Stuart Wildy and to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

Permissions

I give consent to this copy of my thesis when deposited in the University Library being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

The author acknowledges that copyright of published works contained within this thesis (as listed below*) resides with the copyright holder(s) of those works.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

Stuart Wildy Date

---

* Wildy, S., Cazzolato, B., Kotousov, A. and Weisbecker, H. (2010) 'New experimental strain measurement technique utilising a 3D scanning laser vibrometer', *in Proceedings of the Sixth Australasian Congress on Applied Mechanics*, Perth, Australia, pp. 738-747.

Wildy, S., Kotousov, A. and Codrington, J. 2008, 'New passive defect detection technique', *Australian Journal of Mechanical Engineering*, vol. 6, no. 2, pp. 101-105.

Wildy, S., Lee, C. and Yong, S. 2008, 'Monitoring of crack propagation using a cluster of piezo-sensors', *in Proceedings of the Fifth Australasian Congress on Applied Mechanics*, Brisbane, Australia, pp. 366-371.

Acknowledgements

First and foremost, I would like thank my wife. She has been the one constant throughout my PhD candidature; helped me through the many lows, but there also to celebrate the many joys that I have experience during my postgraduate studies at Adelaide University.

More recently, I would also like to thank my good-looking six month old boy Seth. He has provided me with the drive to complete this thesis, but also the necessary distractions required during the long and hard slog of thesis writing and proofing.

Thanks to all my family who have supported me and encouraged me throughout all my studies. Without their help I would be lost.

I would also like to thank my supervisors, who have provided me with support and encouragement over the last 4 years. I would like to very much thank my primary supervisor Andrei Kotousov with his drive for progress and papers, which has lead me to publish a number of journal and conference publications throughout my candidature. Sook Ying Ho, who was there from the beginning of my candidature and provided me with valuable support and feedback. Ben Cazzolato, who introduced me to the 3D Scanning Laser Doppler Vibrometry system mid-way through my candidature, which is now the corner stone of this PhD thesis.

Lastly I would like to thank John “Dr.Cod” Codrington, who became my supervisor near the end of my candidature, but more than that he has become a good friend that has been there for me any time of day to bounce around ideas, and thoughts, but also provide me with reassurance that I was on the right track.

I would like to thank the cohort of Mechanical Engineering Postgraduate Students at Adelaide University for their friendship, as well as the weekly excursions to the Exeter for lunch.

Last but not least I would like to thank the Mechanical and Electrical workshops for their assistance in making and constructing my rigs.

Contents

List of Figures ... **xiii**

List of Tables ... **xxvii**

Nomenclature ... **xxix**

Chapter 1. Introduction ... **1**

1.1 Strain Measurement 3

1.2 Non-Destructive Damage Detection 4

1.3 Aims and Objectives 5

1.4 Outline of Thesis 6

Chapter 2. Literature Review ... **9**

2.1 Strain Measurement 9

2.1.1 Strain Gauge 10

2.1.2 Photoelasticity 12

2.1.3 Moiré and Moiré Interferometry 14

2.1.4 Holographic Interferometry 15

2.1.5 Speckle Methods 17

2.1.6 Digital Image Correlation 19

2.1.7 Thermoelastic Stress Analysis 20

2.1.8 Summary of Strain Measurement Techniques 21

2.2 Damage Detection 22

2.2.1 Visual Inspection 23

2.2.2 Acoustic Emission 24

2.2.3 Ultrasonic 26

2.2.4 Guided Waves 28

2.2.5 Vibration-Based 29

2.2.6 Radiography 31

2.2.7 Eddy-current *32*
2.2.8 Electromechanical Impedance *34*
2.2.9 Summary of Damage Detection Techniques *35*

2.3 Scanning Laser Doppler Vibrometry 36

2.3.1 Strain Measurement Utilising SLDV *37*
2.3.2 Damage Detection Utilising SLDV *41*
2.3.3 Summary of Damage Detection Using SLDV *48*

2.4 Summary 50

2.4.1 Strain Measurement *50*
2.4.2 Damage Detection *51*

2.5 Gap in the Field of Research 51

2.5.1 Strain Measurement *52*
2.5.2 Damage Detection *53*

Chapter 3. Experimental Set Up 55

3.1 Scanning Laser Doppler Vibrometry 55

3.1.1 Optical Configuration *56*
3.1.2 Principle of Heterodyne Interferometry *57*
3.1.3 1D SLDV System *59*
3.1.4 3D SLDV System *61*
3.1.5 Measurement Parameters *64*

3.2 Test Set Up 64

3.2.1 Out-of-Plane Displacement Experiments *65*
3.2.2 In-Plane Displacement Experiments *70*

3.3 Summary 76

Chapter 4. Development of New Strain Measurement Techniques 79

4.1 Sources of Errors in Vibrometry Measurement 80
4.2 In-Plane Strain 82

4.2.1 Previous Studies *83*
4.2.2 Kinematics of Deformation *85*
4.2.3 Investigation of In-Plane Strain *91*
4.2.4 Savitzky-Golay Differentiation Parameters *95*
4.2.5 Mesh Size *98*
4.2.6 Loading *100*
4.2.7 Summary of In-Plane Measurement *102*

4.3 Measurement of Strain at Stress Concentrations 104

4.3.1 Results *106*
4.3.1 Discussion *113*
4.3.2 Summary Strain Measurement at Stress Concentrators *116*

4.4 Bending Strain 116

4.4.1 Previous Studies *117*
4.4.2 Surface Strains for a Plate Loaded in Bending *119*
4.4.3 Investigation of Bending Strain *123*
4.4.4 Savitzky-Golay Differentiation Parameters *125*
4.4.5 Mesh Size *129*
4.4.6 Out-of-plane Loading Amplitude *134*
4.4.7 Summary of Bending Strain Measurement *134*

4.5 Summary 136

Chapter 5. Detection of Crack Damage 141

5.1 Previous Studies 143
5.2 Principles of Deformation in Thin Plates 147

5.2.1 Equations of Equilibrium *147*
5.2.2 Strain Compatibility *149*
5.2.3 Governing Differential Equation *150*
5.2.4 Application of GDE to Damage Detection *151*

5.3 GDE of Plate Displacement Algorithm 154

5.4 Displacement Error Algorithm 157
5.5 Surface Strain Algorithm 162
5.6 Summary 168

Chapter 6. Detection of Delamination in Composite Beams 171

6.1 Principles of Bending in Thin Plates 174

6.1.1 Equations of Equilibrium 174
6.1.2 Governing Differential Equations 177
6.1.3 Application of Principles for Damage Detection 178

6.2 Analytical Delaminated Beam Model 180
6.3 Governing Equation of Beam Deflection 182

6.3.1 Algorithm 183
6.3.2 Filter size 184
6.3.3 Delamination Length 187
6.3.4 Delamination Depth 189

6.4 Beam Deflection Error 192

6.4.1 Previous Studies 192
6.4.2 Algorithm 194
6.4.3 Filter Size 196
6.4.4 Delamination Length 199
6.4.5 Delamination Depth 202

6.5 Beam Curvature Error 205

6.5.1 Previous Studies 205
6.5.2 Algorithm 206
6.5.3 Filter Size 208
6.5.4 Delamination Length 211
6.5.5 Delamination Depth 213

6.6 Summary 216

Chapter 7. Conclusion 223

7.1 Strain Measurement 224

7.1.1 In-Plane Strain 225

7.1.2 Bending Strain 226

7.2 Damage Detection 227

7.2.1 Detection of Crack Damage 228

7.2.2 Detection of Delamination in a Composite Beam 229

7.3 Overview 231

7.4 Further Research Possibilities 232

References 235

Appendix A. Strain Compatibility Equations 245

Appendix B. Savitzky-Golay Differentiating Filter 247

B.1 1D Filter Structure 247

B.2 2D Filter Structure 248

Appendix C. Delaminated Beam Model 251

Appendix D. Damage Detection Results 259

D.1 Governing Differential Equation of In-Plane Displacement 261

D.2 Displacement Error 265

D.3 Surface Strain Algorithm 271

Appendix E. Related Publications 279

E.1 Referred Journal Publications 279

E.2 Refereed Conference Publications 280

E.3 Abstract Reviewed Conference Publication 282

Appendix F. Awards and Achievements 283

List of Figures

Figure 2.1. T-stacked foil strain gauge. 10

Figure 2.2. Photoelastic fringe pattern of a polycarbonate plate with a hole, loaded uniformly (Fiene 2010).. 13

Figure 2.3. Moiré fringe effect.. 15

Figure 2.4. Fringe pattern produced using speckle interferometry (Unknown 2008). 18

Figure 2.5. Thermoelastic stress analysis of a conrod (Stress Photonics, Inc. 2007). 20

Figure 2.6. Diagram of the acoustic emission technique. 25

Figure 2.7. Traditional ultrasonic inspection techniques, (a) pulse-echo, (b) through-transmission, and (c) pitch-catch. 26

Figure 2.8. Schematic illustration of the radiographic imaging technique. 31

Figure 2.9. Schematic illustration of the eddy-current technique. 33

Figure 3.1. Optical configuration of the scanning head. 57

Figure 3.2. Photograph of the 1D vibrometer mounted on a tripod, along with external high-resolution video camera. 60

Figure 3.3. Image of the scan grid used for detecting through-the-width delamination in a composite beam. Note that the scan points are so close together, the scan grid looks like a continuous line. 60

Figure 3.4. 3D vibrometer (PSV-400-3D) mounted on a fix support (PSV-A-T34). 62

Figure 3.5. Three-Dimensional Alignment Specimen (PSV-A-450). 62

Figure 3.6. Illustration of the principle of three-dimensional displacement measurement using a 3D vibrometer. 63

Figure 3.7. Out-of-plane displacement rig. 67

Figure 3.8. Schematic diagram of delaminated composite specimens. 69

Figure 3.9. In-plane displacement rig. 71

Figure 3.10. Experimental setup of out-of-plane measurement experiments. 73

Figure 3.11. Detailed schematics of the in-plane loading specimens. (a) plain, (b) edge crack, (c) centre crack, and (d) notch. 73

Figure 3.12. Experimental setup of in-plane measurement experiments. 75

Figure 3.13. Photograph of the scan grid utilised for (a) determining the accuracy of the in-plane strain measurement technique, and (b) evaluating the strain field near a stress concentrator. The scan grid in (b) is also utilised to investigating the in-plane damage detection techniques. 75

Figure 4.1. Positioning imprecision in the 3D vibrometer due to finite accuracy of the three-dimensional alignment. 81

Figure 4.2. Displacement field uy for a plate highlighting the positioning imprecision for different nodal spatial intervals of (a) 0.22mm and (b) 1.9mm while keeping the zoom of the digital camera constant. 82

Figure 4.3. Photograph of the 3D vibrometer focused at an aluminium dogbone specimen clamped in an Instron 1342 hydraulic test machine (Cazzolato et al. 2008). 84

Figure 4.4. Displacements measured in the x- (top row), y- (middle row) and z-directions (bottom row) on the specimen using triangular (left column) and rectangular elements (right column), excited with a 5Hz frequency (Cazzolato et al. 2008). 86

Figure 4.5. Strains measured in the x- (top row), y- (middle row) and z-directions (bottom row) on the specimen using 3-node triangular linear (left column) and 4-node rectangular bi-linear elements (right column), excited with a 5Hz frequency (Cazzolato et al. 2008). 87

Figure 4.6. The coordinate definition of the spatial position (x) and particle location (X) of a body. 89

Figure 4.7. Illustration of the rectangular plate specimen and mesh used for strain measurement. 93

Figure 4.8. Experimental measured displacements ux, uy and uz (left column) measured on the undamaged plate and their coherence γx2, γy2 and γz2 (right column) to the input voltage. The plate specimen was subjected to a 1.05MPa load and the scan grid (Nx x Ny) consisted of 19x19 data points with a spatial interval (Δx and Δy) of 2mm. 94

Figure 4.9. Variation coefficient (cv,i) of the surfaces strains (εxx and εyy) versus the number of data points used ($2m+1$) in the Savitzky-Golay differentiator with a polynomial approximation of third-order ($n = 3$). The plate specimen was subjected to approximately a 1.05MPa load and the scan grid (Nx x Ny) consisted of 19x19 data points with a spatial interval (Δx and Δy) of 2mm. 97

Figure 4.10. Variation coefficient (cRMSD) of the surfaces strains (εxx and εyy) versus the order of the polynomial order (n) used in the Savitzky-Golay differentiator spanning 9x9 data points ($m = 4$). The plate specimen was subjected to approximately a 1.05MPa load and the scan grid (Nx x Ny) consisted of 19x19 data points with a spatial interval (Δx and Δy) of 2mm. 97

Figure 4.11. Estimated strain fields (εyy) of an acrylic plate subjected a 1Hz quasi-static uniaxial load of 1.05MPa. The red dashed line indicates the limit of the m data points in from the boundary where the edge artefacts occur and the blue dashed circles indicate locations of edge artefacts. A third-order ($n = 3$) Savitzky-Golay differentiating filter was used with a filter size of 9x9 data point ($m = 4$). The scan grid (Nx x Ny) consisted of 19x19 points with spatial interval (Δx and Δy) of 2mm. 99

Figure 4.12. Variation coefficient (cv,i) of the surface strain (εxx and εyy) versus the mesh spatial interval (Δx and Δy) of a scan grid (Nx x Ny) consisted of 19x19 data points. A third-order ($n = 3$). Savitzky-Golay differentiating filter was used with a filter size of 9x9 data point ($m = 4$). The plate specimens were subjected to approximately a 1.03MPa load. 100

Figure 4.13. Variation coefficient (cv,i) of the surface strain (εxx and εyy) versus the load applied to the plate specimen. A third-order ($n = 3$) Savitzky-Golay differentiating filter was used with a filter size of 9x9 data point ($m = 4$). The scan grid (Nx x Ny) consisted of 19x19 data points with a spatial interval (Δx and Δy) of 2mm. 101

Figure 4.14. Illustration of the rectangular plate specimens and meshes used for strain measurement (a) with a notch and (b) with an edge crack. 105

Figure 4.15. The finite element meshes used to simulate the strain field of (a) notched and (b) edge crack specimens. 106

Figure 4.16. Displacements ux, uy and uz and their coherence $\gamma x2$, $\gamma y2$ and $\gamma z2$ with the input voltage, for the rectangular plate with a notch. The plate specimen was subjected to approximately a 0.65 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 107

Figure 4.17. Surface strain (εxx, εyy and γxy) for the rectangular plate with a notch:- (left) FEA and (right) experimental results. A third-order (n = 3) Savitzky-Golay differentiating filter was used with a filter size of 13x13 data point (m = 6). The plate specimen was subjected to approximately a 0.65 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 108

Figure 4.18. Strains extending out from the notch. Comparison of the experimental (red line) and numerical (black dashed line) solution of εxx, (left) and εyy (right) in-line with the notch, (a, b) m = 4, (c, d) m = 5, and (e, f) m = 6. The plate specimen was subjected to approximately a 0.65 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 109

Figure 4.19. Displacements ux, uy and uz and their coherence γx2, γy2 and γz2 with the input voltage, for the rectangular plate with edge crack. The plate specimen was subjected to approximately a 0.78 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 110

Figure 4.20. Surface strain (εxx, εyy and γxy) for the rectangular plate with edge crack:- (left) FEA and (right) experimental results. The plate specimen was subjected to approximately a 0.78 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 111

Figure 4.21. Strain extending out from crack tip. Comparison of the experimental (red line) and numerical (black dashed line) solution of εxx, (left) and εyy (right) in-line with the edge crack, (a, b) m = 4, (c, d) m = 5, and (e, f) m = 6. The plate specimen was subjected to approximately a 0.78 MPa load and the scan grid (Nx x Ny) consisted of 39x39 data points with a spatial interval (Δx and Δy) of 2mm. 112

Figure 4.22. Experimental arrangement used by Ferguson and Carpentier (1989, p. 215) for the comparison between bending strains evaluated using prototype 1D laser Doppler vibrometry and strain gauges. (a) Front view and (b) side view. 118

Figure 4.23. Previous results obtained by Goetsch and Rowlands (1991, p. 431) for the comparison between radial bending strains evaluated using 1D SLDV and strain gauges for a circular aluminium plate. The plate was vibrated at 20Hz at its centre with an unspecified load. 119

Figure 4.24. (a) A plate of constant thickness, (b) a cross-section of the plate before (lower) and after (upper) deflection. 121

Figure 4.25. Experimentally measured deflection (uz) measured on an aluminium beam (blue line) and its coherence (γz2) to the input voltage (green line). The aluminium beam specimen was subjected to a maximum free-end deflection of 50μm and the scan line consisted of 499 points (N) with a spatial interval (Deltax) of 0.97mm. 124

Figure 4.26. Variation coefficient (cRMSD) of RMSD in the bending strain of the aluminium beam specimen versus (a) the number of data points used $(2\mathrm{m}+1)$ in the Savitzky-Golay differentiator $(\mathrm{n}=3)$ and (b) the polynomial order (n) used in the Savitzky-Golay differentiator $(\mathrm{m}=75)$. The aluminium beam specimen was subjected to a maximum free-end deflection of 50μm and the scan grid consisted of 499 points (N) with a spatial interval (Δx) of 0.97mm. 127

Figure 4.27. (a) Estimated bending strain $(\varepsilon\mathrm{xx},\mathrm{SG})$ on the top of an aluminium cantilever beam using a third-order polynomial $(\mathrm{n}=3)$ Savitzky-Golay differentiating filter, utilising 101 data points $(\mathrm{m}=50)$. The aluminium beam specimen was subjected to a maximum free-end deflection of 50μm and the scan grid consisted of 499 points (N) with a spatial interval (Δx) of 0.97mm. The shaded area indicates m data points in from the boundary in which edge artefacts occur. (b) Schematic illustration of the deformed cantilever. 128

Figure 4.28. Close up of Figure 4.26a to clearly show the variation coefficient (cRMSD) of the RMSD in the bending strain of the aluminium beam specimen versus the number of data points $(2\mathrm{m}+1)$ used in the Savitzky-Golay differentiator $(\mathrm{n}=3)$ graph for $2\mathrm{m}+1$ greater than 81 data points. The aluminium beam specimen was subjected to a maximum free-end deflection of 50μm and the scan grid consisted of 499 points (N) with a spatial interval (Δx) of 0.97mm. 129

Figure 4.29. Variation coefficient (cRMSD) of the RMSD in the bending strain evaluated using a Savitzky-Golay differentiator versus the spatial interval between measurement points (Δx). The Savitzky-Golay differentiator utilised a third-order polynomial approximation $(\mathrm{n}=3)$ with (a) $\mathrm{m}=20$ and (b) $2\mathrm{m}+1$ spanning a fixed distance of approximately 143mm, which equal to approximately 30% of the available nodes for each spatial interval. The aluminium beam specimen was subjected to a maximum free-end deflection of 100μm........ 132

Figure 4.30. Schematic illustration of three scan grids used to evaluate bending strain, with the relative size of the filter length used to evaluate the bending strain. (a) shows the filter length of the Savitzky-Golay filter with a fixed 41 measurement nodes ($m = 20$) for each scan grid density, whereas (b) shows the filter length spanning approximately 30% of the scan grid length ($2m + 1 = 0.3N$) for each scan grid density. 133

Figure 4.31. Variation coefficient (cRMSD) of the RMSD in the bending strain versus maximum deflection experienced by the beam where $n = 3$ and $m = 75$. The scan grid consisted of 499 points (N) with a spatial interval (Δx) of 0.97mm. 135

Figure 5.1. Measurement grids used for (a) edge crack, (b) centre crack and (c) crack with a notch. 142

Figure 5.2. Laser scanned area with indicated data grid points used for Lamb wave sensing (Staszewski, Lee, B & Traynor 2007). 144

Figure 5.3. Lamb wave propagation contour plots in the time domain for 75 kHz excitation:- (a) in-plane x-direction, (b) in-plane y-direction and (c) out-of-plane z-direction (Staszewski, Lee, B & Traynor 2007). 145

Figure 5.4. RMS amplitude contour plots for 75 kHz Lamb wave propagation (a) in-plane x-direction, (b) in-plane y-direction and (c) out-of-plane z-direction (Staszewski, Lee, B & Traynor 2007). 146

Figure 5.5. Forces acting on a small element $[dx, dy]$ of a plate in which stresses vary from point to point. 148

Figure 5.6. Illustration of strain compatibility principle. 149

Figure 5.7. Flow diagram of the *GDEP* algorithm RI. 155

Figure 5.8. Acquired residual term of the GDE (RI) for a notch with various crack lengths:- (a) 0 mm, (b) 5 mm, (c) 10 mm, (d) 15 mm, (e) 20 mm and (f) 25 mm. The dark blue and dark red areas on the graph represent the location where RI has exceeded the threshold $\Phi i = 200 m^{-3}$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the notch and crack. A 1st order polynomial ($n = 1$) 2D Savitzky-Golay smoothing filter was used with a filter size of 5x5 point ($m = 2$). 156

Figure 5.9. Flow diagram of the *displacement error* algorithm ui. 158

Figure 5.10. Acquired displacement error in the x-axis (ux) for a notch with various crack lengths:- (a) 0mm, (b) 5mm, (c) 10mm, (d) 15mm, (e) 20mm and (f) 25mm. The dark blue and dark red areas on the graph represent the location where ux has exceeded the threshold Θx = 0.7μm. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the notch and crack. A 1st order polynomial (n = 1) 2D Savitzky-Golay smoothing (rx = ry = 0) filter was used with a filter size of 3x3 point (m = 1). 160

Figure 5.11. Acquired displacement error in the y-axis (uy) for a notch with various crack lengths:- (a) 0mm, (b) 5mm, (c) 10mm, (d) 15mm, (e) 20mm and (f) 25mm. The dark blue and dark red areas on the graph represent the location where uy has exceeded the threshold Θy = 0.7μm. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the notch and crack. A 1st order polynomial (n = 1) 2D Savitzky-Golay smoothing (rx = ry = 0) filter was used with a filter size of 3x3 point (m = 1). 161

Figure 5.12. Flow diagram of the *surface strain* algorithm for (a) normal strains εii and (b) shear strain γxy. 163

Figure 5.13. Acquired mean centred surface strain in the x-axis (εxx) for a notch with various crack lengths:- (a) 0mm, (b) 5mm, (c) 10mm, (d) 15mm, (e) 20mm and (f) 25mm. The dark blue and dark red areas on the graph represent the location where εxx has exceeded the threshold Ψxx = 400μm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the notch and crack. A 2nd order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 165

Figure 5.14. Acquired mean centred surface strain in the y-axis (εyy) for a notch with various crack lengths:- (a) 0mm, (b) 5mm, (c) 10mm, (d) 15mm, (e) 20mm and (f) 25mm. The dark blue and dark red areas on the graph represent the location where εyy has exceeded the threshold Ψyy = 400μm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the notch and crack. A 2nd order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 166

Figure 5.15. Acquired mean centred surface shear strain (γxy) for a notch with various crack lengths:- (a) 0mm, (b) 5mm, (c) 10mm, (d) 15mm, (e) 20mm and (f) 25mm. The dark blue and dark red areas on the graph represent the location where γxy has exceeded the threshold Ψxy = 400μm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the notch and crack. A 2nd order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 167

Figure 6.1. Experimental setup of the cantilever beam, (a) front view of the composite beam shows the scan points (red dots) used by the 1D vibrometer and (b) side view of the composite beam showing the position of the electromagnetic shaker and 1D vibrometer. 172

Figure 6.2. Shear forces and moments acting on a small element *dxdy* of a plate............. 175

Figure 6.3. Dimensioned illustration of the delaminated cantilever beam. 181

Figure 6.4. Flow diagram of the *GDEB* algorithm Rz. 183

Figure 6.5. Acquired residual term of the GDE (Rz) for a delaminated cantilever beam for four different quantities of filter points (2m + 1) utilised within the fourth-order polynomial (n = 4) Savitzky-Golay smoothing (r = 0) filter. The rows denote the filter widths of (i) m = 25, (ii) m = 50, (iii) m = 75 and (iv) m = 100. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. The delamination is located between fourth and fifth layer and 50mm in length. The black line represents Rz, the grey line shows the beam curvature and the shaded section illustrates the location of the delaminated section................ 186

Figure 6.6. Acquired residual term of the GDE (Rz) for a delaminated cantilever beam for four different lengths of delamination between the mid-layers (h2/h = 0.5). The rows denote the lengths of the delamination and are (i) Ld = 25mm, (ii) Ld = 50mm, (iii) Ld = 75mm and (iv) Ld = 100mm. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A fourth-order polynomial (n = 4) Savitzky-Golay smoothing (r = 0) filter was used with a filter width of 151 points (m = 75). The black line represents Rz, the grey line shows the beam curvature and the shaded section illustrates the location of the delaminated section... 188

Figure 6.7. Acquired residual term of the GDE (Rz) for various laminate cantilever beams with a 50mm delamination at different depths. The rows denote the depths of the delamination and are (i) $h2/h = 0.125$, (ii) $h2/h = 0.25$, (iii) $h2/h = 0.375$, (iv) $h2/h = 0.5$, (v) $h2/h = 0.625$, (vi) $h2/h = 0.75$ and (vii) $h2/h = 0.875$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A fourth-order polynomial ($n = 4$) Savitzky-Golay smoothing ($r = 0$) filter was used with a filter width of 151 points ($m = 75$). The shaded section illustrates the location of the delaminated section. 191

Figure 6.8. Flow diagram of the *deflection error* algorithm. 195

Figure 6.9. Acquired error in deflection (uz) for a delaminated cantilever beam for four different quantities of filter points ($2m + 1$) utilised within the third-order polynomial ($n = 3$) Savitzky-Golay smoothing ($r = 0$) filter. The rows denote the filter widths of (i) $m = 25$, (ii) $m = 50$, (iii) $m = 75$ and (iv) $m = 100$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. The delamination is located between fourth and fifth layer and 50mm in length. The shaded section illustrates the location of the delaminated section. 198

Figure 6.10. Acquired error in deflection (uz) for a delaminated cantilever beam for four different lengths of delamination between the mid-layers ($h2/h = 0.5$). The rows denote the lengths of the delamination and are (i) $Ld = 25mm$, (ii) $Ld = 50mm$, (iii) $Ld = 75mm$ and (iv) $Ld = 100mm$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A third-order polynomial ($n = 3$) Savitzky-Golay smoothing ($r = 0$) filter was used with a filter width of 151 points ($m = 75$). The shaded section illustrates the location of the delaminated section. 201

Figure 6.11. Acquired error in deflection (uz) for various laminate cantilever beams with a 50mm delamination at different depths. The rows denote the depths of the delamination and are (i) $h2/h = 0.125$, (ii) $h2/h = 0.25$, (iii) $h2/h = 0.375$, (iv) $h2/h = 0.5$, (v) $h2/h = 0.625$, (vi) $h2/h = 0.75$ and (vii) $h2/h = 0.875$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A third-order polynomial ($n = 3$) Savitzky-Golay smoothing ($r = 0$) filter was used with a filter width of 151 points ($m = 75$). The shaded section illustrates the location of the delaminated section. 204

Figure 6.12. Flow diagram of the *curvature error* algorithm κ. 207

Figure 6.13. Acquired error in curvature (κ) for a delaminated cantilever beam for four different quantities of filter points $(2m + 1)$ utilised within the third-order polynomial $(n = 3)$ Savitzky-Golay smoothing $(r = 0)$ filter. The rows denote the filter widths of (i) $m = 25$, (ii) $m = 50$, (iii) $m = 75$ and (iv) $m = 100$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. The delamination is located between fourth and fifth layer and 50mm in length. The shaded section illustrates the location of the delaminated section............................210

Figure 6.14. Acquired error in deflection (κ) for a delaminated cantilever beam for four different lengths of delamination between the mid-layers $(h2/h = 0.5)$. The rows denote the lengths of the delamination and are (i) $Ld = 25$mm, (ii) $Ld = 50$mm, (iii) $Ld = 75$mm and (iv) $Ld = 100$mm. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A third-order polynomial $(n = 3)$ Savitzky-Golay smoothing $(r = 0)$ filter was used with a filter width of 151 points $(m = 75)$. The shaded section illustrates the location of the delaminated section................212

Figure 6.15. Acquired error in curvature (κ) for various laminate cantilever beams with a 50mm delamination at different depths. The rows denote the depths of the delamination and are (i) $h2/h = 0.125$, (ii) $h2/h = 0.25$, (iii) $h2/h = 0.375$, (iv) $h2/h = 0.5$, (v) $h2/h = 0.625$, (vi) $h2/h = 0.75$ and (vii) $h2/h = 0.875$. The columns show the (a) experimental results, (b) simulated results with equivalent noise and (c) simulated results in the absence of noise. A third-order polynomial $(n = 3)$ Savitzky-Golay smoothing $(r = 0)$ filter was used with a filter width of 151 points $(m = 75)$. The shaded section illustrates the location of the delaminated section. 215

Figure C.1. Delaminated beam model. 251

Figure D.2. Acquired residual term of GDE (RI) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where RI has exceeded the threshold $\Phi I = 200m^{-3}$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A third-order polynomial $(n = 3)$ 2D Savitzky-Golay differentiating filter was used with a filter size of 5x5 point $(m = 2)$..262

Figure D.3. Acquired residual term of GDE (RI) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where RI has exceeded the threshold $\Phi I = 200m^{-3}$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A third-order polynomial ($n = 3$) 2D Savitzky-Golay differentiating filter was used with a filter size of 5x5 point ($m = 2$).. 263

Figure D.4. Acquired displacement error in the x-direction (ux) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where ux has exceeded the threshold $\Theta x = 0.7\mu m$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A first-order polynomial ($n = 1$) 2D Savitzky-Golay smoothing ($rx = ry = 0$) filter was used with a filter size of 3x3 point ($m = 1$).. 266

Figure D.5. Acquired displacement error in the y-direction (uy) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where uy has exceeded the threshold $\Theta y = 0.7\mu m$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A first-order polynomial ($n = 1$) 2D Savitzky-Golay smoothing ($rx = ry = 0$) filter was used with a filter size of 3x3 point ($m = 1$).. 267

Figure D.6. Acquired displacement error in the x-direction (ux) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where ux has exceeded the threshold $\Theta x = 0.7\mu m$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A first-order polynomial ($n = 1$) 2D Savitzky-Golay smoothing ($rx = ry = 0$) filter was used with a filter size of 3x3 point ($m = 1$).. 268

Figure D.7. Acquired displacement error in the y-direction (uy) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where uy has exceeded the threshold $\Theta y = 0.7\mu m$. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A first-order polynomial ($n = 1$) 2D Savitzky-Golay smoothing ($rx = ry = 0$) filter was used with a filter size of 3x3 point ($m = 1$).. 269

Figure D.8. Acquired normalised surface strain in the x-direction (εxx) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where εxx has exceeded the threshold Ψxx = 400mm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 272

Figure D.9. Acquired normalised surface strain in the y-direction (εyy) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where εyy has exceeded the threshold Ψyy = 400mm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 273

Figure D.10. Acquired normalised surface shear (γxy) for an edge crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where γxy has exceeded the threshold Ψxy = 400mm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1)..274

Figure D.11. Acquired normalised surface strain in the x-direction (εxx) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where εxx has exceeded the threshold Ψxx = 400mm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 275

Figure D.12. Acquired normalised surface strain in the y-direction (εyy) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where εyy has exceeded the threshold Ψyy = 400mm/m. At x = 0 and x = 100 represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial (n = 2) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point (m = 1). 276

Figure D.13. Acquired normalised surface shear strains (γxy) for a centre crack with various crack lengths:- (a) 5mm, (b) 10mm, (c) 15mm, (d) 20mm and (e) 25mm. The dark blue and dark red areas on the graph represent the location where γxy has exceeded the threshold $\Psi xy = 400$mm/m. At $x = 0$ and $x = 100$ represents the edges of the plate and the pink line defines the location of the crack. A second-order polynomial ($n = 2$) 2D Savitzky-Golay differentiating filter was used with a filter size of 3x3 point ($m = 1$). 277

List of Tables

Table 3.1. Specimen specification, parameters related to Figure 3.8. 68

Table 3.2. Specimen specifications, parameters related to Figure 3.11.............................. 72

Table 4.1 Comparison of Normalised strains per unit (kN) load and estimated Poisson's Ratio of the test specimen measured using an extensometer and 3D SLDV system (Cazzolato et al. 2008).. 85

Table 4.2. Variation coefficient of the root squared mean deviation ($cRMSD, i$) between the FE and measured x- and y-strain leading up to the tip of the notch specimen ($y = 0$) that are presented in Figure 4.18. The RMSD values are provided in brackets for each case. 115

Table 4.3. Variation coefficient of the root squared mean deviation ($cRMSD, i$) between the FE and measured x- and y-strain leading up to the tip of the edge crack specimen ($y = 0$) that are presented in Figure 4.21. The RMSD values are provided in brackets for each case. 115

Table 4.4. Illustration of the number of data points ($2m + 1$) required within the Savitzky-Golay differentiating filter to encompass a nominal fixed distance of approximately 143mm for each spatial interval (Δx) investigated. 132

Table 4.5. Relationship between the variation coefficient of RMSD in the bending strain (cRMSD) in Figure 4.29a to the ratio of the number of measurement points used ($2m + 1 = 41$) within the Savitzky-Golay filter and total number of measurement points (N). 133

Table 4.6. Comparison of common strain measurement techniques. 138

Nomenclature

a	–	crack length
c_{RMSD}	–	variation coefficient of measured strain
E	–	modulus of elasticity
f	–	beat frequency measured at the photo detector of the laser beam
f_D	–	Doppler shift in laser beam
f_B	–	offset frequency of the reference laser beam
F	–	force applied to end of the cantilever beam
F_x, F_y	–	body force acting on a plate
G	–	shear modulus
h	–	plate or beam thickness
h_2	–	thickness of delaminated section on the front side of beam
h_3	–	thickness of delaminated section on the back side of beam
$h_{t,i}^{n,r}$	–	convolution weights of the Savitzky-Golay differentiation filter
I	–	intensity of the coinciding reference and object beams at the photo detector of the laser vibrometer
$I_{\max}$	–	maximum possible intensity at the photo detector of the laser vibrometer
L	–	length of beam
L_1	–	distance from clamped end of beam to the start of the delaminated zone
L_2	–	distance from clamped end of beam to the end of the delaminated zone
L_d	–	length of the delaminated section of a composite beam
ΔL	–	difference in path length between the reference and object beams
$2m + 1$	–	number of measurement points utilised within a numerical differentiation technique
M_x, M_y, M_{xy}	–	bending moments applied to a small plate element
M_0	–	moment applied to end of the cantilever beam
n	–	polynomial order utilised within a numerical differentiation technique
N	–	total number of measurement points
p	–	uniformly distributed load per unit area applied to a plate

$P_n^{m,r}$	–	Gram polynomial
Q_x, Q_y	–	shear forces applied to a small plate element
r	–	order of differential performed within a numerical differentiation technique
R_I	–	residual term of the governing differential equation for in-plane displacement
R_O	–	residual term of the governing differential equation for out-of-plane displacement
R^2	–	coefficient of determination of a least-squares-fit
$RMSD$	–	root mean standard deviation
SD	–	standard deviation
t	–	time
u_x, u_y	–	displacement field in the x- and y-axes, respectively (in-plane direction)
u_z	–	displacement field in the z-direction (out-of-plane direction)
$\hat{u}_i$	–	displacement error in the i-axes
$u_{z,SG}$	–	fitted displacement in the i-axes
$u_{i,m}$	–	measured displacement in the i-axes
u_E	–	Eulerian displacement
u_L	–	Lagrangian displacement
v	–	object velocity in the direction of the laser beam
$\Delta x, \Delta y$	–	spatial interval between measurement points
$x(X, t)$	–	spatial position of a particle X at the time t
$X(x, t)$	–	the particle located at a spatial position x at time t
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	–	engineering shear strains in x-y, x-z and y-z planes respectively
$\hat{\gamma}_{ij}$	–	mean centre of the surface shear strain
$\gamma_{ij,SG}$	–	estimate of the in-plane surface shear strain using a Savitzky-Golay differentiating filter
$\bar{\gamma}_{ij}$	–	undamaged estimate of the in-plane shear surface strains using a least-squares-fit
$\gamma_x^2, \gamma_y^2, \gamma_z^2$	–	coherence of measured displacement in x-, y- or z-axes to the input vibration voltage
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$	–	normal strains in the x-, y- and z-axes, respectively
$\hat{\varepsilon}_{ij}$	–	mean centre of the surface in-plane strain

$\varepsilon_{ij,SG}$	–	estimate of the in-plane surface strain using a Savitzky-Golay differentiating filter
$\bar{\varepsilon}_{ij}$	–	undamaged estimate of the in-plane surface strains using a least-squares-fit
Θ_i	–	damage detection threshold of the *displacement error* algorithm
$\hat{\kappa}$	–	beam curvature error between the estimated and expected beam curvature
κ_{SG}	–	estimate of beam curvature using a Savitzky-Golay differentiating filter
κ_{LS}	–	undamaged estimate of the in-plane surface strains using a least-squares-fit
λ	–	wavelength of the laser beam
ν	–	Poisson's ratio for an isotropic material
ν_x, ν_y	–	Poisson's ratio for an anisotropic material in the x- and y-axes, respectively
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	–	normal stresses in the x-, y- and z-axes, respectively
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	–	engineering shear stresses in x-y, x-z and y-z planes respectively
ϕ	–	phase difference between the reference and object beams
Φ_I	–	damage detection threshold of the governing differential equation algorithm
Ψ_{ij}	–	damage detection threshold of the surface strain error algorithm