Very Steep Solitary Waves in Two-Dimensional Free Surface Flow

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Abstract

Very steep solitary waves on the surface of a flow in a channel have been widely studied, in part due to their connection with the onset of wave breaking. An important extension to the study of solitary waves in a channel is the effect of localised forcing on their evolution. Forced very steep waves are the focus of this study, as they have received only limited attention in the past, and are of importance in the latter stages of wave breaking. Two types of forcing are considered, a localised pressure disturbance applied to the free surface, or a localised change to the otherwise flat topography of the channel, such as a bump or a trench. Gravity is the only body force considered, as surface tension is neglected. Boundary-integral methods are used to determine solutions to the free surface, whose evolution is described by a fully nonlinear potential flow model.

It is shown that for both types of forcing, like for unforced waves, the waves approach a Stokes limiting configuration as the wave-height is increased, and the solutions also exhibit non-uniqueness with respect to quantities such as the wave energy. The stability of the forced solutions is investigated here using a weakly nonlinear theory, valid in the limit when the wave-height and the steepness are small. The time-evolution of a perturbed steady solution is computed, and linearised stability analysis is performed numerically. It is shown that the unstable solutions may emit a solitary wave ahead of the forcing, and attain the form of the stable solution near the forcing location.

Signed Statement

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