Deformation retractions from spaces of continuous maps onto spaces of holomorphic maps

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Abstract

A fundamental property of an Oka manifold Y is that every continuous map from a Stein manifold X to Y can be deformed to a holomorphic map. In a recent paper, Lárusson [19] considers the natural question of whether it is possible to simultaneously deform all continuous maps f from X to Y to holomorphic maps, in a way that depends continuously on f and does not change f if f is holomorphic to begin with. In other words, is $\mathscr{O}(X, Y)$ a deformation retract of $\mathscr{C}(X, Y)$? Lárusson provided a partial answer to this question. In this thesis we further develop the work of Lárusson on the topological relationship between spaces of continuous maps and spaces of holomorphic maps from Stein manifolds to Oka manifolds, mainly in the context of domains in $\mathbb{C}$. The main tools we use come from complex analysis, Oka theory, algebraic topology and the theory of absolute neighbourhood retracts. One of our main results provides a large supply of infinitely connected domains X in $\mathbb{C}$ such that $\mathscr{O}(X, \mathbb{C}^*)$ is a deformation retract of $\mathscr{C}(X, \mathbb{C}^*)$.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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